term - duration of investment/loan

interest - money earned on investment/ paid on a loan

principal - original amount invested/loaned

future value (amount) - future worth of investment

maturity - contracted end date

1.2 & 1.3(i) – COMPOUND INTEREST: FUTURE VALUE

Goal: Compare simple interest with compound interest.

Example 1: Developing a compound interest formula

Both Eugene and Francine received a $1000 prize in a math contest.

- Eugene bought a $1000 simple interest GIC (guaranteed investment certificate) with his prize money. It has a 5-year term and earns 3.6% paid annually.
- Francine bought a $1000 compound interest GIC (guaranteed investment certificate) with her prize money. It has a 5-year term and earns 3.6% paid annually.

compound interest: interest is earned on both the principal and the accumulated interest.

Compare the future values of Eugene's and Francine's investments at maturity.

Simple Interest:

amount (future value)

$A = P(1+rt)$

principal, term, rate (decimal)

$A = 1000(1+0.036(5))$

$= \$1180$

Compound Interest:

Year	Value
1	$A = 1000(1+0.036(1)) = 1000(1.036) = \$1036$
2	$A = 1036(1+0.036(1)) = 1036(1.036) = \$1073.30$
3	$A = 1073.30(1.036) = \$1111.93$
4	$A = 1111.93(1.036) = \$1151.96$
5	$A = 1151.96(1.036) = \$1193.44$

$1193.44 - 1180 = \$13.44$ more

$A = 1000(1.036)^5$

The formula for compound interest:

$A = P(1+i)^n$

A - amount

P - principal

i - interest rate per pay period

n - number of pay periods

Calculate the future values of Eugene's and Francine's investment if the term is 10 years instead.

Eugene

$A = 1000(1+0.036(10))$

$= \$1360$

Francine

$A = 1000(1+0.036)^{10}$

$= \$1424.29$

$1424.29 - 1360 = \$64.29$

Example 2:Determining the future value of an investment with semi-annual compounding

Gerald invested his inheritance of $20 000 in an account that earns 9.2% compounded semi-annually. The interest rate is fixed for 10 years. He plans to use the money for a sport car in 5 to 10 years.

Compare the future value of his investment after 5 years and 10 years. What if his investment were to earn simple interest instead?

$A = P(1+i)^n$

5 yrs. $A = 20000\left(1+\frac{0.092}{2}\right)^{5\times2}$
$= \$31\,357.89$

10 yrs. $A = 20000(1+0.046)^{20}$ ← 10×2, $0.092/2$
$= \$49165.87$

Example 3: Comparing interest on investments with different compounding periods

Hanna wants to invest $3000 so that she can renovate her living room in about 3 years; she has the following investment options (annual/semi-annual/monthly/weekly/daily) at 4.8%:

			↑ interest rate / year		
Principal ($)	3000	3000	3000	3000	3000
Interest Rate	0.048 0.048	0.048 0.024	0.048 0.004	0.048	0.048
Periods / Year	1 3	2 3×2=6	12 3×12=36	52 3×52=156	365 3×365=1095
Value at End of Year	$3000(1+0.048)^3$	$3000(1+0.024)^6$	$3000(1+0.004)^{36}$		
0	3000	3000	3000	3000	3000
1					
2					
3	3453.07	3458.76	3463.66	3464.42	3464.62

Example 4: Estimating doubling times for investments

Ivan and Jenny invested $4000 by purchasing CSBs. Ivan's earns 8% compounded annually, while Jenny's earn 9% compounded annually.

↑ Canada Savings Bond

a. Estimate the doubling time for each CSB.
b. Verify the estimate by determining the doubling time for each CSB.

Ivan $72/8 = 9$ years

Jenny $72/9 = 8$ years

Assignment: read p. 29; p. 19 #1 – 3; p. 30 #3 – 11 (odds)