1.4 – COMPOUND INTEREST: PRESENT VALUE

Goal: Determine the principal or present value of an investment, given its future value and compound interest rate.

Example 1: Determining the present value of investments earning compound interest

Kenneth is 22 years old. He has inherited some money from a relative. Kenneth wants to invest some of the money to buy a home in ~~Maple Ridge~~ Burnaby when he turns 35. He estimates that he will need about ~~$225 000~~ 2 225 000 to buy a home.

a. How much does he have to invest now, at 5.5% compounded annually?

$A = P(1+i)^n$

$2\,225\,000 = P(1+0.055)^{13}$ (35 − 22)

$2\,225\,000 = P(2.00577\ldots)$

$P = 2\,225\,000 / 2.00577\ldots$

$P = \$1\,109\,297.52$

b. What is the ratio of future value to present value for Kenneth's investment?

$\frac{A}{P} = \frac{2\,225\,000}{1\,109\,297.52} = 2.00577\ldots$

Rule of 72: $72 / 5.5 = 13.0\overline{9}$

c. How would the ratio change if the interest rate increased to 6% but was compounded semi-annually?

$A = P(1+i)^n$

$A/P = (1+i)^n$

$= \left(1+\frac{0.06}{2}\right)^{13\times 2} = 2.156\ldots$

Example 2: Determining the present value of an investment that is compounded quarterly

Lana and Matt are computer scientists. They researched the costs to set up a software company. They estimate that $40 000 will be enough.

They plan to set up the company in 3 years and have invested money at 9.6%, compounded quarterly, to save for it.

a. How much money should they have invested?

$A = P(1+i)^n$

$40000 = P\left(1+\frac{0.096}{4}\right)^{3\times 4}$

$40000 = P(1.024)^{12}$

$P = 40000 / (1.024)^{12}$

$= \$30092.66$

b. How much interest will they earn over the term of their investment?

$I = A - P = 40000 - 30092.66$

$= \$9907.34$

Example 3: Determining an unknown interest rate and unknown term

Niko has invested $14 400 in a Registered Education Savings Plan (RESP). He wants his investment to grow to at least $50 000 by the time his newborn enters university, in 18 years.

a. What interest rate, compounded annually, will result in a future value of $50 000? Round your answer to two decimal places.

$A = P(1+i)^n$

$50000 = 14400\,(1+i)^{18}$

$\frac{50000}{14400} = (1+i)^{18}$

$3.47\overline{2} = (1+i)^{18}$

$\sqrt{3.47\overline{2}} = \sqrt{(1+i)^{18}}$

$1.0716 \doteq 1+i$

$0.0716 = i$

$i = 7.16\%$

b. Suppose that Niko wants his $14 400 to grow to at least $60 000 at the interest rate from part a. How long will this take?

$60000 = 14400\,(1+0.0716)^n$

$\frac{60000}{14400} = (1.0716)^n$

$4.1\overline{6} = (1.0716)^n$

$n = \frac{\log(4.1\overline{6})}{\log(1.0716)} \doteq 20.637$ years

0.637×12 months/yr

$20.637 - 18 = 2.637$ years

$= 2$ years 8 months

(actually it's just 3 years)

$n = \frac{\log\left(\frac{A}{P}\right)}{\log(1+i)}$

Assignment: p. 40 #1, 3, 5 – 7, 10, 11, 13