F Math 12

3.1 Types of Sets and Set Notation p. 146

Name _____

Date _____

Goal: Understand sets and set notation.

1. **set**: A collection of distinguishable objects; for example, the set of whole numbers is $W = \{0, 1, 2, 3, ...\}$.

2. **element**: An object in a set; for example, 3 is an element of D, the set of digits.

3. **universal set**: A set of all the elements under consideration for a particular context (also called the sample space); for example, the universal set of digits is $D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

4. **subset**: A set whose elements all belong to another set; for example, the set of odd digits, $O = \{1, 3, 5, 7, 9\}$, is a subset of D, the set of digits. In set notation, this relationship is written as: $O \subset D$.

5. **complement**: All the elements of a universal set that do not belong to a subset of it; for example, $O' = \{0, 2, 4, 6, 8\}$ is the complement of $O = \{1, 3, 5, 7, 9\}$, a subset of the universal set of digits, D. The complement is denoted with a prime sign, O'.

6. **empty set**: A set with no elements; for example, the set of odd numbers divisible by 2 is the empty set. The empty set is denoted by { } or ∅.

7. **disjoint**: Two or more sets having no elements in common; for example, the set of even numbers and the set of odd numbers are disjoint.

8. **finite set**: A set with a countable number of elements; for example, the set of even numbers less than 10, $E = \{2, 4, 6, 8\}$, is finite.

9. **infinite set**: A set with an infinite number of elements; for example, the set of natural numbers, $N = \{1, 2, 3,...\}$, is infinite.

10. **mutually exclusive**: Two or more events that cannot occur at the same time; for example, the Sun rising and the Sun setting are mutually exclusive events.

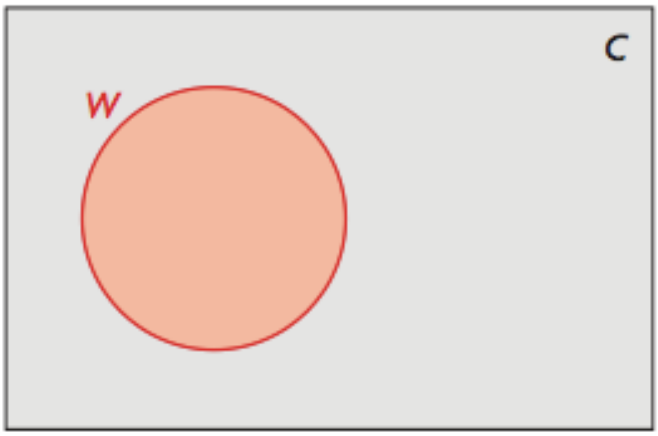
INVESTIGATE the Math

Jasmine is studying the provinces and territories of Canada. She has decided to categorize the provinces and territories using **sets** .

How can Jasmine use sets to categorize Canada's regions?

A. List the elements of the universal set of Canadian provinces and territories, *C*.

B. One subset of *C* is the set of Western provinces and territories, *W*. Write *W* in set notation.



C. The Venn diagram above represents the universal set, *C*. The circle in the Venn diagram represents the subset *W*. The **complement** of *W* is the set *W′*.

i. Describe what *W′* contains.

ii. Write *W′* in set notation.

iii. Explain what *W′* represents in the Venn diagram.

D. Jasmine wrote the set of Eastern provinces as follows: *E* = {NL, PE, NS, NB, QC, ON} Is *E* equal to *W′*? Explain.

E. List *T*, the set of territories in Canada. Is *T* a subset of *C*? Is it a subset of *W*, or a subset of *W′*? Explain using your Venn diagram.

F. Explain why you can represent the set of Canadian provinces south of Mexico by the **empty set** .

G. Consider sets C, W, W', and T. List a pair of disjoint sets. Is there more than one pair of **disjoint** sets?

H. Complete your Venn diagram by listing the elements of each subset in the appropriate circle.

Communication | *Notation*

The following is a summary of notation introduced so far.

Sets are defined using brackets. For example, to define the universal set of the numbers 1, 2, and 3, list its elements:

$U = \{1, 2, 3\}$

To define the set A that has the numbers 1 and 2 as elements:

$A = \{1, 2\}$

All elements of A are also elements of U, so A is a subset of U:

$A \subset U$

The set A', the complement of A, can be defined as:

$A' = \{3\}$

To define the set B, a subset of U that contains the number 4:

$B = \{\}$ or $B = \varnothing$

$B \subset U$

Communication | *Notation*

The phrase "from 1 to 5" means "from 1 to 5 inclusive."

In set notation, the number of elements of the set X is written as $n(X)$.

For example, if the set X is defined as the set of numbers from 1 to 5:

$X = \{1, 2, 3, 4, 5\}$

$n(X) = 5$

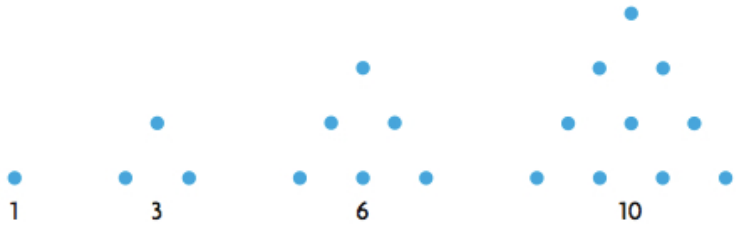
Example 1: Sorting numbers using set notation and a Venn diagram (p.148)

a) Indicate the multiples of 5 and 10, from 1 to 500, using set notation. List any subsets.

b) Represent the sets and subsets in a Venn diagram.

Example 2: Determining the number of elements in sets (p. 149)

A triangular number, such as 1, 3, 6, or 10, can be represented as a triangular array.



a) Determine a pattern you can use to determine any triangular number.

b) Determine how many natural numbers from 1 to 100 are

   i) even and triangular,
   ii) odd and triangular, and
   iii) not triangular.

c) How many numbers are triangular?

Example 3: Describing the relationships between sets (p. 151)

Alden and Connie rescue homeless animals and advertise in the local newspaper to find homes for the animals. They are setting up a web page to help them advertise the animals that are available. They currently have dogs, cats, rabbits, ferrets, parrots, lovebirds, macaws, iguanas, and snakes.

a) Design a way to organize the animals on the web page. Represent your organization using a Venn diagram.

b) Name any disjoint sets.

c) Show which sets are subsets of one another using set notation.

d) Alden said that the set of fur-bearing animals could form one subset. Name another set of animals that is equal to this subset.

Example 4: Solving a problem using a Venn diagram (p.152)

Bilyana recorded the possible sums that can occur when you roll two four-sided dice in an outcome table:

	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

- Display the following sets in one Venn diagram:
  - rolls that produce a sum less than 5
  - rolls that produce a sum greater than 5
- Record the number of elements in each set.
- Determine a formula for the number of ways that a sum less than or greater than 5 can occur. Verify your formula.

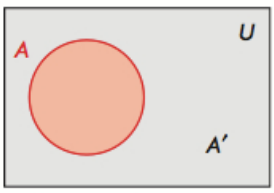
In Summary

Key Ideas

- You can represent a set of elements by:
  - listing the elements; for example, $A = \{1, 2, 3, 4, 5\}$
  - using words or a sentence; for example, $A = \{\text{all integers greater than 0 and less than 6}\}$
  - using set notation; for example, $A = \{x \mid 0 < x < 6, x \in \mathbb{I}\}$
- You can show how sets and their subsets are related using Venn diagrams. Venn diagrams do not usually show the relative sizes of the sets.
- You can often separate a universal set into subsets, in more than one correct way.

Need to Know

- Sets are equal if they contain exactly the same elements, even if the elements are listed in different orders.
- You may not be able to count all the elements in a very large or infinite set, such as the set of real numbers.



- The sum of the number of elements in a set and its complement is equal to the number of elements in the universal set:

$$n(A) + n(A') = n(U)$$

- When two sets A and B are disjoint,

$$n(A \text{ or } B) = n(A) + n(B)$$

HW: 3.1 p. 154-158 #4, 6, 8, 9, 11, 12, 14, 15, 16 & 19