F Math 12

3.3 Intersection and Union of Two Sets p. 162

Name ____________________

Date ____________________

Goal: Understand and represent the intersection and union of two sets.

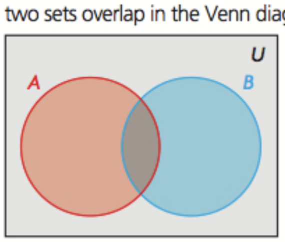
1. **intersection**: The set of elements that are common to two or more sets. In set notation, $A \cap B$ denotes the intersection of sets A and B; for example, if $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$, then $A \cap B = \{3\}$.

2. **union**: The set of all the elements in two or more sets; in set notation, $A \cup B$ denotes the union of sets A and B; for example, if $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$, then $A \cup B = \{1, 2, 3, 4, 5\}$.

3. **Principle of Inclusion and Exclusion**: The number of elements in the union of two sets is equal to the sum of the number of elements in each set, less the number of elements in both sets; using set notation, this is written as $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

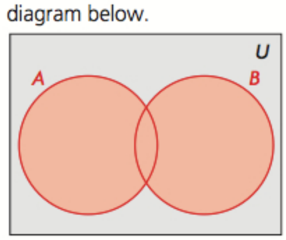
Communication | *Notation*

In set notation, $A \cap B$ is read as "intersection of A and B." It denotes the elements that are common to A and B. The intersection is the region where the two sets overlap in the Venn diagram below.



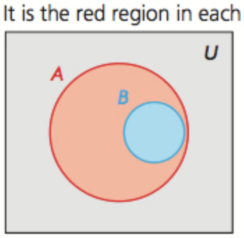
$A \cap B$

$A \cup B$ is read as "union of A and B." It denotes all elements that belong to at least one of A or B. The union is the red region in the Venn diagram below.

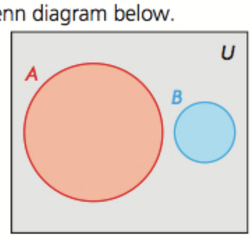


$A \cup B$

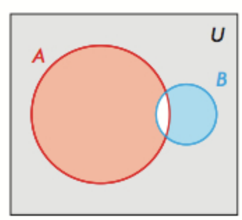
$A \setminus B$ is read as "A minus B." It denotes the set of elements that are in set A but not in set B. It is the red region in each Venn diagram below.



$A \setminus B$ **when** $B \subset A$



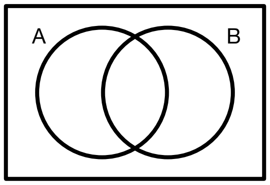
$A \setminus B$ **when they are disjoint**



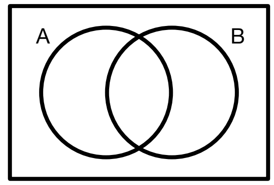
$A \setminus B$ **when they intersect**

Venn Diagrams & Notation

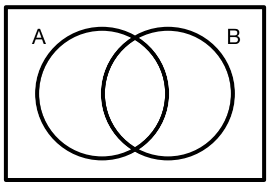
Shade the region that contains the elements that belong.



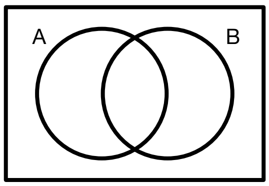
A



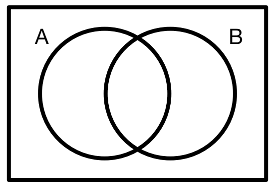
B



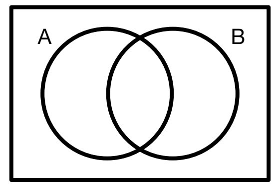
A'



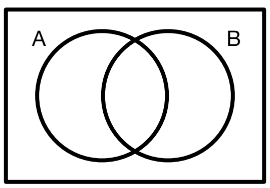
B'



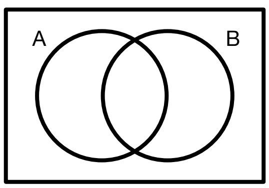
$A \cup B$



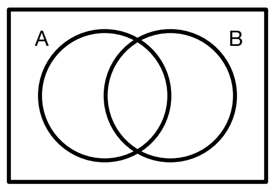
$(A \cup B)'$



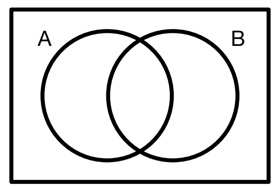
$A \cap B$



$(A \cap B)'$



$A \backslash B$



$B \backslash A$

Example 1: Determining the union and intersection of disjoint sets (p.164)

If you draw a card at random from a standard deck of cards, you will draw a card from one of four suits: clubs (*C*), spades (*S*), hearts (*H*), ordiamonds (*D*).

a) **Describe** sets *C*, *S*, *H*, and *D*, and the universal set *U* for this situation.

b) Determine *n*(*C*), *n*(*S*), *n*(*H*), *n*(*D*), and *n*(*U*).

$U =$ ____________________ $n(U) =$ ______

$S =$ ____________________ $n(S) =$ ______

$H =$ ____________________ $n(H) =$ ______

$C =$ ____________________ $n(C) =$ ______

$D =$ ____________________ $n(D) =$ ______

c) **Describe** the union of *S* and *H*. Determine $n(S \cup H)$.

d) **Describe** the intersection of *S* and *H*. Determine $n(S \cap H)$.

e) Determine whether the events that are described by sets *S* and *H* are mutually exclusive, and whether sets *S* and *H* are disjoint.

f) **Describe** the complement of $S \cup H$.

Example 2: Determining the number of elements in a set using a formula (p. 166)

The athletics department at a large high school offers 16 different sports:

badminton	hockey	tennis
basketball	lacrosse	ultimate
cross-country running	rugby	volleyball
curling	cross-country skiing	wrestling
football	soccer	
golf	softball	

Determine the number of sports that require the following types of equipment:

a) a ball and an implement, such as a stick, a club, or a racquet

b) only a ball

d) either a ball or an implement

c) an implement but not a ball

e) neither a ball nor an implement

Principle of Inclusion and Exclusion

The number of elements in the union of two sets is equal to the sum of the number of elements in each set, less the number of element in ___________________________

Subtract the elements in the intersection, so they are not counted twice, once in $n(A)$ and once in $n(B)$

If two sets, A and B are disjoint, they contain no common elements.

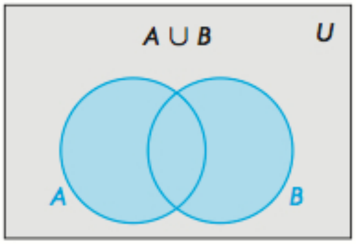
Example 3: Determining the number of elements in a set by reasoning (p. 168)

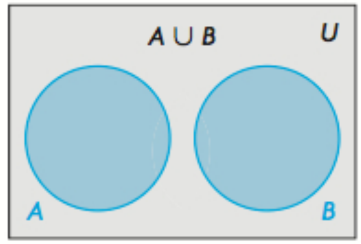
Jamaal surveyed 34 people at his gym. He learned that 16 people do weight training three times a week, 21 people do cardio training three times a week, and 6 people train fewer than three times a week. How can Jamaal interpret his results?

In Summary

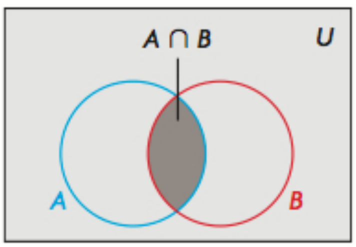
Key Ideas

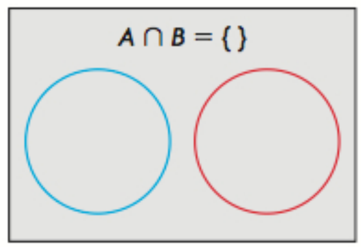
- The **union** of two or more sets, for example, $\boldsymbol{A \cup B}$, consists of all the elements that are in at least one of the sets. It is represented by the entire region of these sets on a Venn diagram. It is indicated by the word "**or**."





- The **intersection** of two or more sets, for example, $\boldsymbol{A \cap B}$, consists of all the elements that are common to these sets. It is represented by the region of overlap on a Venn diagram. It is indicated by the word "**and**."



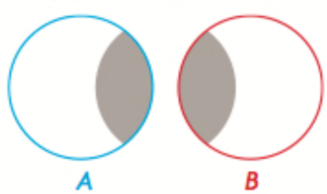


Need to Know

- If two sets, A and B, contain common elements, the number of elements in A or B, $n(A \cup B)$, is:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

This is called the Principle of Inclusion and Exclusion. To calculate $n(A \cup B)$, subtract the elements in the intersection so they are not counted twice, once in $n(A)$ and once in $n(B)$.



- If two sets, A and B, are disjoint, they contain no common elements:

$$n(A \cap B) = 0 \text{ and}$$
$$n(A \cup B) = n(A) + n(B)$$

- Elements that are in set A but not in set B are expressed as $A \setminus B$. The number of elements in A or B, $n(A \cup B)$, can also be determined as follows:

$$n(A \cup B) = n(A \setminus B) + n(B \setminus A) + n(A \cap B)$$

HW: 3.3 p. 171-175 #5, 6, 7, 9, 10, 11, 12, 16, 17 & 18