F Math 12

3.5 Conditional Statements and Their Converse p. 195

Name ____________________

Date ____________________

Goal: Understand and interpret conditional statements.

1. **conditional statement**: An “if–then” statement; for example, “If it is Monday, then it is a school day.”

2. **hypothesis**: An assumption; for example, in the statement “If it is Monday, then it is a school day,” the hypothesis is “It is Monday.”

3. **conclusion**: The result of a hypothesis; for example, in the statement “If it is Monday, then it is a school day,” the conclusion is “it is a school day.”

4. **counterexample**: An example that disproves a statement; for example, “If it is Monday, then it is a school day” is disproved by the counterexample that there is no school on Thanksgiving Monday. Only one counterexample is needed to disprove a statement.

5. **converse**: A conditional statement in which the hypothesis and the conclusion are switched; for example, the converse of “If it is Monday, then it is a school day” is “If it is a school day, then it is Monday.”

6. **biconditional**: A conditional statement whose converse is also true; in logic notation, a biconditional statement is written as “*p* if and only if *q*.” For example, the statement “If a number is even, then it is divisible by 2” is true. The converse, “If a number is divisible by 2, then it is even,” is also true. The biconditional statement is “A number is even if and only if it is divisible by 2.”

Communication | *Notation*

$p \Rightarrow q$ is notation for “If *p*, then *q*.”
$p \Rightarrow q$ is read as “*p* implies *q*.”

Communication | *Notation*

$p \Leftrightarrow q$ is notation for “*p* if and only if *q*.”
This means that both the conditional statement and its converse are true statements.

LEARN ABOUT the Math

James and Gregory like to play soccer, regardless of the weather. Their coach made this **conditional statement** about today's practice: "If it is raining outside, then we practise indoors."

When will the coach's conditional statement be true, and when will it be false?

Example 1: Verifying a conditional statement (p.195)

Verify when the coach's conditional statement is true or false.

Hypothesis: ___________________________________

Conclusion: ___________________________________

Each of these statements is either true or false, so to verify this conditional statement, consider four cases.

Case 1: The hypothesis is true and the conclusion is true.

When the hypothesis and conclusion are both true, a conditional statement is _________

Case 2: The hypothesis is false, and the conclusion is false.

When the hypothesis and conclusion are both false, a conditional statement is _________

Case 3: The hypothesis is false, and the conclusion is true.

When the hypothesis is false and conclusion is true, a conditional statement is _________

Case 4: The hypothesis is true, and the conclusion is false.

When the hypothesis is true and conclusion is false, a conditional statement is _________

This ___________________ shows that the conditional statement is __________

Use a Truth Table to Summarize the Observations

Let p represent the hypothesis: *It is raining outside*.
Let q represent the conclusion: *We practise indoors.*

p	q	$p \Rightarrow q$

When the hypothesis is false, regardless of whether the conclusion is true or false, the conditional statement is **true**

From the truth table, I can see that the **only** time a **conditional statement** will be **false** is when the **hypothesis** is __________ and the **conclusion** is __________.

Example 2: Writing conditional statements (p. 200)

"A person who cannot distinguish between certain colours is colour blind."

a) Write this sentence as a conditional statement in "if *p*, then *q*" form.

b) Write the converse of your statement.

c) Is your statement biconditional? Explain.

The first statement is _________

The converse is _______________

The statement can be written:

"A person is colour blind __________________________ that person cannot distinguish between certain colours."

Example 5: Verifying a biconditional statement (p. 200)

Reid stated the following biconditional statement: “A quadrilateral is a square if and only if all of its sides are equal.” Is Reid’s biconditional statement true? Explain.

Conditional Statement:

Converse:

In Summary

Key Ideas

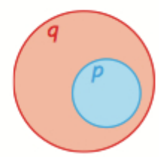
- A conditional statement consists of a hypothesis, p, and a conclusion, q. Different ways to write a conditional statement include the following:
  - If p, then q.
  - p implies q.
  - $p \Rightarrow q$
- To write the converse of a conditional statement, switch the hypothesis and the conclusion.

Need to Know

- A conditional statement is either true or false. A truth table for a conditional statement, $p \Rightarrow q$, can be set up as follows:

p	q	$p \Rightarrow q$
T	T	T
F	F	T
F	T	T
T	F	F

  A conditional statement is false only when the hypothesis is true and the conclusion is false. Otherwise, the conditional statement is true, even if the hypothesis is false.
- You can represent a conditional statement using a Venn diagram, with the inner oval representing the hypothesis and the outer oval representing the conclusion. The statement "p implies q" means that p is a subset of q.



- Only one counterexample is needed to show that a conditional statement is false.
- If a conditional statement and its converse are both true, you can combine them to create a biconditional statement using the phrase "if and only if."

HW: 3.5 p. 203-206 #1-8 & 12