4.1

4.1 Patterns

(handwritten) P.121

$\overset{+4}{\frown}\overset{+4}{\frown}\overset{+4}{\frown}\overset{+4}{\frown}\overset{+4}{\frown}$ 5, 9, 13, 17, 21, 25 ?

Can you predict the next number in the pattern 5, 9, 13, 17, ...? If you said 21, you are right. What if you were asked to find the 100th number? This would be more difficult. In this section, we will develop a linear equation to represent all values of the pattern.

To find the 100th number of the pattern above, a general equation is needed.

The common difference is 4.

1st number: 5 = 1 + 4(1)
2nd number: 9 = 1 + 4(2)
3rd number: 13 = 1 + 4(3)
4th number: 17 = 1 + 4(4)

So the 100th number is 1 + 4(100) = 401. A general equation is $t = 1 + 4n$, where n is the number of the term, and t is the term itself.

(handwritten)

Ex 1, $\overset{+7}{\frown}\overset{+7}{\frown}$ 6, 13, 20, 27 ....   Q, 200th term?

$t_1 \quad t_2 \quad t_3 \quad t_4$

(t₁) 1st term = 6 = 6 + (0)·7   ← 1st term / difference

2nd term = 13 = 6 + (1)·7

3rd term = 20 = 6 + (2)·7

4th term = 27 = 6 + (3)·7

Let:
a = 1st term
d = difference
n = term number

Arithmetic Sequence : $t_n = a + (n-1)\cdot d$

Example 1

Determine the perimeter of the 10th set of blocks.

(handwritten)
$t_{200} = 6 + (200-1)\cdot 7$ (199)
$= 6 + 1393$
$t_{200} = 1399$

▶ **Solution:** The perimeters are 4, 6, 8, ... with a common difference of 2.

Set 1: Perimeter = 2 + 2(1) = 4
Set 2: Perimeter = 2 + 2(2) = 6
Set 3: Perimeter = 2 + 2(3) = 8
Set 4: Perimeter = 2 + 2(4) = 10

let P = perimeter and n = number of squares (or length of rectangle)

The general equation is $P_n = 2 + 2n$

Therefore the 10th set of blocks has a perimeter of $P_{10} = 2 + 2(10) = 22$.

(handwritten Ex2)
Ex2) $a = 10$ $d = -2$ $t_{24} = ?$

$\overset{-2}{\frown}$ (10), 8, 6, 4, ... $t_n = a + (n-1)\cdot d$

$t_{24} = 10 + (24-1)\cdot(-2)$

$= 10 - (23)(2)$

$= 10 - 46$

$t_{24} = \boxed{-36}$

$\begin{array}{r} 23 \\ \times\ 2 \\ \hline 46 \end{array}$

Note: It is possible for a sequence to have a common difference that is negative. For example, the sequence: 13, 8, 3, − 2... has a common difference of 8 − 13 = − 5.

(handwritten) P.122 $t_n = \overset{\text{first term}}{ⓐ} + (n-1)\cdot \overset{\text{difference}}{d}$ ← term

Example 2

Write an equation relating t to n.

n	0	1	2	3	4
t	4	6	8	10	12

▶ **Solution:** As n goes up by 1 unit, t goes up by 2 units.

Therefore $t = 2n + b$, with b an unknown value.

If $n = 0$, then $t = 2(0) + b$
$= b$

Therefore b must be 4, and $t = 2n + 4$.

(handwritten Ex3)
Ex3) (8), 5.5, 3, 0.5, -2 ....

$a = 8$ $d = -2.5$

Q) $t_{31} = ?$ $n = 31$

$t_{31} = \overset{a}{8} + (\overset{n}{31} - 1)\cdot(\overset{d}{-2.5})$

$= 8 - 30(2.5)$

$= 8 - 75$

$= \boxed{-67}$

$\begin{array}{r} 30 \\ \times\ 2.5 \\ \hline 600 \\ 150 \\ \hline 75.0 \end{array}$

(top right handwritten) $\overset{-2.5}{\frown}$ 8, 5.5, 3, 0.5

Example 3

Find the 100th term of the pattern: 7, 4, 1, − 2, − 5, ...

▶ **Solution:** This pattern is decreasing by 3.

Therefore $t = -3n + b$, with b an unknown value.

If the first term is $n = 1$, with $t = 7$, then $7 = -3(1) + b \rightarrow b = 10$

Therefore $t = -3n + 10$.

The 100th term can be found by substituting 100 for n: $t = -3(100) + 10$

Therefore $t = -3n + 10$.

The 100th term can be found by substituting 100 for n: $t = -3(100) + 10$
$$= -300 + 10$$
$$= -290$$

The 100th term is -290.

Rent-A-Wreck rents a car for $30.00 per day, plus 20¢ per kilometre driven.

    a) Write an equation relating cost to kilometres driven per day.

    b) What is the cost if the car was driven 120 kilometres for one day.

    c) If $57.40 was charged to a customer for a one day rental, how many kilometres were driven?

▶ Solution: a) $C = 0.20n + 30$

    b) $C = 0.20(120) + 30$
        $= 54$

    A customer would be charged $54.00.

    c) $54.70 = 0.20n + 30$
       $0.20n = 57.40 - 30$
       $0.20n = 27.40$
          $n = 137$

    The customer drove 137 kilometres.

P123

4.1 Exercise Set

1. In the equation $A = 3p$, determine A when p is:

    a) 3 _____   b) 5 _____

    c) 11 _____   d) 17 _____

    e) −6 _____   f) $x + 2$ _____

2. In the equation $S = 4n - 2$, determine S when n is:

    a) 4 _____   b) 9 _____

    c) 18 _____   d) 23 _____

    e) 41 _____   f) $x - 1$ _____

P.123

3. Determine the common difference in each linear pattern.
  $d = ?$

    a) 2, 5, 8, 11, ... _____   b) 6, 10, 14, 18 ... _____

    c) −4, −1, 2, 5, ... _____   d) 15, 10, 5, 0, ... _____

    e) −5, −11, −17, −23, ... _____   f) −8, −3, 2, 7, ... _____

    Ext
    g) 1, 2, 3, 4, ... $\sqrt{2}$ h) $\sqrt{2} + \sqrt{3}, \sqrt{2}, \sqrt{2} - \sqrt{3}$ ...
    $+\partial$

HW P.123

Q 1-4, 6 (Lett)

4. Determine the next three numbers in each linear pattern.

    a) 2, 5, 8, 11, ... ____, ____, ____   b) 6, 10, 14, 18, ... ____, ____, ____

    c) −4, −1, 2, 5, ... ____, ____, ____   d) 15, 10, 5, 0, ... ____, ____, ____

    e) −5, −11, −17, −23, ... ____, ____, ____   f) −8, −3, 2, 7, ... ____, ____, ____

    g) 11, 8, 5, 2, ... ____, ____, ____   h) −4.1, −3.7, −3.3, ... ____, ____, ____

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(handwritten top right) 1st term → term
$$t_n = a + (n-1) \cdot d$$
↳ difference
6, 10, 14, 18, ... *(with +4 +4 +4)*

5. Write an equation relating *t* to *n*. (It must work for every pair of values)   *(handwritten: P.124)*

a)
n	1	2	3	4
t	2	5	8	11

b)
n	t₁	t₂	t₃	t₄
t	6	10	14	18

(handwritten) $d = 4$

$t_n = 6 + (n-1) \cdot (4) = 6 + 4n - 4$

$t_n = 4n + 2$

c)
n	1	2	3	4
t	−4	−1	2	5

d)
n	1	2	3	4
t	15	10	5	0

(handwritten) $d = -5$

$t_n = 15 + (n-1) \cdot (-5) = 15 - 5n + 5$

$t_n = 20 - 5n$

e)
n	1	2	3	4
t	−5	−11	−17	−23

f)
n	1	2	3	4
t	1	1.5	2	2.5

6. Determine the 50th term of the following linear pattern.

(handwritten: Try)
a) 2, 5, 8, 11, ... $t_n = a + (n-1) d$
(handwritten) n = 50, a = , d =

b) 6, 10, 14, 18, ...

c) −4, −1, 2, 5, ...

d) 15, 10, 5, 0, ...   *(handwritten: −5)*
t₁ t₂ t₃
a = 15
d = −5
n = 50

$t_{50} = a + (n-1) \cdot d$
$t_{50} = 15 + (50-1) \cdot (-5)$
$= 15 - 49 \cdot (5)$
$= 15 - 245$
$= -230$

(handwritten side)
49
× 5
245 *(shown as 4 9 × 45 245)*

e) −5, −11, −17, −23, ...

f) 0, ½, 1, 3/2, ...

7. In the following patterns, determine the number of sides needed to produce 10 polygons.

a)

b)

c)

d)
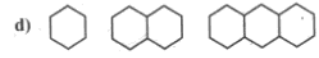
(handwritten: P.125) $t_n = a + (n-1) \cdot d$ Section 4.1 - Patterns ♦ 125

8. The total cost of publishing a school yearbook is a fixed price, plus a cost for each yearbook printed.

(handwritten: Fixed cost)
C	300	500	700	900
t	0	100	200	300

(handwritten)
fixed cost: $300
per unit: 100 Books = $500 $200 for 100 book
200 Books = $700 $2/book = unit price.

a) Using the data provided, write an equation relating the total cost (C) to the number of yearbooks printed (n).

(handwritten)
Total Cost = Fixed + Unit Cost × Units
C = $300 + $2 · n ↳ books

b) Calculate the cost of printing 650 yearbooks.

(handwritten)
Cost = $300 + $2 × 650 books
C = $300 + $1300
C = $1600

9. The cost of renting a car is $50 plus 20¢ per kilometer travelled.   *(handwritten: $0.20)*

(handwritten: Try)
a) Write an equation relating cost (C) to the number (n) of kilometers travelled.

(handwritten)
Total Cost = Fixed + Unit cost × Units
C = $50 + $0.20 × d ↳ distance.

b) Calculate the cost of travelling 480 kilometers.

(handwritten)
C = $50 + $0.20 × 480 km
C = $50 + $96
C = $146

(handwritten side)
480
× 0.2
96.0

10. The cost of sending the school's basketball team to a tournament is ...

$C = \$50 + \$0.20 \times d$
$\quad\quad\quad\quad\quad\hookrightarrow \text{distance.}$

$C = \$50 + \$96$
$\boxed{C = \$146}$

10. The cost of sending the school's basketball team to a tournament is a fixed entrance fee, plus a cost per student. If 10 players cost $260, and 15 players cost $300

?

a) What is the cost per student?

$\$40 \begin{cases} \$260 = 10\,ppl \\ \$300 = 15\,ppl \end{cases} \Big)+5\,ppl$

$\dfrac{\$40}{5\,ppl}$
$\Downarrow$
$\boxed{\begin{array}{l}\text{unit} \\ \text{cost}\end{array}\quad \$8/\text{person}}$

b) What is the entrance fee to the tournament?

$10\,ppl \times \$8/p = \$80$

but we were charged $260, why?

Entrance fee $= \$260 - \$80 = \boxed{\$180}$

11. In 1994, 45% of students graduated from university in less than 5 years. In 2000, 41% of students graduated from university in less than 5 years.

a) Write an equation relating the year of graduation to the number of years spent in university.

b) If this linear trend continues, what percent of university students will graduate in less than 5 years at the end of 2012?

12. The sequence of numbers 0, 1, 1, 2, 3, 5, 8, 13, . . . is known as the Fibonacci sequence. Determine the next five numbers of the Fibonacci sequence.