Q: **Why does a set of _n_ objects containing some identical objects have fewer permutations than a set of _n_ different objects?**

A: Since all permutations of a set of objects must be different, all repeated arrangements caused by any identical objects in the set have to be eliminated. Think about how you could arrange a set of three tiles: two red and one blue. If you list the possible arrangements using R_1, R_2, and B to represent the tiles, you get six arrangements. However, some of those are the same, so you really have only three different arrangements, or permutations.

R_1R_2 B R_2R_1 B R_1B R_2 R_2B R_1 B R_1R_2 B R_2R_1

RRB   RBR   BRR
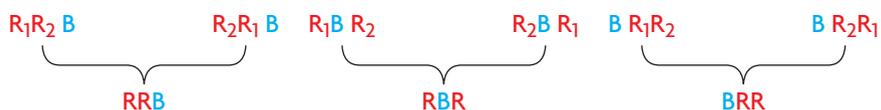
To determine the number of permutations, determine the number of permutations if the three tiles are all different, 3!, and divide by 2! (2 is the number of objects that are identical). The number of permutations of a set of three tiles with two that are identical can be represented as

$$_3P_2 = \frac{3!}{2!} \text{ or } 3$$

Q: **Why are there fewer combinations of a set of _n_ objects than permutations of the same _n_ objects?**

A: Order does not matter when counting combinations, but it does matter for permutations. Think about how you could arrange a set of three different letters (A, B, and C) in a row of three letters. There are six permutations. However, because order does not matter when you count combinations, those six permutations are all the same combination.

Six permutations: **One combination:**

ABC, ACB, BAC, BCA, CAB, CBA   ABC

To determine the number of three-letter combinations for this set, determine the number of permutations, 3!, and then divide by the number of ways the letters can be arranged, 3!. The number of three-letter combinations of a set of three letters can be represented as

$$_3C_3 = \frac{3!}{3!} \text{ or } 1$$

Q: How do you decide what strategy to use to solve a counting problem?

A: First, determine if order is important. If it is, use a permutation model. If not, use a combination model.

- Look for conditions. Consider these first as you develop your solution.
- If there is a repetition of r of the n objects to be eliminated, it is usually done by dividing by $r!$.
- If a problem involves multiple tasks that are connected by the word AND, then the Fundamental Counting Principle can be applied: multiply the number of ways that each task can occur.
- If a problem involves multiple tasks that are connected by the word OR, the Fundamental Counting Principle does not apply: add the number of ways that each task can occur. This typically is found in counting problems that involve several cases.

PRACTISING

Lesson 4.1

1. When is the Fundamental Counting Principle used to solve a counting problem? Use an example in your explanation.

2. Create a tree diagram to show all the possible ways that three coins can land when you flip a quarter, a toonie, and a loonie all at the same time.

3. A student is writing a 10-question multiple-choice test. Each question has 4 choices: A to D. How many different sets of answers can the student give?

Lesson 4.2

4. Solve each equation. State the restrictions on the variable.

a) $\dfrac{(n + 2)!}{n!} = 20$

b) $\dfrac{(n + 1) \cdot n!}{(n - 1)!} = 132$

5. Which expression has a larger value: $_6P_6$ or $\dfrac{8!}{6!}$? Explain how you know.

6. Chorale Saint-Jean from Edmonton, Alberta, is the largest and most active francophone choir in Western Canada. If the singers have rehearsed 12 different songs for an upcoming tour, in how many different orders could they perform the 12 songs?

Lesson 4.3

7. How many different ways can a director of education, a superintendent of curriculum, and a superintendent of finance be selected from a group of 25 candidates?

8. A website offers an online practice driving test that consists of 10 questions selected from a bank of 25 questions. The level of each question ranges from easy to difficult. How many different ways can the test be created in each of the following situations?
   a) There are no conditions.
   b) The easiest question of the 25 is always first and the most difficult question is always last.

9. Suppose you draw five cards from a standard deck of cards and arrange them in a row from left to right in the order you draw them. How many different five-card arrangements are possible?

Lesson 4.4

10. Consider the 11 letters in the word MATHEMATICS. How many different arrangements are possible in the following situations?
    a) All the letters are used.
    b) All the letters are used, but each arrangement must begin with the C.

11. Tina is playing with a tub of building blocks. The tub contains 3 red blocks, 5 blue blocks, 2 yellow blocks, and 4 green blocks. How many different ways can Tina stack the blocks in a single tower in each situation below?
    a) There are no conditions.
    b) There must be a yellow block at the bottom of the tower and a yellow block at the top.

Lesson 4.5

12. Which expression results in the greatest value: $_{10}C_5$ or $_{11}C_7$ or $\binom{15}{2}$?

13. How many different selections of 4 books can Ruth choose from a box of 20 different books at her neighbour's garage sale?

14. Liz claims that if you have a set of n different objects and you select r of them, the number of combinations you can make will always be greater than the number of permutations. Do you agree? Justify your decision.

Lesson 4.6

15. The town council is forming a committee, and 9 men and 10 women have volunteered. How many different ways can a committee of 4 people be chosen in each situation below?
    a) There are no conditions.
    b) There must be an equal number of men and women on the committee.
    c) No men can be on the committee.

16. How many different ways can 15 teachers be divided into 3 groups of 5 for an activity at a staff meeting?

Lesson 4.7

17. If 12 points are arranged in a circle, how many different ways can the points be joined to form straight lines?

18. Kim, Kandice, and Kerry are female triplets. They and their 10 cousins are posing for a series of photographs. One pose involves all 13 children. How many ways can the 7 boys and 6 girls be arranged in one row under each of the following conditions?
    a) The boys and girls must alternate positions.
    b) The triplets must stand next to each other.

19. How many different five-card hands with at least two face cards can be dealt from a standard deck of playing cards?