6.1 - Exploring the Graphs of Polynomial Functions

Foundations of Mathematics 12 – 6.1

6.1 – EXPLORING THE GRAPHS OF POLYNOMIAL FUNCTIONS

A polynomial function consist of one or more terms, which are separated by + or – signs.

The degree of a polynomial function is the value of the highest exponent in the function. If a polynomial function includes a term with no variable, this term is called a constant term.

ex, $y = x$ Deg 1 $y = x^2$ Deg 2 $y = x^3$ Deg 3 $y = x^2 - 3$ Deg 2

Determine the Degree and the Constant

Example 1: Determine the degree and the constant of each polynomial function.

a. $f(x) = x^2 + 4x - 5$ Deg 2

constant −5

b. $g(x) = 3x - 7$

deg 1 Const = −7

$x^0 = 1$

c. $h(x) = 8$

Deg = ∅

d) $y = 2x^2 + 4x^4 - 3x + 4$

Deg = 4

A number that multiplies the variable in a polynomial is called a coefficient. The leading coefficient is the number that multiplies the term with the highest power.

ex, $y = 3x^2 - 4x + 2$

Leading Coeff.

Determine the Leading Coefficient

Example 2: Determine the leading coefficient of each polynomial function.

a. $f(x) = x^2 + 4x - 5$

leading Coefficient = 1

b. $g(x) = 3x - 7$

L.C. = 3

ex, $y = x^2 - 5x + 4x^3 - 1$ Leading Coeff. = $\boxed{+4}$

The terms in a polynomial function are normally written so that the powers are in descending order.

For example, $f(x) = 2x^3 + 3x^2 - 2x + 5$

$y = 4x^3 + x^2 - 5x - 1$

right order

Example 3: Write a polynomial function in descending order that satisfies the following conditions.

a. degree 2, leading coefficient –3

$y = -3x^2$ also $y = -3x^2 + 7$

b. degree 2, leading coefficient 7, two terms

1 term 2 term

$y = 7x^2 + 27$

c. degree 1, leading coefficient 1

$y = 1x$ $y = x$

d. degree 0

$y = 11$

e. degree 3, constant term –8

$\boxed{y = x^3 - 8}$

Leading Coefficient constant term

f.

∈ ℝ

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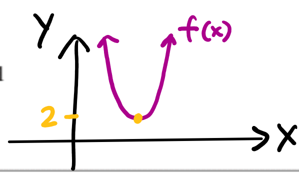
$\boxed{x \in \mathbb{R}}$



Domain "x" can be all of the Real number

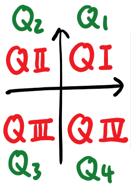
The domain is the set of all possible x-values which will make the function "work" and will output real y-values.

Range ex $\boxed{y \geq 2}$ "y" is greater than or equal to 2

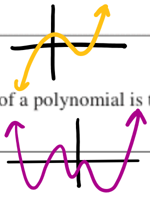
The range of a function is the complete set of all possible **resulting** y-values of the dependent variable.

Range $2x$ $\boxed{y \geq 2}$ "y" is greater than or equal to 2

The range of a function is the complete set of all possible **resulting** y-values of the dependent variable.



End behaviour



QIII → QI ✓

Read from Left → Right.

The end behaviour of a polynomial is the description of the shape of the graph, from left to right, on the coordinate plane.

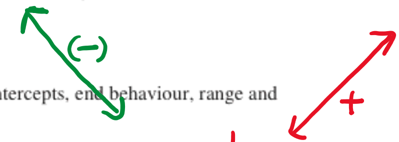
QII → QI

Turning Point

A turning point is any point where the graph of a function changes from increasing to decreasing or from decreasing to increasing.

Polynomial functions are named according to their degree. Polynomial functions of degrees 0, 1, 2, and 3 are called constant, linear, quadratic, and cubic functions, respectively.

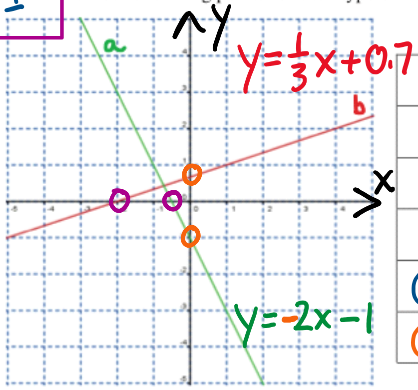
Characteristics of Polynomial Functions



$\boxed{\text{Degree } 1}$

Example 4: Determine the type of function, the degree, the x-intercepts, y-intercepts, end behaviour, range and number of turning points for each type of function.

a.



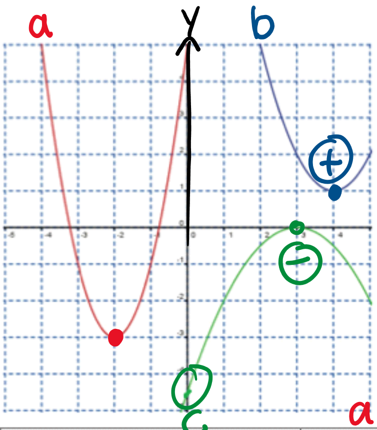
	a	b
Type of function	Linear	Linear
Degree	1	1
Number of x-ints (x-intercept)	1	1
Number of y-ints	1	1
(x) Domain	$x \in \mathbb{R}$	$x \in \mathbb{R}$
(y) Range	$y \in \mathbb{R}$	$y \in \mathbb{R}$

	a	
End Behaviour	QII → QIV	Q3 → Q1
Number of Turning Points	none	none

Foundations of Mathematics 12 – 6.1 $y = x^2$ $y = 3x^2 - 7$ $y = -4x^2 + 5x + 2$

$\boxed{\text{Deg}: 2}$

b.



	a	b	c
Type of function	quadratic		
Degree	2	2	2
Number of x-ints	2	∅	1
Number of y-ints	1	1	1
Domain	$x \in \mathbb{R}$	$x \in \mathbb{R}$	$x \in \mathbb{R}$
Range	$y \geq -3$	$y \geq 1$	$y \leq \emptyset$

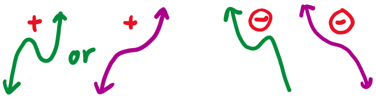
bigger than less than

	a	b	c
End Behaviour	Q2 → Q1	Q2 → Q1	Q3 → Q4
Number of Turning Points	1	1	1

End Behaviour	$Q_2 \to Q_1$	$Q_2 \to Q_1$	$Q_3 \to Q_4$
Number of Turning Points	1	1	1

↑ also called "vertex"

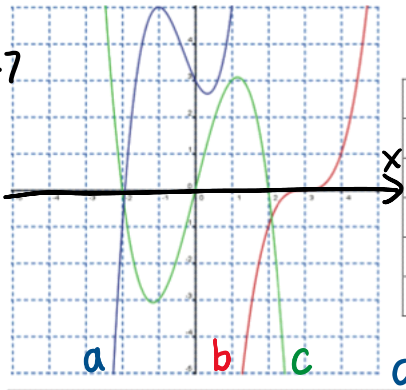
Deg 3



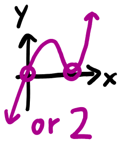
$y = x^3 - 2x + 7$

⊕/⊖

c.

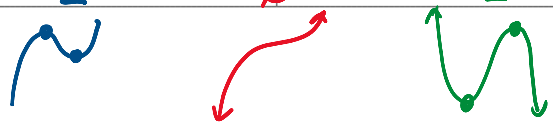


	a	b	c
Type of function	Cubic		
Degree	3	3	3
Number of x-ints	1	1	3
Number of y-ints	1	1	1
Domain	$x \in \mathbb{R}$		
Range	$y \in \mathbb{R}$		



or 2

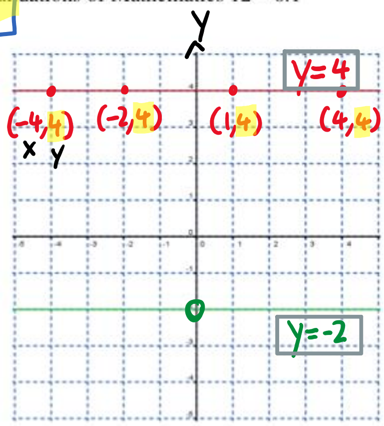
	a	b	c
End Behaviour	$Q3 \to Q1$	$Q3 \to Q1$	$Q2 \to Q4$
Number of Turning Points	2	∅	2



Deg ∅ → no "x" - variable

horizontal Line!!

d.

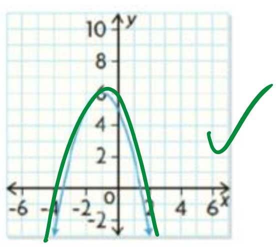


	$y=4$	$y=-2$
Type of function	constant	constant
Degree	∅	∅
Number of x-ints	∅	∅
Number of y-ints	1	1
Domain	$x \in \mathbb{R}$	$x \in \mathbb{R}$
Range	$y=4$	$y=-2$

	$y=4$	$y=-2$
End Behaviour	QII → QI	QIII → QIV
Number of Turning Points	none	none

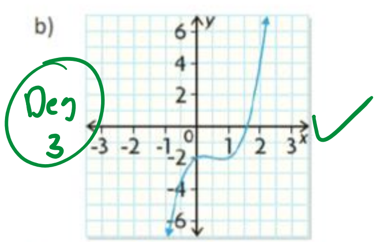
Example 5: Which of the following graphs might represent polynomial functions?

a) Deg 2



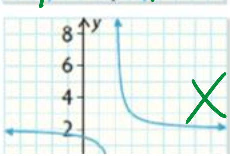
✓

b) Deg 3



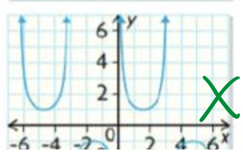
✓

c)



✗

d)



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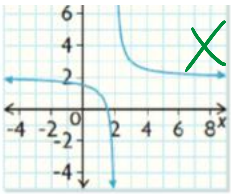
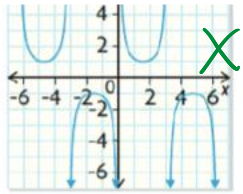
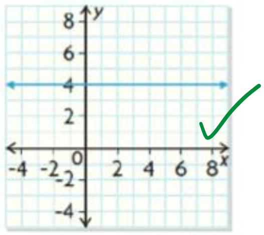


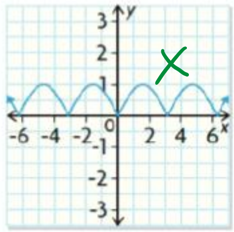


e)

Deg
∅



f)



Assignment: p. 383 #1 – 4