Show all work where possible for full marks.

1. Without graphing, which of the following functions increases at a faster rate? Explain why.

a. $y = \log(x)$ base 10   b. $y = \ln(x)$ Ln base "e" ≈ 2.72....

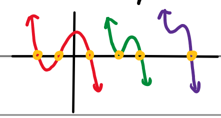
Ln(x) inc faster ! because it is base "e" instead of "10"

2 marks

2. Compare the number of intercepts and end behavior of an exponential function in the form $y = A(b)^x$, where $A > 0$, and $0 < b < 1$ to the polynomial where the highest degree term is $-2x^3$ and the constant term is 4.

neg deg 3

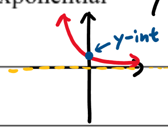
Polynomial $y = -2x^3 + 3x^2 + 4$ ← y-int



$\boxed{\text{one y-int}}$ $\boxed{\text{1 or 2 or 3 x-int}}$ $\boxed{Q\,II \rightarrow Q\,IV \text{ dec}}$

2 marks

Ch.7 Exponential $y = 3(0.5)^x$ exp. dec.



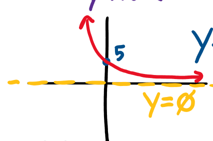
$\boxed{\text{one y-int}}$ $\boxed{\text{none x-int}}$ $\boxed{Q\,II \rightarrow Q\,I \text{ dec}}$

2 marks

$y = \boxed{A}(b)^x$ y-int, base

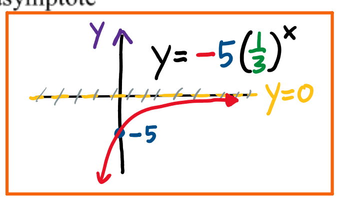
3. Using the equation of this function describe the following characteristics of its graph: *(7 marks)*

$f(x) = -5\left(\frac{1}{3}\right)^x$



i) Base

ii) Equation of the asymptote



iii) x-intercepts

iv) y–intercept

v) End behavior

vi) Domain

vii) Range

i) 1/3

ii) y=∅

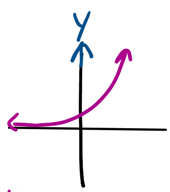
iii) none

iv) y=-5

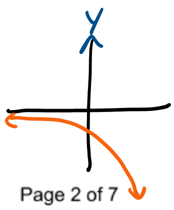
v) Q III → Q IV (Inc)

vi) $x \in \mathbb{R}$

vii) $y < \varnothing$



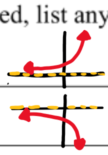
Range $y > 0$

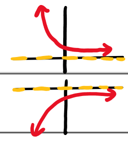


Range $y < 0$

$y = a \cdot (b)^x$

b. What parameter could you manipulate to change the direction of the function? If the direction was changed, list any new characteristics of the graph.

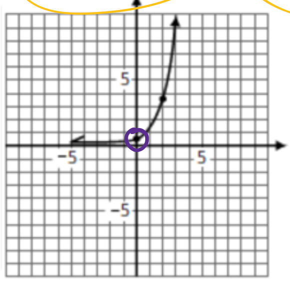
"b > 1"  if "+a" / if "−a"

"0 < b < 1"  if "+a" / if "−a"

2 marks

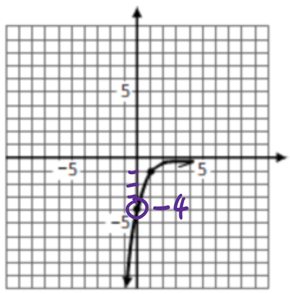
4. Match each function with the given graph. Explain your reasoning. *(2 marks each)*

i) $y = -4\left(\frac{1}{4}\right)^x$ *ii)* $y = \frac{1}{2}(e)^x$ *iii)* $y = \frac{1}{3}\ln(x)$ *iv)* $y = \log(x) + 4$

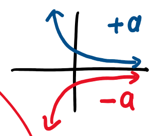


Function: $y = \frac{1}{2}e^x$

Reason: y-int : $\frac{1}{2}$ ⊕ base $= e > 1$ (2.7) inc

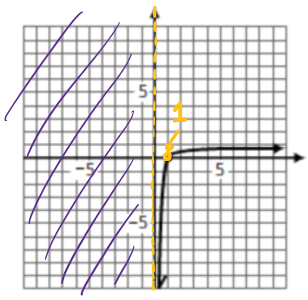


Function: $y = -4\left(\frac{1}{4}\right)^x$



+a / −a

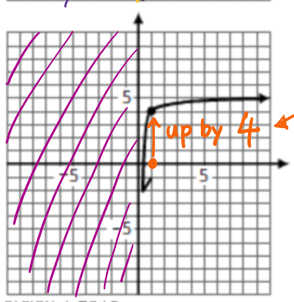
Reason: y-int : −4 0 < b < 1 $\boxed{\text{dec}}$ ⊖ invert



Function: $y = \frac{1}{3}\text{Ln}(x)$

⊕ inc

Reason: x-int : 1 , graph only in "+X" side.



Function: $y = \text{Log}\,x + 4$

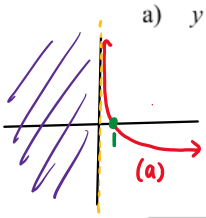
Reason: ⊕ inc , only on +x side

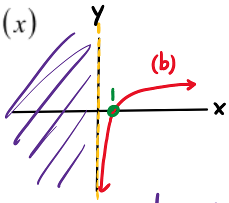
de.

5. List the similarities and differences in the two functions below in terms of the x-intercept(s), the y-intercepts, domain, range, base, equation of the asymptote and end behaviour for the following:

(5 marks)

a) $y = -2\log(x)$ b) $y = \frac{4}{3}\ln(x)$

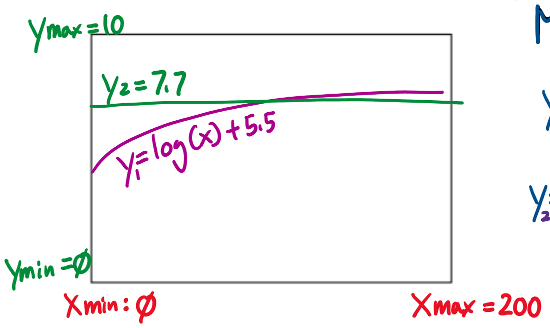




Similarities	**Differences**
X-int (X = 1)	-Log : QI → QIV (dec)
no y-int	Ln : QIV → QI (inc)
Domain : X > 0	
Range : Y ∈ IR	

6. An aftershock measuring 5.5 on the Richter scale occurred south of Christchurch, New Zealand in June 2011. The magnitude, M of an earthquake that is T times more intense than an earthquake measuring 5.5 on the Richter scale can be modeled by the following function: $M = \log(T) + 5.5$. For every one unit of increase on the Richter scale, the intensity of an earthquake increases 10 fold.

a) Graph the function, [show window] and determine how much more intense an earthquake measuring 7.7 that occurred on Queen Charlotte Islands in October 2012 was than the aftershock in New Zealand.



Richer Scale intensity

$M = \text{Log}(T) + 5.5$

$Y_1 = \text{Log}(x) + 5.5$

$Y_2 = 7.7$ X = ___?

$\boxed{\text{Intersect}}$

a) 158.5 times

2 marks

b) Determine the value of the function at $T = 3$, explain the answer in the context of this question.

X = 3 Y = 5.977 ?

"Value"

b) Richter Scale "6"

2 marks

Ch.7 Test on Wednesday Jan 17th

c) How many times as intense would an earthquake measuring 8.5 on the Richter scale be then one measuring 5.5? Express your answer as a power with base 10.

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$Y_2 = 8.5$

"Intersect"

"set"

$X_{max} = 1600$

$\boxed{X = 1000 \text{ times intensity}}$

c) ____________ *2 marks*

7. A \$5000 investment that grows at annual rate of 6% can be modeled by the function: $y = 5000\,(1.06)^x$

a) Determine the value of the investment after 20 years.

$Y_1 = 5000(1.06)^x$ ← time

\$ ← y-int

$X = 20\text{yr}$

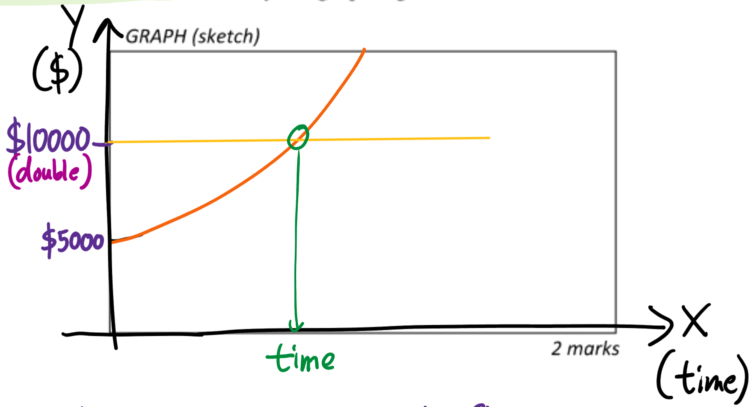
a) $Y = \$16035.68$ *2 marks*

Value

b) How long would it take for the investment to double? Use your graphing calculator and show window dimensions and graph.

WINDOW	
Xmin =	∅
Xmax =	30
Xscl =	
Ymin =	∅
Ymax =	15000
Yscl =	
Xres =	

GRAPH (sketch)



2 marks

$Y_2 = 10{,}000$ $X = 11.9$

b) ~12 years *2 marks*

c) How long would it take the investment to double using the rule of 72? Explain any differences.

c) time to double $\approx \frac{72}{6} = \boxed{12 \text{ yrs}}$ *2 marks*

8. a) Would the future value of a \$1000 investment be better represented by an increasing exponential function or an increasing logarithmic function? Give reasons for your choice.

$A = P(1+r)^n$ we should use exponential function.

\$ time

2 marks

b) What would the dependent and independent variables be in either case? Give reasons for your choice.

(x) Independent = time

(y) dependent = Amount of Money

2 marks

c) Would there have to any restrictions on the domain or range in the function you used? Give reasons for your choice.

time always positive $X_{min} = \emptyset$

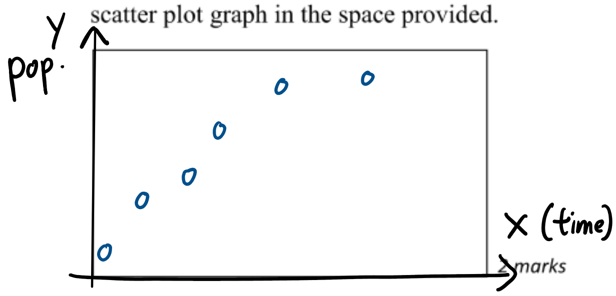
Money should start with positive $Y_{min} = \emptyset$

2 marks

9. The population of Surrey, BC is shown in the table below for the years 1986 through 2011 in five year increments. Surrey incorporated in 1879 with fewer than two hundred citizens.

Year after 1985	Year	Population
L_1		L_2
1	1986	154193
6	1991	208706
11	1996	267300
16	2001	347820
21	2006	394980
26	2011	408251

a) Create a scatter plot of this data to show how the population is related to the year. Sketch the scatter plot graph in the space provided.



2 marks

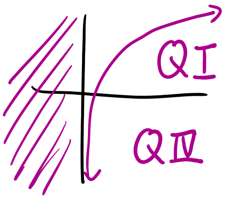
Ln Reg. L_1, L_2, Y_1

b) Determine an equation for a logarithmic regression function that would model this data.

$y = 79646 \ln(x) + 120793$

b) ______________ *1 mark*

c) Describe the end behaviour.



c) QIV → QI

(Inc) but slowing down

1 mark

d) Determine any restrictions on the Domain and Range.

time
Domain: $x > 0$

1 mark

population
Range: $y \geq \emptyset$

1 mark

e) Estimate the population for the year 2025.

2025 − 1985 = 40 years   Xmax = 50

X = 40   Y = $\boxed{414{,}597 \text{ ppl}}$

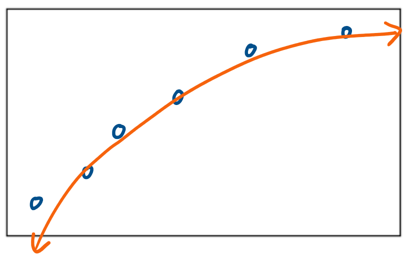
e) ______________ *1 mark*

10. The net debt of the Canadian government is shown in the table below from 1960 to 2010.

Year after 1950 ← you pick / not

L_1	Year	Debt [Billions] L_2
10	1960	20
20	1970	175
30	1980	370
40	1990	450
50	2000	561
60	2010	600

a) Create a scatter plot of this data to show how the debt is related to the year. Sketch the scatter plot graph in the space provided.



2 marks

b) Determine an equation for a regression function that would model this data.

$y = 338.6 \ln(x) - 788.4$

b) ______________ *1 mark*

c) In what year will the year the debt will go over 800 billion dollars?

$Y_2 = 800$ X = 109 yrs after 1950   +109 yr

Ymax = 1000   Xmax = 150

c) $\boxed{\text{Year } 2059}$ *1 mark*