Circular Motion Review Key

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Uniform Circular Motion Review WS + Artificial Gravity

1. (Hori) A race car going 32 m/s rounds a curve 56 m in radius. Calculate the car's centripetal acceleration.

$$a_c = \frac{v^2}{r} = \frac{32^2}{56} = \boxed{18.3\ m/s^2}$$

2. (Hori) A runner of mass 67 kg moving at a speed of 8.8 m/s rounds a bend with radius of 25 m. Calculate the centripetal force acting on the runners.

$$F_c = \frac{mv^2}{r} = \frac{67(8.8)^2}{25} = \boxed{207.5\ N}$$

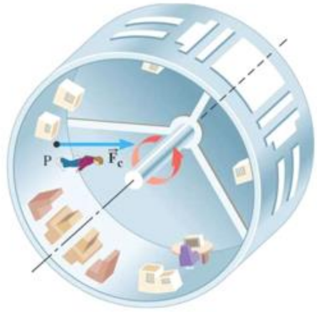
3. (Hori) An athlete whirls a 7.00 kg hammer tied to the end of a 1.3 m chain in a horizontal circle. The hammer makes one revolution in 1.0 second
   a. What is the centripetal acceleration of the hammer?
   b. What is the tension in the chain?

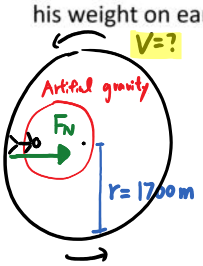
a) $$a_c = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2(1.3)}{(1)^2} = \boxed{51.3\ m/s^2}$$

b) $F_c = T \qquad T = ma_c$

$T = 7(51.3) = \boxed{359N}$

4. (Space: Artificial G) One of the problems astronauts who live on the international space station experience after long period of time is muscle atrophy because of living in a content state of free fall and apparent (not actual) weightlessness. One potential solution in the future would be to spin modules of a space station to induce artificial gravity through centripetal acceleration. Because of rotational motion, any person or object inside would feel a force directed towards the center. At what speed must the surface of a cylinder-shaped space station with a radius of 1700 m have to move so that an astronaut experiences a force equal to his weight on earth?





$F_c = F_N$ ($F_g = mg$)

Earth F_N F_g

$$\frac{\cancel{m}v^2}{r} = \cancel{m}g$$

$$\boxed{g = \frac{v^2}{r}}$$

strength of Artificial Gravity.

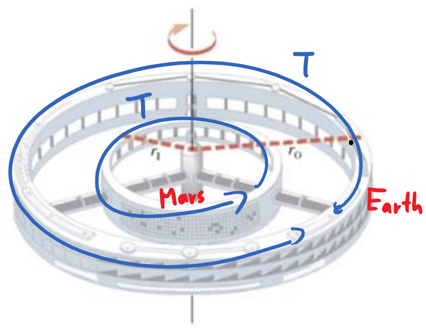
normal.

ex) $g = 9.8$

$$9.8 = \frac{v^2}{1700}$$

$$\boxed{V = 129\ m/s}$$

5. (Space: Artificial G) A space laboratory is rotating to create artificial gravity. Its period is chosen so that the outer ring ($r = 2150\ m$) simulates the acceleration due to gravity on Earth. What should the radius of the inner ring be so that it simulates the acceleration due to gravity on the surface of Mars ($3.72\ m/s^2$)



* Same Period for inner and outer rings !!!

Earth : Outer

$$F_c = F_g \qquad \frac{\cancel{m}4\pi^2 r}{T^2} = \cancel{m}g \quad (9.8\ m/s^2)$$

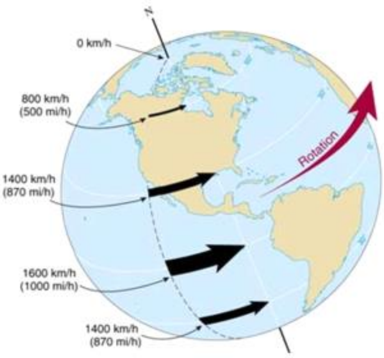
$$\boxed{T = 93\ sec}$$

Inner Ring : Mars

$$\frac{4\pi^2 r}{T^2} = g \quad (3.72\ m/s^2)$$

$$\boxed{r = 816\,m}$$

6. (Space: Artificial G) Early sceptics of the idea of a rotating Earth said that the fast spin of the Earth would throw people at the equator into space.
   a. Calculate the speed of a 97 kg person standing on the equator.
   b. Calculate the force needed to accelerate this person in the circle
   c. Calculate the force of gravity acting on the person
   d. What is the person's apparent weight?
   e. If the rotation of the Earth was increased so that the centripetal acceleration was equal to the acceleration due to gravity, how fast would the person be moving?
   f. How man minutes long would one day be in this case?

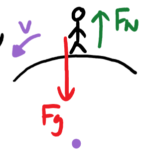


a, equator : $V = \frac{2\pi r}{T_{rotation}} = \frac{2\pi\, 6.38 \times 10^6}{8.61 \times 10^4} = 465.6\ m/s$

b, $F_c = \frac{mv^2}{r} = \frac{97(465.6)^2}{6.38 \times 10^6} = 3.3\ N$

c, $F_g = mg = 97(9.8) = 950.6\ N$

d,

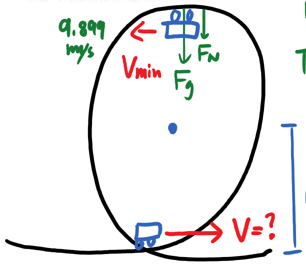


$F_c = F_g - F_N$

$3.3 = 950.6 - F_N$

$\boxed{F_N = 947.3\ N} \rightarrow 96.7\ kg$

7. (Vert) A popular daredevil trick is to complete a vertical loop on a motorcycle. This trick is dangerous, however, because if the motorcycle does not travel with enough speed, the rider falls off the track before reaching the top of the loop. What is the minimum speed (at the bottom) necessary for a rider to successfully go around a vertical loop of radius 10.0 m?



Top: $F_c = F_g + F_N$

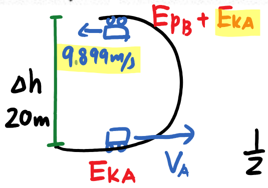
Top (min speed): $F_c = F_g$

$F_N = \emptyset$

$\frac{\cancel{m}v^2}{r} = \cancel{m}g$

$\boxed{V_{min\,Top} = \sqrt{rg}}$

$V_{min\,Top} = \sqrt{10_m \times 9.8} = 9.899\ m/s$



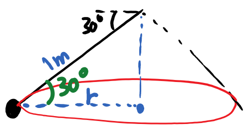
Conservation of Energy.

$E_{KA} = E_{KB} + E_{PB}$

$\frac{1}{2}\cancel{m}V_A^2 = \frac{1}{2}\cancel{m}(9.899)^2 + \cancel{m}g\,h$ (20m)

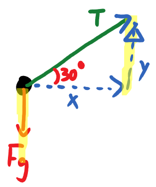
$\therefore V_A = 22.1\ m/s$ min @ bottom

8. (Vert) A 0.25 kg mass attached to a string of length 1 m is spun in a horizontal circle. At what speed must it travel so that the angle the sting makes with horizontal is 30 degree? (hint: start by drawing a free body diagram)



$r = 1\cos 30°$

$r = 0.866\ m$



$\boxed{Y}$: $T_y = F_g = mg$

$\boxed{X}$: $F_c = T_x$

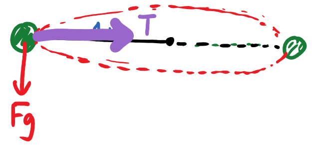
$\tan 30° = \frac{T_y}{T_x}$

$T_x = T_y / \tan 30°$

$\frac{\cancel{m}v^2}{r} = \frac{\cancel{m}g}{\tan 30°}$

0.866 m

$\boxed{V = 3.83\ m/s}$



1) 18 m/s^2	2) 210 N	3) 51 m/s^2; 360 N	4) 130 m/s
5) 816 m	6) 470 m/s; 3.3 N; 950 N; 947 N; 7900 m/s; 85 mins	7) 9.9 m/s	8) 3.83 m/s

More Gravitation and Circular Motion Review: $\frac{r_u^3}{T_u^2} = 1 \frac{A.U.}{\text{Earth yr.}}$

9. (Space: Kepler)

(a) Uranus has an orbital period of 84 years. What is its average distance from the sun? T_u

$$\frac{r_u^3}{(84)^2} = 1 \qquad r_u^3 = 7056 \qquad r_u = 19.18\ A.U. = 19.18 \times (1.5\times10^{11}\,m) = \boxed{2.88\times10^{12}\,m}$$

1 A.U, yr

(b) Neptune's orbit is 50% bigger than the orbit of Uranus. What is the orbital period for Neptune?

$$r_N = 1.5\, r_u \qquad \frac{r_N^3}{T_N^2} = \frac{r_u^3}{T_u^2} \Rightarrow \frac{(1.5 r_u)^3}{T_N^2} = \frac{r_u^3}{T_u^2} \Rightarrow \frac{3.375\,\cancel{r_u^3}}{T_N^2} = \frac{\cancel{r_u^3}}{T_u^2}$$

$$3.375\, T_u^2 = T_N^2 \qquad T_N = 1.837\, T_u \ (84\,yr) \rightarrow 154.3\ yr. \text{ Earth} \Rightarrow \boxed{4.876\times10^{9}\,sec}$$

Neptune Period.

10. (Space) A 5800 kg object is lifted from the Earth's surface to a radius of 2.3×10^7 m. What is the gravitational field strength of the Earth at this location?

$$r_{\text{from center}} = 2.3\times10^7 \qquad g = \frac{GM}{r^2} = \frac{G(5.98\times10^{24})}{(2.3\times10^7)^2} = \boxed{0.754\,m/s^2}$$

11. (Space: Kepler) A 1500 kg satellite orbits the earth at a radius of 8.2×10^6 m. What is the period (time to complete one orbit) for the space shuttle? what else orbits the Earth? → moon.

$$\text{orbit} \quad \frac{r_{sat}^3}{T_{sat}^2} = \frac{r_{moon}^3}{T_{moon}^2} \qquad \frac{(8.2\times10^6)^3}{T_{sat}^2} = \frac{(3.84\times10^8)^3}{(2.36\times10^6)^2} \qquad \boxed{T_{sat} = 7364\ sec}$$

12. (Space) A 6800 kg space probe is in a circular orbit of radius 8.0×10^9 m about our Sun.

a. What is the orbital speed of this space probe?

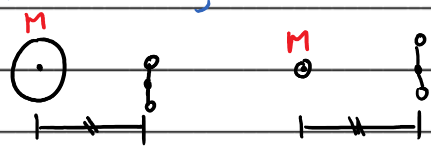
$$F_c = F_g \qquad \frac{\cancel{m}V^2}{r} = \frac{GM\cancel{m}}{r^2} \qquad V_{orbit} = \sqrt{\frac{GM_{sun}}{r}} \quad M_{sun} = 1.98\times10^{30}\,kg,\ r = 8\times10^9\,m \qquad \boxed{V = 128{,}484\ m/s}$$

b. Suppose the Sun was to suddenly collapse to 1/10 of its present radius ***without*** any change in mass. The space probe's orbital speed about the Sun will:

____ Increase ____ Decrease ✓ Stay the same

c. Using principles of physics explain your answer.

Same mass, same distance to the center ⇒ Same V_{orbit}.



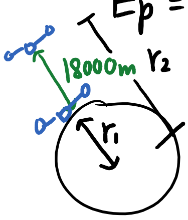
13. (Space) A 2500 kg rocket is lifted from the surface of the earth to a vertical height of 18000 m above the earth's surface.

a. What is the potential energy of the rocket while it is sitting on the Earth's surface?

on surface

$$E_p = \frac{-GMm}{r_1} = \frac{-G(5.98\times10^{24})(2500)}{6.38\times10^6\,m} = \boxed{-1.563\times10^{11}\,J}$$

$G = 6.67\times10^{-11}$, r_E



$$r_2 = r_1 + 18000\,m = 6.38 \times 10^6 + 18000 = 6.398 \times 10^6$$

b. What is the potential energy of the rocket when it is at 18000 m above the Earth's surface?

$$E_{p_2} = \frac{-G(5.98 \times 10^{24})(2500)}{6.398 \times 10^6\,m} = \boxed{-1.55856 \times 10^{11}\,J}$$

c. What work did the rockets on the rocket do in lifting this rocket from the Earth's surface to a height of 18000 m?

$$W = \Delta E_p = E_{p_2} - E_{p_1} = \boxed{4.397 \times 10^8\,J}$$

14. (Space) A 6500 kg satellite is in geostationary orbit around the Earth at a radius of 4.2 x 10^7 m/s.

a. What does Geostationary mean? *(1 mark)*

The orbital period of the satellite matches the rotational period of Earth so that the satellite always stay at a fixed point above the surface.

b. What is the total energy of this satellite at this distance from the earth? *(3 marks)*

$$E_T = \frac{1}{2} E_p = -\frac{1}{2}\frac{GMm}{r} = -\frac{1}{2}\frac{G(5.98 \times 10^{24})(6500)}{4.2 \times 10^7} = \boxed{-3.086 \times 10^{10}\,J}$$