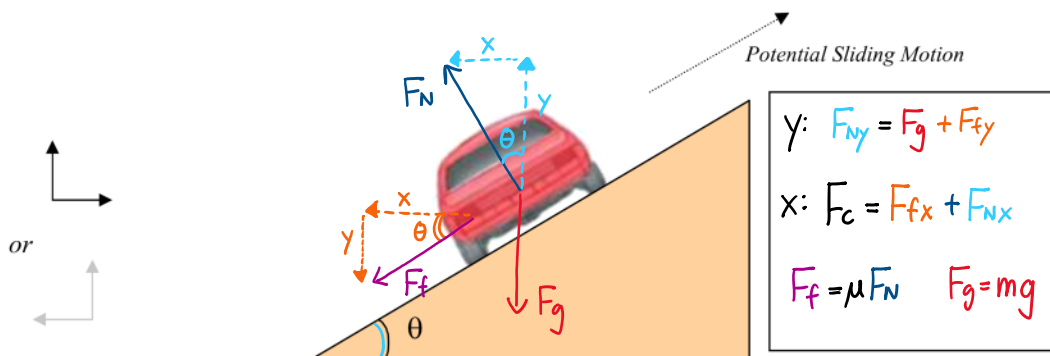


Level 3 - Cars Traveling Around a Banked Curve (w/friction)

Ex.

Find the **maximum** speed a car of mass m traveling along a banked curve (whose path is the shape of a circle of radius r) can have in order to make the curve without sliding up the incline.



Determine the motion in each direction using Newton's 2nd law and the force diagram.

$$\begin{aligned} \text{X: } F_c &= F_{Nx} + F_{fx} \\ \frac{mv^2}{r} &= F_N \sin \theta + F_f \cos \theta \\ \frac{mv^2}{r} &= F_N [\sin \theta + \mu \cos \theta] \\ \frac{mv^2}{r} &= \frac{mg [\sin \theta + \mu \cos \theta]}{[\cos \theta - \mu \sin \theta]} \end{aligned}$$

$$\begin{aligned} \text{Y: } F_{Ny} &= F_g + F_{fy} \\ F_N \cos \theta &= mg + F_f \sin \theta \\ F_N \cos \theta - \mu F_N \sin \theta &= mg \\ F_N [\cos \theta - \mu \sin \theta] &= mg \\ F_N &= \frac{mg}{[\cos \theta - \mu \sin \theta]} \end{aligned}$$

Level 3

$$V_{max} = \sqrt{rg \frac{\sin \theta + \mu \cos \theta}{\cos \theta - \mu \sin \theta}}$$

no Friction $\mu = 0$ → $V = \sqrt{rg \tan \theta}$ (Level 2)

no bank $\theta = 0$ → $V = \sqrt{\mu rg}$ (Level 1)

Including friction, this is the restriction on the speed of the car to go around a banked curve without sliding up the incline.

If $v_{car} > v_{max}$, the car will slide up the incline

not on test!!

In terms of the angle:

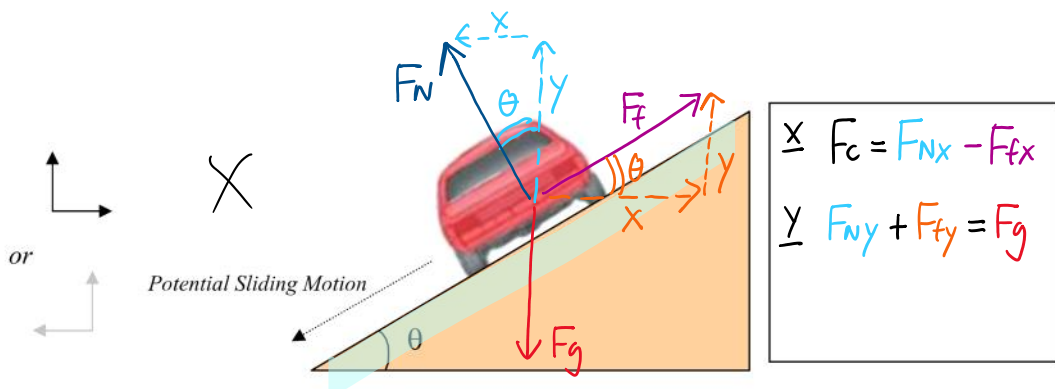
$$\tan \theta = \frac{\left(\frac{v^2}{gr}\right) - \mu}{1 + \left(\frac{\mu v^2}{gr}\right)}$$

* There is also a *minimum* speed the car must have when you include friction or it will start to slide down the incline.

For this scenario, the potential motion is down the incline. This means the static friction force points up the incline.

Ex.

Find the **minimum** speed a car traveling along a banked curve (whose path is the shape of a circle) can have in order to make the curve without sliding down the incline.



Determine the motion in each direction using Newton's 2nd law and the force diagram.

$$v_{\min} = \sqrt{rg \frac{\sin \theta - \mu \cos \theta}{\cos \theta + \mu \sin \theta}}$$

Including friction, this is the restriction on the speed of the car to go around a banked curve without sliding down the incline.

If $v_{\text{car}} < v_{\min}$, the car will slide down the incline