1.1 – SIMPLE INTEREST

Goal: Solve problems that involve simple interest.

Example 1:Solving a simple interest problem

Aaron is 18 years old and needs money to pay for college. When he was born, his grandparents bought him a $500 Canada Savings Bond (CSB) with a *term* of 10 years. They chose a CSB as an investment because they like the security of loaning money to the government. The *interest* earned was determined using a *fixed interest rate* of 6% per year on the original investment and was paid at the end of each year until Aaron's 10^{th} birthday.

Determine the *simple interest* earned on the *principal* and the *future value* of the investment when it reaches *maturity*.

To understand the question, we need to know:

term: _____

interest: _____

fixed interest rate: _____

simple interest: _____

principal: _____

future value: _____

maturity: _____

To answer the question, we will organize our calculation into a table like the one below:

Year	Value of Investment at Start of Year ($)	Simple Interest Earned Each Year ($)	Accumulated Interest ($)	Value of Investment at End of Year ($)
0				
1				
2				
3				
4				

Note that the interest does not change in a scenario involving simple interest at a fixed interest rate. So instead we can simply calculate a year's interest and multiply it by the number of years in the term, or:

$i = Prt$

Adding the principal allows us to determine the future value of the investment, A

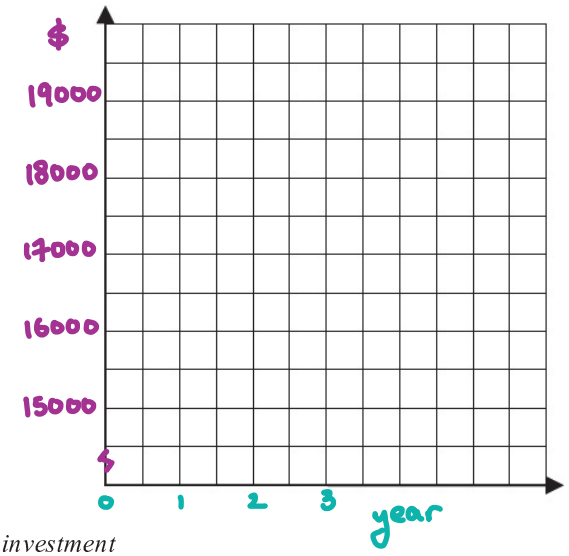
$A = P + Prt = P(1 + rt)$

Example 2: Representing the growth of a simple interest investment

Betty invested $15 000 in a savings account. Betty earned a simple interest rate of 8%, paid semi-annually on her investment. She intends to hold the investment for 3.5 years, when she will use the money to buy a car. Determine the value of the investment at each half year until she withdraws the money.

Year	Value of Investment ($)
0	
0.5	
1	
1.5	
2	
2.5	
3	
3.5	

Value of Investment over Time



Example 3: Determining the duration of a simple interest investment

Charles invested his savings of $5000 at 6% simple interest, paid annually.

How long will it take for the future value of the investment to grow to $6500?

What is Charles *rate of return*?

rate of return: _____

Example 4: Determining the rate of interest n a simple interest investment

Danielle invested $25 000 in a simple interest CSB that paid interest annually. If the future value of the CSB is $29 375 at the end of the term, what interest rate does the CSB earn? Given that rate, how much would the investment be worth if she withdrew after four years instead?

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Assignment: p. 14 #4 – 8, 11 – 14

1.2 & 1.3(i) – COMPOUND INTEREST: FUTURE VALUE

Goal: Compare simple interest with compound interest.

Example 1: Developing a compound interest formula

Both Eugene and Francine received a $1000 prize in a math contest.

- Eugene bought a $1000 simple interest GIC (guaranteed investment certificate) with his prize money. It has a 5-year term and earns 3.6% paid annually.
- Francine bought a $1000 compound interest GIC (guaranteed investment certificate) with her prize money. It has a 5-year term and earns 3.6% paid annually.

compound interest: _____

_____

Compare the future values of Eugene's and Francine's investments at maturity.

Simple Interest: Compound Interest:

Year	Value
1	
2	
3	
4	
5	

The formula for compound interest:

$A = P(1+i)^n$ A

P

i

n

Calculate the future values of Eugene's and Francine's investment if the term is 10 years instead.

Example 2:Determining the future value of an investment with semi-annual compounding

Gerald invested his inheritance of $20 000 in an account that earns 9.2% compounded semi-annually. The interest rate is fixed for 10 years. He plans to use the money for a sport car in 5 to 10 years.

Compare the future value of his investment after 5 years and 10 years. What if his investment were to earn simple interest instead?

Example 3: Comparing interest on investments with different compounding periods

Hanna wants to invest $3000 so that she can renovate her living room in about 3 years; she has the following investment options (annual/semi-annual/monthly/weekly/daily) at 4.8%:

Principal ($)	3000	3000	3000	3000	3000
Interest Rate	0.048	0.048	0.048	0.048	0.048
Periods / Year	1	2	12	52	365
Value at End of Year					
0	3000	3000	3000	3000	3000
1					
2					
3					

Example 4: Estimating doubling times for investments

Ivan and Jenny invested $4000 by purchasing CSBs. Ivan's earns 8% compounded annually, while Jenny's earn 9% compounded annually.

a. Estimate the doubling time for each CSB.
b. Verify the estimate by determining the doubling time for each CSB.

Assignment: read p. 29; p. 19 #1 – 3; p. 30 #3 – 11 (odds)

1.4 – COMPOUND INTEREST: PRESENT VALUE

Goal: Determine the principal or present value of an investment, given its future value and compound interest rate.

Example 1: Determining the present value of investments earning compound interest

Kenneth is 22 years old. He has inherited some money from a relative. Kenneth wants to invest some of the money to buy a home in Maple Ridge when he turns 35. He estimates that he will need about $225 000 to buy a home.

a. How much does he have to invest now, at 5.5% compounded annually?

b. What is the ratio of future value to present value for Kenneth's investment?

c. How would the ratio change if the interest rate increased to 6% but was compounded semi-annually?

Example 2: Determining the present value of an investment that is compounded quarterly

Lana and Matt are computer scientists. They researched the costs to set up a software company. They estimate that $40 000 will be enough.

They plan to set up the company in 3 years and have invested money at 9.6%, compounded quarterly, to save for it.

a. How much money should they have invested?

b. How much interest will they earn over the term of their investment?

Example 3: Determining an unknown interest rate and unknown term

Niko has invested $14 400 in a Registered Education Savings Plan (RESP). He wants his investment to grow to at least $50 000 by the time his newborn enters university, in 18 years.

a. What interest rate, compounded annually, will result in a future value of $50 000? Round your answer to two decimal places.

b. Suppose that Niko wants his $14 400 to grow to at least $60 000 at the interest rate from part a. How long will this take?

Assignment: p. 40 #1, 3, 5 – 7, 10, 11, 13

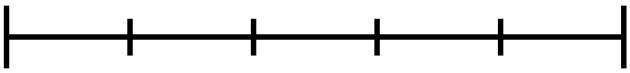
1.5 – INVESTMENT INVOLVING REGULAR PAYMENTS

Goal: Determine the future value of an investment that earns compound interest involving regular payments.

Example 1: Determining the future value of an investment involving regular deposits.

Darva is saving for a trip to Australia in 5 years. She deposits $500 into her savings account at the end of each 6-month period from what she earns as a server. The account earns 3.8%, compounded semi-annually. How much money will be in the account at the end of 5 years? How much of this money will be earned interest?

N =



I% =

PV =

PMT =

FV =

P/Y =

C/Y =

Example 2: Comparing a regular payment investment with a single payment investment.

Adam made a $200 payment at the end of each year into an investment that earned 5%, compounded annually. Blake made a single investment at 5%, compounded annually. At the end of 5 years, their future values were equal.

1. What was their future value?
2. What principal amount did Blake invest 5 years ago?
3. Who earned more interest? Why?

N =	N =
I% =	I% =
PV =	PV =
PMT =	PMT =
FV =	FV =
P/Y =	P/Y =
C/Y =	C/Y =

Example 3: Determining the interest rate of a regular payment investment.

Jeremiah deposits $750 into an investment account at the end of every 3 months. Interest is compound quarterly, the term is 3 years, and the future value is 10 059.07/ What annual rate of interest does Jeremiah's investment earn?

N =

FV =

I% =

P/Y =

PV =

C/Y =

PMT =

Example 4: Determining the regular payment amount of an investment.

Celia wants to have $300 000 in 20 years so that she can retire. Celia has found a trust account that earns a fixed rate of 10.8%, compounded annually.

a. What regular payments must Celia make at the end of each year to meet her goal of $300 000?
b. How much interest will she earn over the 20 years?

N =

FV =

I% =

P/Y =

PV =

C/Y =

PMT =

Example 5: Determining the term of a regular payment investment.

On Luis's 20th birthday, he started making regular $1000 payments into an investment account at the end of every 6 months. He wants to save for a down payment on a home. His investment earns 3.5%, compounded semi-annually. At what age will he have more than $18 000?

N =

FV =

I% =

P/Y =

PV =

C/Y =

PMT =

Assignment: p. 55 #1 – 17 (odds)

1.6 – SOLVING INVESTMENT PORTFOLIO PROBLEMS

Goal: Analyze, compare, and design investment portfolios that meet specific financial goals.

Example 1: Determining the future value and doubling time of an investment portfolio.

Phyllis started to build an investment portfolio for her retirement. She purchased a $500 Canada Savings Bond (CSB) at the end of each year for 10 years. The first five CSBs earned a fixed rate of 4.2%, compounded annually. The next five CSBs earned a fixed rate of 4.6% compounded annually. Three years ago, she also purchased a $4000 GIC that earned 6%, compounded monthly.

a. What was the value of Phyllis's portfolio 10 years after she started to invest?
b. Phyllis found a savings account that earned 4.90%, compounded semi-annually. She redeemed her portfolio and invested all the money in the savings account. About how long will it take her to double her money?

N =
I% =
PV =
PMT =
FV =
P/Y =
C/Y =

N =
I% =
PV =
PMT =
FV =
P/Y =
C/Y =

N =
I% =
PV =
PMT =
FV =
P/Y =
C/Y =

Example 2: Comparing the rates of return of two investment portfolios.

Jason and Malique are each hoping to buy a house in 10 years. They want their money to grow so they can make a substantial down payment.

Jason's portfolio:

- A 10-year $2000 GIC that earns 4.2%, compounded semi-annually

N =
I% =
PV =
PMT =
FV =
P/Y =
C/Y =

- A savings account that earns 1.8%, compounded weekly, where he saves $55 every week

N =
I% =
PV =
PMT =
FV =
P/Y =
C/Y =

- A 5-year $4000 bond that earns 3.9%, compounded quarterly, which he will reinvest in another bond at an interest rate of 4.1%

N =
I% =
PV =
PMT =
FV =
P/Y =
C/Y =

Malique's portfolio:

- A tax-free savings account (TFSA) that earns 2.2%, compounded monthly, and has a current balance of $5600

N =
I% =
PV =
PMT =
FV =
P/Y =
C/Y =

- The purchase. at the end of each year, of a 10-year $500 CSB that earns 3.6%, compounded annually

N =
I% =
PV =
PMT =
FV =
P/Y =
C/Y =

- A savings account that earns 1.6%, compounded monthly. where she saves $200 every month

N =
I% =
PV =
PMT =
FV =
P/Y =
C/Y =

Example 3: Designing and adjusting an investment portfolio to meet a financial goal.

Stan plays in a band. Next year, he wants to have enough money to buy a new guitar. The new guitar costs $1750, including taxes and shipping. Stan works part time and can afford to save $15 every week. As well, he has $300 left from his summer job. He needs an investment portfolio so that he can save money to buy the guitar in a year.

a. Why might Stan include a GIC and a high-interest savings account in his portfolio?
b. If the GIC earns 5%, compounded annually, and the savings account earns 2.9%, compounded weekly, will he have enough money in a year? If not how much does he have to save each week?

N =
I% =
PV =
PMT =
FV =
P/Y =
C/Y =

N =
I% =
PV =
PMT =
FV =
P/Y =
C/Y =

Assignment: p. 65 #2 – 4, 6, 8, 9

CHAPTER 1 REVIEW

1. Hal invested $40 000 at an interest rate of 6%, compounded annually. He wants to know how long it will take for the investment to double.

   a. Estimate the doubling time. Verify your answer.

   b. How long would it take for the investment to double if the interest was simple

2 Val has $12 000 and wants it to grow to $50 000. She has narrowed the possibilities to the following two investment options:

   a. 6% compound semi-annually

   b. 5.1% compounded quarterly

   Which option should she choose? Why?

N =	FV =	N =	FV =
I% =	P/Y =	I% =	P/Y =
PV =	C/Y =	PV =	C/Y =
PMT =		PMT =	

3. Warren started investing when he was 5 years old. He deposited $5 from his allowance at the end of every month into a savings account that earned 5.80%, compounded monthly. He did this until he was 25 years old.

   a. How much did he invest altogether? What is the current value of his investment at age 25? What is his rate of return?

N =
I% =
PV =
PMT =
FV =
P/Y =
C/Y =

b. Suppose that Warren had wanted his investment to have the same value as in part a) at age 25, but had started investing when he was 20. What would his monthly payments have been?

N =
I% =
PV =
PMT =
FV =
P/Y =
C/Y =

4. Both Alex and Jamie have an investment portfolio.

a. What is the current value of each portfolio?
b. Who has the greater rate of return? Explain.

Alex's portfolio:

- A 10-year $5000 GIC, purchased 9 years ago, that earns 2.6%, compounded annually
- A 5-year $2000 CSB, purchased 4 years ago, that earns 3. 1%, compounded semi-annually
- A savings account at 1.4%, compounded weekly. into which he has been making weekly deposits of $l5 for 5 years

Jamie's portfolio:

- A 10-year $3000 bond, purchased 9 years ago, that earns simple interest at 2.7%
- A 3-year $700 CSB. Purchased 3 years ago. that earns 2.8%, compounded semi-annually
- A high-interest savings account, at 1.7%, compounded monthly, into which she has been making monthly deposits of $100 for 6 years

N =
I% =
PV =
PMT =
FV =
P/Y =
C/Y =

N =
I% =
PV =
PMT =
FV =
P/Y =
C/Y =

N =
I% =
PV =
PMT =
FV =
P/Y =
C/Y =

Assignment: p. 71 #1, 2, 4, 6 – 8, 10 – 12