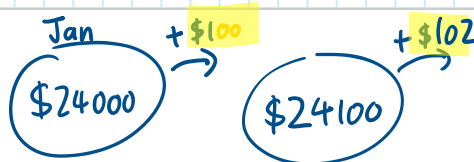


2-1 Analyzing Loans

January 11, 2021 12:41 PM



Foundations of Mathematics 12 – 2.1

2.1 – ANALYZING LOANS

Goal: Solve problems that involve single payment loans and regular payment loans.

Example 1: Solving for the term and total interest of a loan with regular payments

Lex borrowed \$24 000 at 5% interest, compounded monthly, to fund his research into a viable kryptonite weapon. Lex negotiated regular loan payments of \$400 at the end of each month until the loan is paid off.

- In which month will Lex have at least half of the loan paid off?
- How long will it take Lex to pay off the loan?
- How much interest will Lex have paid by the time he has paid off the loan?

a) halt

* $N = ?$ $37.07 \Rightarrow 38$ months

* $N = ?$ 69.2 (70 months)

I% = 5%

I% = 5%

Total Payment: $\$400 \times 69.18744 = \$27,674.98$

Load PV = \$24000

PV = \$24000

Repayment PMT = -\$400

PMT = -\$400

Interest = $\$27,674.98 - \$24,000 = \$3,674.98$

Still owes half FV = -\$12000

owes FV = \emptyset (pay off)

Total payment loan Cost of borrowing!!

P/Y = 12 (monthly)

P/Y = 12

C/Y = 12

C/Y = 12

End

Example 2: Solving for the future value of a loan with a single loan payment

After years of exposure, Lex is in need of treatment to cure himself of kryptonite poisoning.

Lex's fellow evildoers loaned him \$100 000 at a fixed interest rate of 13% compounded annually. The loan is to be repaid in a single payment on the maturity date, which is at the end of 5 years.

How much will Lex need to pay his fellow evildoers on the maturity date? What is the accumulated interest on the loan?

N = 5yr

I% = 13%

Loan PV = \$100,000

PMT = \emptyset

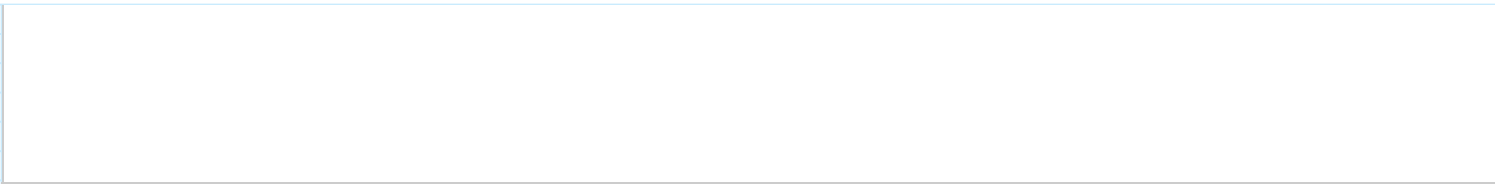
Interest: \$84,243.52

One time (Owes at the end) FV = -\$184,243.52

P/Y = 1

C/Y = 1

End



Foundations of Mathematics 12 – 2.1

Example 3: Solving for the present value and interest of a loan with a single payment

Focusing his attention instead on genetics research, Lex wants a loan to renovate his LexCorp Laboratory. His bank will charge him 3.6% interest, compounded quarterly. He already has an investment that will mature in 5 years. When his investment matures, Lex wants to use the money to repay the renovation loan with one payment. He wants the amount of the payment to be no more than \$2 000 000.

- How much can he borrow?
- How much interest will he pay?

$N = 5 \text{ yrs}$
 $I\% = 3.6\%$
 $\text{Loan}^* \text{PV} = \$1,671,886.04$
 $\text{PMT} = 0$
 $\text{FV} = -\$2,000,000$
 $\text{P/Y} = 1$
 $\text{C/Y} = 4$
 End

Interest : \$328,113.96

Example 4: Solving for the payment and interest of a loan with regular payments

Lex's genetic research has gone awry and the result is the complete destruction of his lab.

Lex is now negotiating with his bank for a mortgage on a new lab. He has been told that he needs to make a 10% down payment on the purchase price of \$2 250 000. Then the bank will offer a mortgage loan for the balance at 3.75%, compounded semi-annually, with a term of 20 years and with monthly mortgage payments.

- How much will each payment be?
- How much interest will Lex end up paying by the time he has paid off the loan, in 20 years?
- How much will he pay altogether?

$N = 20 \text{ yr} \times 12 = 240 \text{ payments}$
 $I\% = 3.75\%$
 $\text{PV} = \$2025,000$
 $\text{PMT} = ?$ \$11975.49/month
 $\text{FV} = 0 \text{ (pay off)}$
 $\text{P/Y} = 12$
 $\text{C/Y} = 2$

$\text{down payment} = 0.1 \times \$2250,000 = \$225,000$
 $\text{Loan} : \$2,250,000 - \$225,000 = \$2,025,000$
 $\text{Total Payment } \$11,975.49 \times 240 = \$2,874,117.6$
 $\text{Interest} : \$2,874,117.6 - \$2,025,000 = \$849,117.60$
 $\text{Down} \quad \text{Total monthly payment}$
 $\text{c) } \$225,000 + \$2,874,117.6 = \boxed{\$3,099,117.60}$

Foundations of Mathematics 12 – 2.1

Example 5: Relating payment and compounding frequency to interest charged

Deciding to turn to politics, Lex is running for the presidency and is in need of funds.

A minor, yet unnamed contributor has offered Lex the following two loan options for borrowing \$8000. What advice would you give?

Option A: He can borrow at 4.06% interest, compounded annually, and pay off the loan in payments of 1800.05 at the end of each year.

Option B: He can borrow at 4.06% interest, compounded weekly, and pay off the loan in payments of \$34.62 at the end of each week.

CIBC
 $*N = ?$ 4.99... 5yr
 $I\% = 4.06\%$
 (Loan) $PV = \$8000$
 Reg Payment $PMT = -\$1800.05$
 One time $\cancel{FV} = \emptyset$
 $P/Y = 1$
 $C/Y = 1$
 End
 Total payment: $\$1800.05 \times 5$ times
CIBC $\$9000.25$
 borrowed \$8000 pay back

$*N = 254.9 \rightarrow 255$ weeks
 $I\% = 4.06\%$
 $PV = \$8000$
 $PMT = -\$34.62/\text{weekly}$
 $\cancel{FV} = \emptyset$
 $P/Y = 52$
 $C/Y = 52$
 Total Payment: $\$34.62 \times 254.92987$
TD $\$8825.67$ ✓ better
 pay back less \$\$



Assignment: p. 92 #5, 7, 9, 10, 12, 14, 15, 17

2-2 Credit Card

January 11, 2021 2:05 PM

Limit = \$1000



Foundations of Mathematics 12 – 2.2

2.2 – EXPLORING CREDIT CARD USE

Goal: Compare credit options that are available to consumers.

Incentives or promotions are sometimes offered to entice people to apply for and use credit cards to make purchases. Examples of such rewards include rewards points programs, no annual fees, and cash rebates on either the first purchase (especially with store credit cards) or specific types of purchases (grocery, gas).

If there is no outstanding balance from the previous month (billing cycle) and the new balance is paid off in full by the payment due date, no interest is charged however, if there is an outstanding balance, then there would be a minimum amount that must be paid, based on a percent of the outstanding balance, and interest will be calculated based on the remaining balance.

If a credit card does NOT have an outstanding balance AND it is used for a single purchase, it can be treated as a loan. The purchase price is the principal borrowed, and regular payments can be made until the balance is paid off.

Example 1:

Jayden saw the new sound system he wanted on sale for \$2623.95, after taxes. He had to buy it on credit and has two options:

- Use his new bank credit card, which has an interest rate of 14.5%, compounded daily.
- Apply for the store credit card, which offers an immediate rebate of \$100 on the price but has an interest rate of 19.3%, compounded daily.

Jayden cannot afford to pay the balance in full. Both credit cards require a minimum monthly payment of 2.1% on the outstanding balance, but Jayden is confident that he can make regular monthly payments of \$110.

Which credit card is the better option for Jayden, and why?

RBC: $*N = ?$ 28.3412

$I = 14.5\%$

$PV = \$2623.95$

$PMT = -110$

$FV = \emptyset$

$P/Y = 12$

$C/Y = 365$

End

Total Payment: $PMT \times N$
 $\$110 \times 28.3412$

RBC $\$3117.53$

better!! (cost less)

Best buy: $N = ?$ 28.9247

$I = 19.3\%$

$PV = \$2623.95 - \100 (rebate!!)

$PMT = -\$110$

$PV = \emptyset$

$P/Y = 12$

$C/Y = 365$

End

$\$110 \times 28.9247 =$
 $\$3181.72$

Best buy

Foundations of Mathematics 12 – 2.2

Example 2:

Kevin is flying home this summer to visit his family. He has to put his plane ticket on a credit card. The ticket cost \$786.34. He can afford payments of \$150 monthly. He chooses a credit card that charges 12.9%, compounded daily, with an annual fee of \$50. How much will he pay in total for his plane ticket home?

$$* N = ? 5.7816... \quad P/Y = 12 \text{ (monthly)}$$

$$I = 12.9\% \quad C/Y = 365 \text{ (daily)}$$

$$\text{(Loan)} \quad PV = \$786.34 + \$50$$

$$PMT = -\$150$$

$$FV = \emptyset$$

End.

• We will pay off in 6 months *round for time !!*

• we paid $\$150 \times 5.7816... = \867.24 *don't round for \$\$*

$\$150 \ \$150 \ \$150 \ \$150 \ \$150 \ \117.24
 $\underline{\hspace{10em}} \ \750

Assignment: p. 100 #1ab, 2, 4a

2-3 Problem involving Credit

January 13, 2021 12:43 PM

Foundations of Mathematics 12 – 2.3

2.3 – SOLVING PROBLEMS INVOLVING CREDIT

Goal: Solve problems that involve credit.

Example 1: Solving a credit problem that involves overall cost and number of payments.

Meryl and Kyle are buying furniture worth \$1075 on credit. They can make monthly payments of \$75 and have two credit options. Which option should they choose? Explain.

Option A: **IKEA** The furniture store credit card, which is offering a \$100 rebate off the purchase price and an interest rate of 18.7% compounded daily.

Option B: **RBC** A new bank credit card, which has an interest rate of 15.4%, compounded daily, but no interest for the first year.

IKEA

$$N = 14.6542$$

$$I\% = 18.7\%$$

$$PV = \$1075 - \$100$$

$$PMT = -\$75$$

$$FV = \emptyset$$

$$P/Y = 12$$

$$C/Y = 365$$

\$1099.07

RBC

$$*N = 2.3845$$

$$I = 15.4\%$$

$$PV = \$1075 - \$900$$

$$PMT = -\$75$$

$$FV = \emptyset$$

$$P/Y = 12$$

$$C/Y = 365$$

1 Year

$$\$75 \times 12 = \$900$$

\therefore We pay back \$900 before Interest

$$\$75 \times 2.3845 = \$178.84$$

$$\text{Total} = \$900 + \$178.84$$

\hookrightarrow **\$1078.84**

Cheaper!!!

Example 2: Solving a credit problem that involves payment amount and overall cost.

Ed wants to buy a car and needs to use credit to finance it. The cost, with taxes and shipping, is \$24 738. Ed wants to repay his loan in 4 years using monthly payments and has two credit options:

- His secured line of credit at 1.7%, compounded monthly, above the Bank of Canada rate, which is currently ~~5.95%~~ **5.95%** $1.7\% + 5.95\% = 7.65\%$
 - The dealership's financing plan at ~~5.22%~~ **5.22%**, compounded daily
- a. Which option should he choose? Why?
- b. Suppose that the Bank of Canada rate changed to 1.1% after 2 years. How would this affect his line of credit payments if he still wanted to pay off the loan in 4 years?

Line of Credit

$$N = 4\text{yr} \times 12 = 48$$

$$I\% = 7.65\%$$

$$PV = \$24738$$

$$*PMT = \$599.87 \text{ monthly.}$$

$$FV = \emptyset$$

$$P/Y = 12$$

$$C/Y = 12$$

End.

Honda

$$N = 48$$

$$I\% = 5.22$$

$$PV = \$24738$$

$$*PMT = \$572.29$$

$$FV = \emptyset$$

$$P/Y = 12$$

$$C/Y = 365$$

\uparrow
Better!!

Foundations of Mathematics 12 – 2.3

$\$1500 - \4000

Example 3: Solving a problem that involves interest amount and rate

Jon's ~~\$\$\$~~ $\$2000$ car insurance payment is due. He does not have enough cash to make the payment, so he is considering these two credit options: **Total paid = $\$2400$** $\$400$

- Borrow the money from a **payday loan company** for a ~~$\$100$~~ fee if it is paid back in full within 2 months.
- Get a cash advance on his credit card, which is carrying a zero balance. The interest charged for cash advances is 19.99% , compounded **daily**, and **takes effect immediately**. He can afford to pay the required $\$5$ minimum payment after the first month and then plans to pay off the balance in full at the end of the second month.

- Which is the better option for Jon? Explain.
- What annual interest rate would equate to the fee charged by the payday loan company?

Cash Advance

One time \swarrow 2 months \searrow Year. \swarrow

$N = \frac{2}{12}$ yr.

debt. **better**

$*FV = \$2067.74$

$I\% = 19.99$ $P/Y = 1$

$PV = 2000$ $C/Y = 365$

$PMT = \emptyset$

Payday load

$N = (\frac{2}{12})$ yr.

$*I\% = 114.5\%$

$PV = 2000$

$PMT = \emptyset$

$FV = -2400$

$P/Y = 1$

$C/Y = 12$

Example 4: Solving a **debt consolidation** problem that involves an interest amount

Nicki wants to be debt-free in 5 years. She has two credit cards on which she makes monthly payments:

- Card A has a balance of $\$2436.98$ and an interest rate of 18.5% , compounded daily.
- Card B has a balance of $\$3043.26$ and an interest rate of 19% , compounded daily.

Nicki has a line of credit at her bank with an interest rate of 9.6% , compounded monthly, and a credit limit of $\$6000$. How much interest will she save if she uses her line of credit to pay off both credit cards?

	<u>Card A</u>	<u>Card B</u>	<u>Bank</u>
N	$5 \times 12 = 60$	60	$N = 60$
I%	18.5%	19%	$I = 9.6\%$
PV	2436.98	3043.26	$PV = \$5480.24$
PMT	$*62.73$	$*79.19$	$PMT = \$115.36$
FV	\emptyset	\emptyset	$FV = \emptyset$
P/Y	12	12	$P/Y = 12$
C/Y	365	365	$C/Y = 12$
	$\$62.73 \times 60$	$\$79.19 \times 60$	Consolidation $\$115.36 \times 60 = \6921.60
	$\$3763.80$	$\$4751.4$	\rightarrow save \approx $\$1593.60$
	$\$3763.80 + \$4751.4 = \$8515.20$		

Assignment: p. 115 #4, 6ab, 7 – 10, 12, 13

Ch 2.
Assignment Q 1-3 (now)

Q 4-5 (next class)

Foundations of Mathematics 12 – 2.4

2.4 – BUY, RENT, OR LEASE?

Goal: Solve problems by analyzing renting, leasing, and buying options.

- Lease: A contract for purchasing the use of property, such as a building or vehicle, from another, the leaser, for a specified period.
- Equity: The difference between the value of an item and the amount still owing on it; it can be thought of as the portion owned.
- Asset: An item or a portion of an item owned; also known as property.

- Appreciation: Increase in the value of an asset over time. $\$ \uparrow$
- Depreciation: Decrease in the value of an asset over time. $\$ \downarrow$

Rent : fixed \$ - short term ^(daily) (month to month)
 Buy : - cash → one-time
 - finance → Regular Payment.
 Lease : longer term rent (for car: with option to buy out at the end)

Example 1: Solving a problem that involve leasing, buying, or renting a vehicle

Mande needs a vehicle for work, on average, 2 days each month. She has three options:

- She could lease a vehicle, which requires a down payment of \$4000 and lease payments of \$380 per month plus tax. She would need insurance at \$1220 each year (which could be paid monthly) and would have to pay for repairs and some maintenance, which would average \$50 each month. For the 4-year lease she is looking at, she would have no equity in the vehicle at the end of the term.
- She could buy a vehicle for \$32 800 and finance it for 4-year term at 4.5% interest, compounded monthly. She would have the same insurance, repairs, and maintenance costs that she would have with leasing. However, the equity of the vehicle would be considered an asset.
- She could continue to rent at \$49.99 per day, plus tax, with unlimited kilometres.

Buy
 $N = 4 \times 12 = 48$
 $I\% = 4.5$
 $PV = 32800$
 $*PMT = -\$747.95$
 $FV = \emptyset$
 $P/Y = 12$
 $C/Y = 12$
 End

$$\frac{\$747.95 + \$50 + \frac{\$1220}{12}}{12}$$

Buy = \$899.62/month
 have a car !!

leasing: $\$380/\text{mon} \times 12 \times 4\text{yr} = \$18,240$

down payment: \$4000

insurance: $\$1220 \times 4\text{yr} = \4880

maintenance: $\$50 \times 12 \times 4 = \2400

total: $\$29,520$ per month = $\$615/\text{month}$
 $\hookrightarrow \div 4\text{yr} \div 12\text{months}$

renting: $\$49.99 \times 12\text{days/month} = \599.88

Foundations of Mathematics 12 – 2.4

Example 2: Solving a problem that involve leasing or buying a water heater

The 10-year-old hot water heater in Tom's home stopped working, so he needs a new one. Tom works for minimum wage. After paying his monthly expenses, he has \$35 disposable income left. He has an unused credit card that charges 18.7%, compounded daily. He has two options:

- Tom could lease from his utility company for \$17.25 per month. This would include parts and service.
- He could buy a water heater for \$712.99, plus an installation fee of \$250.

What do you recommend? What if the water heater will last for 8 years?

* $N = 36.3074$
 $I\% = 18.7\%$
 $PV = \$712.99 + \250
 $PMT = -\$35$
 $FV = \emptyset$
 $P/Y = 12$
 $C/Y = 365$
 End

Total $\$35 \times 36.3074$
 $\$1270.76$
 buy.
 need to pay for repair.

leasing: $\$17.25 \times 12 \text{ month} \times 8 \text{ yr}$
 $\$1656$ for 8 years.
 ↳ includes maintenance.

dift. $\sim \$400$
 If the repair cost exceeds the difference, then leasing is better.

Example 3: Solving a problem that involves leasing or buying office space

Lance started his own construction business 2 years ago. His business has grown quickly, and his home office is no longer big enough. He is considering these two options:

- He could sign a 3-year lease on office space, with monthly rental payments of \$2000, and a refundable damage deposit of \$2000, but there is a penalty for breaking the lease.
 - He could purchase a house for \$2 850 000 and renovate so it could be used as an office. A 5% down payment would be required, and he would take out a 15-year mortgage at 5%, compounded semi-annually, with monthly payments. Assume appreciation of 2% year.
- Lease: $\$2000 \times 12 \times 3 \text{ yr} = \$72,000$

What do you recommend for Lance?

Buying: $5\% \text{ down} = 0.05 \times \$2850000 = \$142,500$
 $PV(\text{loan}) = \$2850000 - \$142,500 = \$2,707,500$

$N = 15 \text{ yr} \times 12 = 180$
 $I\% = 5\%$
 $PV = 2,707,500$
 * $PMT = -21338.45$
 $FV = \emptyset$
 $P/Y = 12$
 $C/Y = 2$

Total cost: $\$21338.45 \times 180 + \$142,500$
 $= \$3,983,421$

Value of Office: $A = P(1+i)^n$
 $A = 2850000(1+0.02)^{15} = \$3,835,724.76$

Actual Cost: $\$3,983,421$
 $- \$3,835,724.76$
 $\$147,696.24$

Assignment: p. 130 #4 – 8, 10 – 12

Ch 2 Review

January 19, 2021 12:52 PM
 Foundations of Mathematics 12 – Chapter 2 Review

CHAPTER 2 REVIEW

Example 1:

A year ago, Ryan borrowed \$5000 to pay for a new roof on his garage. He borrowed at 7.25% compounded quarterly, for 2 years. He will repay the loan in a single payment at the end of the term.

- a. What amount will be due at the maturity date? How much of this amount is interest?
- b. Suppose that the term, interest rate, and compounding were the same, but Ryan decided to make regular monthly payments over the 2 years instead. How much would he end up paying altogether? How much of this would be interest?

$N = 2 \text{ yr}$
 $I\% = 7.25$
 $PV = \$5000$
 $PMT = \emptyset \leftarrow \text{single payment only}$
 $* FV = \$5772.70$
 $P/Y = 1$
 $C/Y = 4$
 End

Interest : $FV - PV = \$772.70$

One time (FV)

$N = 2 \text{ yr} \times 12 = 24$
 $I\% = 7.25\%$
 $PV = \$5000$
 $* PMT = -224.43 \leftarrow \text{monthly payment}$
 $FV = \emptyset$
 $P/Y = 12$
 $C/Y = 4$

Total Paid $\rightarrow \$224.43 \times 24 = \5386.36

Interest = $5386.36 - 5000$
 $Int = \$386.36$

Example 2:

Allison's grandmother is coming to live with her. Allison needs to renovate her home to make it wheelchair-accessible. Allison has signed a promissory note for a loan from a cousin at 5.5%, compounded monthly. She will repay the loan with a single payment in 4 years.

If she wants to repay no more than \$15 000, what is the most she can borrow?

$N = 4 \text{ yr}$
 $I\% = 5.5\%$ $\leftarrow \text{we can borrow !!}$
 $(\text{Loan}) PV = ? \quad \12043.83
 $\text{Reg Payment } PMT = \emptyset$
 $\text{One repayment } FV = -15000$
 $P/Y = 1$
 $C/Y = 12$
 End.

(FV)
 FV
 $\text{Loan } (PV) = ?$

Canadian Prime rate !! 7.2%
 BOC

Foundations of Mathematics 12 – Chapter 2 Review

Example 3:

Christian wants to upgrade his backpacking equipment to prepare for a summer of trekking. The cost of the new equipment is \$1265, and he has two options to pay for it:

- A new credit card with an introductory offer of 3% off the first purchase and an interest rate of 14.5%, compounded daily
 - A line of credit, with an interest rate of 6.8%, compounded daily
- a. Christian intends to pay off what he borrows in 1 year, with monthly payments. How much would his payments be for each option?
- b. Should he use the new credit card or his line of credit? Explain.

C.C.
 $N = 1 \text{ yr} \times 12 = 12$
 $I\% = 14.5\% \quad 1 - 0.03$
 (Loan) $PV = \$1265 \times (0.97) = 1227.05$
 $* \text{PMT} = ? - \$110.51$
 $FV = \emptyset$
 $P/Y = 12$
 $C/Y = 365$
End

Line of Credit
 $N = 12 \text{ months}$
 $I\% = 6.8\%$
 $PV = \$1265$
 $* \text{PMT} = -\$109.35$
 $FV = \emptyset$
 $P/Y = 12$
 $C/Y = 365$
End

↑ pick LOC b/c the monthly payment is lower.

Example 4:

Erin, an avid sailor, is spending two months at her parents' cottage near Lac La Biche, Alberta. She wants to sail every day.

- Rent.** At the marina, she can rent a Laser sailboat for \$60 per day or \$375 per week.
- The marina is selling a used Laser for \$2200. Erin can arrange a 6-month bank loan at 5.2%, compounded monthly, with monthly loan payments. Lasers depreciate at 25% each year.

Recommend whether Erin should rent daily, rent weekly, or buy used.

Buy
 unit → $N =$
 $I\% = 5.2\%$
 $PV = \$2200$
 $* \text{PMT} = -372.25$
 $FV = \emptyset$
 $P/Y =$
 $C/Y = 12$

$> \text{Total paid} = \$372.25 \times 6 = 2233.50$
 $\text{resale value} \Rightarrow A = P(1-i)^n = 2200(1-0.25)^1 = \1650
 $\text{actual cost} \Rightarrow \$2233.50 - \$1650 = \583.50
 so much cheaper !!

$\text{Rent (daily)} = \$60 \times 61 \text{ days} = \3660

$\text{Rent (weekly)} = \$375 \times 8 \text{ weeks} = \3000

Assignment: p. 136 #1 – 7