5.1 – EXPLORING PROBABILITY

Goal: Use probability to make predictions.

Trial

A trial is any operation whose outcome cannot be predicted with certainty. e.g. a coin is tossed, a die is rolled.

Experiment

An experiment consists of one or more trials. e.g. a coin is tossed, two dice are rolled, a coin is tossed and a die is rolled.

Outcome

An outcome is the result of the carrying out an experiment. e.g. H, 6 and 4, T and 4.

Sample Space

The sample space (S) of an experiment is the set of all possible outcomes. e.g. {H,T}, {(1,1),(1,2), ... ,(6,6)}, {(H,1),(H,2), ... ,(T,5),(T,6)}

Event

An event is a subset of the sample space. It consists of one or more of the possible outcomes of an experiment. . e.g. {H}, {(1,4)}, {(T,2)}

Probability

To denote the probability of an event A, we write $P(A)$, which is read as "the probability of A ". Probabilities are usually represented by decimals or fractions between 0 and 1, but sometimes percentages are used. For any event A, $0 < P(A) \leq 1$.

If the event X does not include any of the outcomes in the sample space, then the event X is impossible and we write $P(X) = 0$. e.g. P(rolling a 9 on a standard die) = 0.

If the event Y includes all the outcomes in the sample space, then the event Y is certain and $P(Y) = 1$. e.g. P(rolling a natural number less than 7 on a standard die) = 1

Complimentary Events

The probability of an event and the probability of the complement will always add to 1 (100%). $P(A) + P(A') = 1$

The *theoretical probability* of an event A is determined from the formula:

$$P(A) = \frac{\text{\# of outcomes favourable to } A}{\text{total \# of outcomes in the sample space}} = \frac{n(A)}{n(S)}$$

Example 1:

Four black counters and five white counters are placed in a bag. One of the counters is selected at random.

a) State the probability that the counter selected is black (B).

$P(B) = \frac{4}{4+5} = \frac{4}{9}$

b) State the probability that the counter selected is not black (B').

$P(B') = P(W) = \frac{5}{9}$ $\qquad P(B) + P(B') = \frac{4}{9} + \frac{5}{9} = 1$

The *experimental probability* of an event A consisting of multiple trials is determined from the formula:

$$P(A) = \frac{\text{\# of times event } A \text{ occurs}}{\text{total \# of trials}} = \frac{n(A)}{n(T)}$$

Example 2:

Simulate rolling a die ~~10~~ 12 times and recording the number of times a 5 appears. From your simulation, what is the experimental probability of getting a 5?

$P(5) = \frac{1}{6}$ ← theoretical $\qquad P(5) = \frac{3}{12} = \frac{1}{4}$ ← from one experiment

Example 3: test question!

Sasha and Mika have to invent a **fair game** for a class project. Sasha suggests this game:

- Two people play. Each player has a container.
- Both players put three identical slips of paper, numbered 1, 2, and 3, into their own container.
- For each turn, both players draw one slip of paper from their container.
    - Player 1 scores a point if the product of the two numbers drawn is less than their sum.
    - Player 2 scores a point if the product of the two numbers drawn is greater than their sum.
- Neither player gets a point if the product and sum are equal.
- After each turn, the players return their slip of paper to their container.
- A game consists of 10 turns.

Is Sasha's game a fair game?

player 1

player 2

2 \ 1	1	2	3
1	2 ✓ 1	3 ✓ 2	4 ✓ 3
2	3 ✓ 2	4 ✓ 4	5 ✓ 6
3	4 ✓ 3	5 ✓ 6	6 ✓ 9

+ sum
× product

$P(\text{player 1 wins}) = \frac{5}{9}$

$P(\text{player 2 wins}) = \frac{3}{9} = \frac{1}{3}$ not a fair game

$P(\text{tie}) = \frac{1}{9}$

Assignment: p. 303 #1 – 4

5.2 – PROBABILITY AND ODDS

Goal: Understand and interpret odds, and relate them to probability.

Odds in Favour
The ratio of the probability that an event will occur to the probability that the event will not occur, or the ratio of the number of favourable outcomes to the number of unfavourable outcomes.

The odds in favour of event A occurring are given by the ratio $\frac{P(A)}{P(A')}$ or $P(A):P(A')$.

Odds Against
The ratio of the probability that an event will not occur to the probability that the event will occur, or the ratio of the number of unfavourable outcomes to the number of favourable outcomes.

The odds against event A occurring are given by the ratio $\frac{P(A')}{P(A)}$ or $P(A'):P(A)$.

Determine the Odds Using Sets

Example 1:

Bailey holds all the cards from a standard deck of 52 playing cards. He asks Morgan to choose a single card without looking.

a) Determine the odds in favour of Morgan choosing a face card.

12 face cards : 40 not-face cards

less likely

more likely

12:40 → 3:10 odds in favours

10:3 odds against

b) Determine the odds against Morgan drawing a face card.

40:12 → 10:3

$P(\text{face}) = \frac{12}{52} = \frac{3}{13}$ probability

$P(A')$ is the probability of the complement of A, where $P(A') = 1 - P(A)$.

Example 2:

Research shows that the probability of an expectant mother, selected at random, having twins is 1/32.

a) What are the odds in favour of an expectant mother having twins?

$P(\text{twins}) = \frac{1}{32}$

1:31 odds in favour of twins

$P(\text{twins}') = 1 - \frac{1}{32} = \frac{31}{32}$ twins' = not twins

b) What are the odds against an expectant mother having twins?

31:1 odds against twins

If the odds in favour of event A occurring are $m : n$, then $P(A) = \dfrac{m}{m+n}$.

Determine the Probability from Odds

Example 3:

A computer randomly selects a university student's name from the university database to award a $100 gift certificate for the bookstore. The odds against the selected student being male are 57 : 43. Determine the probability that the randomly selected university student will be male.

43 : 57 odds in favour

$P(\text{male}) = 43/(43+57)$

$= 43/100 = 43\%$

Make a Decision Based On Odds and Probability

Example 4:

A hockey game has ended in a tie after a 5 min overtime period, so the winner will be decided by a shootout. The coach must decide whether Ellen or Brittany should go first in the shootout. The coach would prefer to use her best scorer first, so she will base her decision on the players' shootout records. Who should go first?

Player	Attempts	Goals Scored
Ellen	13	8
Brittany	17	10

Ellen: $8/13 = 0.6153\ldots$

Brittany: $10/17 = 0.5882\ldots$

odds in favour:

$8:5 \xrightarrow{\times 7} 56:35$

$10:7 \xrightarrow{\times 5} 50:35$

Ellen should go first.

Ellen should go first.

Interpret Odds Against and Make a Decision

Example 5:

A group of Grade 12 students are holding a charity carnival to support a local animal shelter. The students have created a dice game that they call Bim and a card game that they call Zap. The odds against winning Bim are 5 : 2, and the odds against winning Zap are 7 : 3. Which game should Madison play?

Bim odds for winning: 2 : 5

$P(\text{win}) = 2/(2+5)$

$\doteq 0.2857$

Zap odds for winning: 3 : 7

$P(\text{Win}) = 3/(3+7)$

$= 0.3$

Play Zap!

Assignment: p. 310 #5 – 17 (odds)

5.3 – PROBABILITIES USING COUNTING METHODS

Goal: Solve probability problems that involve counting techniques.

You may be able to use the Fundamental Counting Principle and techniques involving permutations and combinations to solve probability problems with many possible outcomes. The context of each particular problem will determine which counting techniques you will use.

Use permutations when order is important in the outcomes. Use combinations when order is not important in the outcomes.

$$P(A) = \frac{\text{\# of outcomes favourable to } A}{\text{total \# of outcomes in the sample space}} = \frac{n(A)}{n(S)}$$

Solve a Probability Problem Using Tree Diagrams and Counting Techniques

Example 1:

Two cards are drawn without replacement from a well shuffled deck of 52 cards. Calculate the probability that the two cards drawn are both aces.

4 A's

$$P(2A\text{'s}) = \frac{{}_{4}C_{2} \leftarrow \text{Choose 2}}{{}_{52}C_{2} \leftarrow \text{Choose 2}} = \frac{6}{1326} \doteq 0.45\%$$

52 cards

Example 2:

A bag of marbles contains 5 red, 3 green and 6 blue marbles. If a child grabs 3 marbles from the bag, determine the probability that exactly 2 are blue.

Choose 2 Blue

Choose 1 Not Blue

$$P(2\,\text{Blue}) = \frac{{}_{6}C_{2} \times {}_{8}C_{1}}{{}_{14}C_{3}} = \frac{15 \times 8}{364} = {}^{120}/_{364} \doteq 32.97\%$$

3 out of 14 total

Try:

Two seeds are chosen from a packet containing 10 seeds; 3 that will produce red flowers, 4 that will produce white flowers and 3 that will produce blue flowers. What is the probability that both seeds produce red flowers?

$$P(2\,\text{red}) = {}_{3}C_{2} / {}_{10}C_{2} = 3/45 = 1/15 \doteq 6.67\%$$

Example 3:

Euchre is played with a deck of 24 cards that is similar to a standard deck of 52 playing cards, but with only the ace (A), 9, 10, jack (J), queen (Q), king (K) for all four suits. Each player is dealt five cards.

Determine the probability that a dealt hand will contain the following:

a. A, K, Q, J of the same suit

$$P(A,K,Q,J \text{ same suit}) = \frac{\overset{\text{4 suits}}{{}_4C_1} \times \overset{A}{{}_1C_1} \times \overset{K}{{}_1C_1} \times \overset{Q}{{}_1C_1} \times \overset{J}{{}_1C_1} \times \overset{\text{anyone of the 20 cards left.}}{{}_{20}C_1}}{{}_{24}C_5} = \frac{80}{42504} \doteq 0.188\%$$

b. Five cards of the same colour

$$P(\text{same colour}) = \frac{\overset{\text{2 colours}}{{}_2C_1} \times \overset{\text{12 cards of the same colour}}{{}_{12}C_5}}{{}_{24}C_5} = \frac{2 \times 792}{42504} = \frac{1584}{42504} \doteq 3.73\%$$

c. Four of a kind (that is, four cards of the same rank)

$$P(\text{4 of a kind}) = \frac{\overset{\text{6 ranks}}{{}_6C_1} \times \overset{\text{4 cards}}{{}_4C_4} \times \overset{\text{1 leftover out of 20}}{{}_{20}C_1}}{{}_{24}C_5} = 120/42504 \doteq 0.282\%$$

Example 4:

Three prizes are awarded in a raffle during a halftime show at a school basketball game. Ben, Janelle, James and 17 other students each have one ticket.

a. If the raffle has a first, second, and third prize, determine the probability, as an exact value, that Ben wins first prize, Janelle wins second price, and James wins the third prize.

$$P(A) = \frac{\overset{\text{1st}}{{}_1C_1} \times \overset{\text{2nd}}{{}_1C_1} \times \overset{\text{3rd}}{{}_1C_1}}{\underset{\text{order matter}}{{}_{20}P_3}} = 1/6840$$

b. If the raffle has three identical prizes, determine the probability that Ben, Janelle, and James win the prizes.

$$P(B) = \frac{\overset{\text{Ben, Janelle, James}}{{}_3C_3}}{\underset{\text{order does not matter}}{{}_{20}C_3}} = 1/1140$$

Try:

Three people are randomly chosen from a fellowship of 14 people to be president, treasurer, and secretary. What is the probability that Frank, Sam and Aaron will be ones chosen?

$$P(C) = \frac{{}_3P_3}{{}_{14}P_3} = \frac{6}{2184} \doteq 0.275\%$$

Solve a Probability Problem Using Fundamental Counting Principle

Example 5:

Access to a particular online game is password protected. Every player must create a password that consists of two capital letters followed by three digits. If repetitions are allowed in a password, determine the probability that a password chosen at random will contain the letters S and Q.

SQ or QS ($_2P_2$)

$$P(A) = \frac{2! \times \cancel{10^3}}{26^2 \times \cancel{10^3}}$$

($_{26}C_1 \times {}_{26}C_1$)

$$= \frac{2}{676} = 0.296\%$$

Try:

A bank card personal identification number consists of any four digits. Repeat digits are allowed and code can start with zero. What is the probability that a code begins and ends with the digits 5?

choose 5

$$P(B) = \frac{1 \times 10 \times 10 \times 1}{10 \times 10 \times 10 \times 10} = \frac{100}{10000} = 1\%$$

Example 6:

A computer has rearranged all the letters in the word SURREY into every possible order. The computer picks one of these arrangements at random. What is the probability that the chosen arrangement begins with two Rs?

$$P(RR\_\_\_\_) = \frac{1 \times 4!}{\left(\frac{6!}{2!}\right)} = \frac{24}{360} \doteq 6.67\%$$

S, U, E, Y

2R's repeat

Try:

Beau hosts a morning radio show in Saskatoon. To advertise his show, he is holding a contest at a local mall. He spells out SASKATCHEWAN with letter tiles. Then he turns the tiles face down and mixes them up. He asks Sally to arrange the tiles in a row and turn them face up. If the row of tiles spells SASKATCHEWAN, Sally will win a new car. Determine the probability that Sally will win the car.

12 letters

2 s

3 a

$$P(\text{win}) = \frac{1}{\left(\frac{12!}{2!3!}\right)} = \frac{1}{39916800}$$

Assignment: p. 321 #5 – 17 (odds)

5.4 – MUTUALLY EXCLUSIVE EVENTS

Goal: Understand and solve problems that involve mutually exclusive and non-mutually exclusive events.

In any experiment, if the events A and B have no common outcomes. The events A and B are called mutually exclusive. If the events A and B have common outcomes, then the events A and B are not mutually exclusive.

Distinguish Mutually Exclusive and Non-Mutually Exclusive Events

Example 1: In each case, state whether the events A and B are mutually exclusive or not.

a. Experiment – a die is rolled

Event A – an even number is rolled

Event B – an odd number is rolled

mutually exclusive

b. Experiment – a card is drawn from a standard deck

Event A – a face card is selected

Event B – a heart is selected

J, Q, K of hearts

not mutually exclusive

c. Experiment – two dice are thrown

Event A – the dice both show the same value

Event B – the total is 11

mutually exclusive

Try: Consider the four events shown below involving randomly drawing a card from a standard deck of 52 cards. Which of these events are mutually exclusive?

F: the card is a face card

K: the card is a King

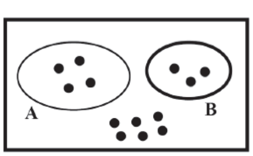
S: the card is a spade

H: the card is a heart

S and H are mutually exclusive

Analyze Mutually Exclusive Events

Example 2: If each of the 13 outcomes in the sample space is equally likely, find the following by counting the outcomes.



A and B are mutually exclusive

$P(A) = 4/13$

$P(B) = 3/13$

union - or

$P(A \cup B) = (4+3)/13 = 7/13$

$P(A \cap B) = 0$

intersection - and

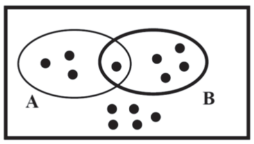
What is the relationship between $P(A)$, $P(B)$ and $P(A \cup B)$?

$\frac{4}{13} + \frac{3}{13} = \frac{7}{13} \rightarrow P(A) + P(B) = P(A \cup B)$

only for mutually exclusive events

Analyze Non-Mutually Exclusive Events

Example 3: *If each of the 13 outcomes in the sample space is equally likely, find the following by counting the outcomes.*



$P(A) = \frac{4}{13}$

$P(B) = \frac{5}{13}$

$P(A \cup B) = \frac{8}{13}$

$P(A \cap B) = \frac{1}{13}$

What is the relationship between $P(A)$, $P(B)$ and $P(A \cup B)$?

$P(A) + P(B) - P(A \cap B) = P(A \cup B)$

You can represent the favourable outcomes of two mutually exclusive events, *A* and *B*, as two disjoint sets.

You can represent the favourable outcomes of two non-mutually exclusive events, *A* and *B*, as two intersecting sets.

Represent the probability that either *A* or *B* will occur by: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Determine the Probability of Events that Are Mutually Exclusive

Example 4: Jane and Victor are playing a board game. To move on this turn, Jane must roll either doubles or a sum of 7 with the two standard dice. What is the probability that Jane will move on this turn?

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

$P(\text{doubles}) = \frac{6}{36} = \frac{1}{6}$

$P(7\text{'s}) = \frac{6}{36} = \frac{1}{6}$

$P(\text{doubles} \cup 7\text{'s}) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$

Determine the Probability of Events that Are Not Mutually Exclusive

Example 5: Jane and Victor are playing a board game. According to a different rule, if a player rolls a sum that is greater than 8 or a multiple of 5, the player gets a bonus of 100 points. Determine the probability that Victor will receive a bonus of 100 points on his next roll.

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

$P(>8) = \frac{10}{36}$

$P(5x) = \frac{7}{36}$

$P(>8 \cap 5x) = \frac{3}{36}$

$P(>8 \cup 5x) = P(>8) + P(5x) - P(>8 \cap 5x) = \frac{10}{36} + \frac{7}{36} - \frac{3}{36} = \frac{14}{36} = \frac{7}{18}$

Example 6: A card is drawn from a standard deck. Find the probability of getting:

a. a face card or a club

$$\underbrace{\tfrac{12}{52}}_{\text{faces}} + \underbrace{\tfrac{13}{52}}_{\text{clubs}} - \underbrace{\tfrac{3}{52}}_{\text{club faces}} = \tfrac{22}{52} = \tfrac{11}{26}$$

b. an eight or a spade

$$\tfrac{4}{52} + \tfrac{13}{52} - \tfrac{1}{52} = \tfrac{16}{52} = \tfrac{4}{13}$$

c. a black card or a diamond

$$\tfrac{26}{52} + \tfrac{13}{52} = \tfrac{39}{52} = \tfrac{3}{4}$$

d. a Jack or a red card

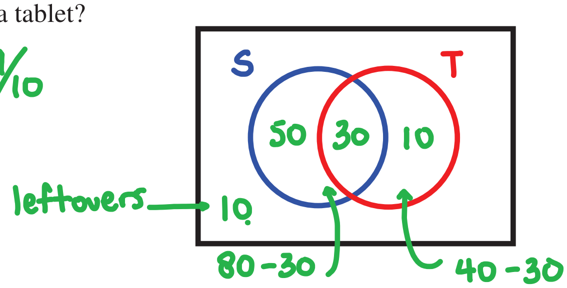
$\tfrac{7}{13}$, but how?

Determine a Venn Diagram to Solve a Probability Problem that Involves Two Events

Example 7: 100 people in a community participated in a technology survey. It was found that 80 people have a smartphone, 40 people have a tablet, and 30 people have both a smartphone and a tablet. If one of these people is chosen at random, what is the probability that the person has

a. Smartphone or a tablet?

$$\tfrac{90}{100} = \tfrac{9}{10}$$



b. neither?

$$\tfrac{10}{100} = \tfrac{1}{10}$$

Assignment: p. 339 #5 – 15 (odds)

5.5 – DEPENDENT EVENTS AND CONDITIONAL PROBABILITY

Goal: Understand and solve problems that involve dependent events and conditional probability.

Dependent Events

Events whose outcomes are affected by each other; for example, if two cards are drawn from a deck without replacement, the outcome of the second event depends on the outcome of the first event (the first card drawn).

Distinguish Independent and Dependent Events

Example 1: Classify the following events as dependent or independent.

a. The experiment is choosing two bills without replacement from a purse containing four \$5 bills, five \$10 bills and three \$20 bills. The first event is that the first bill is \$5 and the second event is that the second bill is \$5.

dependent events

b. The experiment is rolling a die and rolling the die again. The first event is that the number on the first roll is a six and the second event is that the number on the second roll is two.

independent events

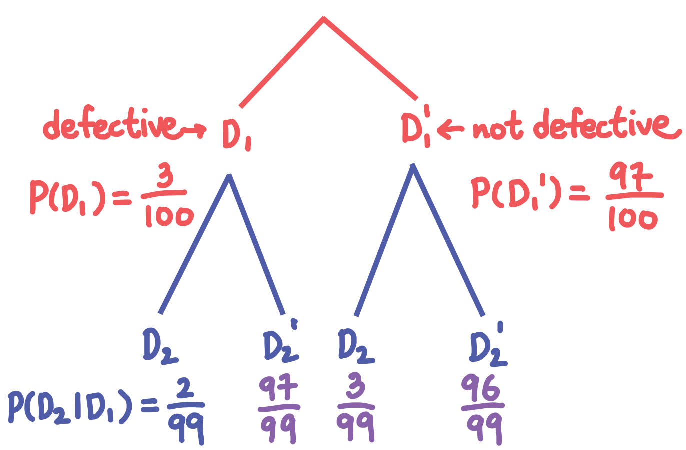
probability of B given A has occured

Dependent Events

$P(B \mid A)$ is the notation for a conditional probability. It is read "the probability that event B will occur, given that event A has already occurred".

Calculate the Probability of Two Dependent Events

Example 2: A computer manufacturer knows that, in a box of 100 chips, 3 will be defective. Joyce will draw 2 chips, at random, from a box of 100 chips. Determine the probability that Joyce will draw 2 defective chips.



and

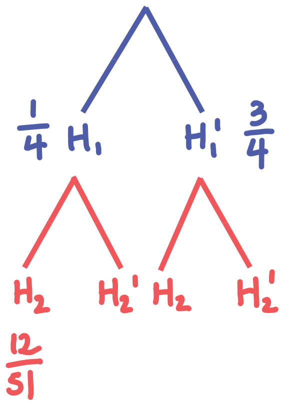
$$P(A \cap B) = P(A) \times P(B \mid A)$$

$$P(D_1 \cap D_2) = P(D_1) \times P(D_2 \mid D_1)$$

$$= \frac{3}{100} \times \frac{2}{99} = \frac{6}{9900} = \frac{1}{1650}$$

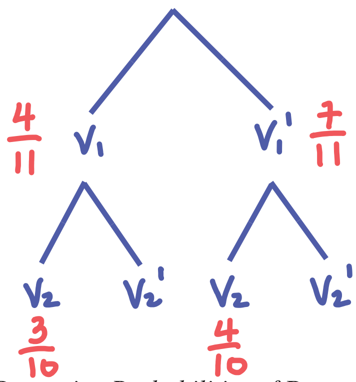
$$\doteq 0.06\%$$

Example 3: Two cards are drawn from a well-shuffled of 52 cards. What is the probability that the first card is a heart and the second card is a heart if the experiment is carried out without replacement?



$$P(H_1 \cap H_2) = P(H_1) \times P(H_2 | H_1)$$
$$= \frac{1}{4} \times \frac{12}{51}$$
$$= \frac{1}{17} \doteq 5.88\%$$

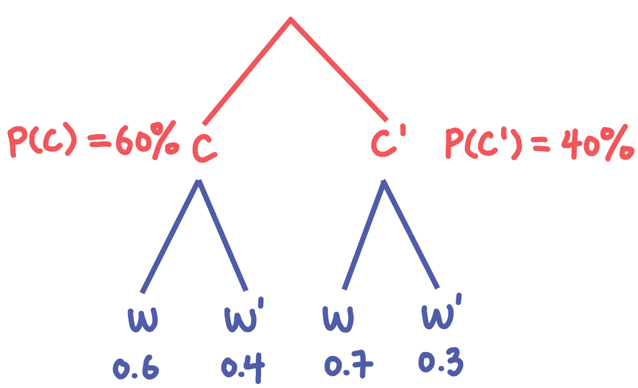
Example 4: Each of the 11 letters from the word MATHEMATICS is placed on a separate card. A card is drawn and not replaced. A second card is drawn. What is the probability that the ~~2 cards chosen are both vowels~~? 2^{nd} card is a vowel?



$$P(V_2) = P(V_2 \cap V_1) + P(V_2 \cap V_1')$$
$$= P(V_2 | V_1) \times P(V_1) + P(V_2 | V_1') \times P(V_1')$$
$$= \frac{3}{10} \times \frac{4}{11} + \frac{4}{10} \times \frac{7}{11}$$
$$= \frac{12}{110} + \frac{28}{110} = \frac{40}{110} = \frac{4}{11} \doteq 36.\overline{36}\%$$

Determine Probabilities of Dependent Events with More Than One Case

Example 5: Hillary is the coach of a junior ultimate team. Based on the team's record, it has a 60% chance of winning on calm days and a 70% chance of winning on windy days. Tomorrow, there is a 40% chance of high winds. There are no ties in ultimate. What is the probability that Hillary's team will win tomorrow?

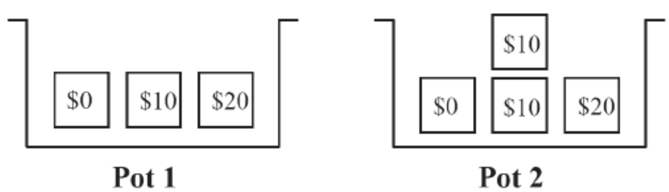


$$P(W) = P(W \cap C) + P(W \cap C')$$
$$= P(W|C)P(C) + P(W|C')P(C')$$
$$= (0.6) \cdot (0.6) + (0.7)(0.4)$$
$$= 0.36 + 0.28 = 0.64 = 64\%$$

example 6, 7 : answer = $41.\overline{6}\%$

Example 6: Bag A contains 1 black and 2 white marbles, and Bag B contains 1 white and 2 black marbles. A marble is randomly chosen from Bag A and placed in Bag B. A marble is then randomly chosen from Bag B. Determine the probability that the marble selected from Bag B is white.

Example 7: A pot is randomly selected, then a bill is randomly chosen from that pot. Suppose it is not known from which pot a bill came. What is the probability that a $10 bill is selected?



Assignment: p. 350 #1 – 19 (odds)

5.6 – INDEPENDENT EVENTS

Goal: Understand and solve problems that involve independent events.

Independent Events

Two events are independent if the knowledge that one event has occurred has no effect on the probability of the other event occurring. For example, tossing tails with a coin and drawing the ace of spades from a standard deck of 52 playing cards are independent events.

Distinguish Independent and Dependent Events

Example 1: Classify the following events as dependent or independent.

a. Two cards are selected from a well-shuffled deck of cards and the experiment is carried out without replacement. The first event is drawing a jack. The second event is drawing another jack.

dependent (use conditional probability from 5.5)

b. Two cards are selected from a well-shuffled deck of cards and the experiment is carried out with replacement. The first event is drawing an ace of hearts. The second event is drawing a black 5.

independent

c. A fair die is rolled and a fair coin is tossed. The first event is rolling an odd number on the die. The second event is obtaining a tail on a flip of the coin.

independent

The probability that two independent events, A and B, will both occur is the product of their individual probabilities:
$\cancel{P(A \cap B)} = P(A) \times P(B)$
$P(A \cap B)$

Determine Probabilities of Independent Events

Example 2:

The probabilities that Emma will pass Grade 12 math and Grade 12 physics this semester are 0.85 and 0.75 respectively. If these events are independent, determine the probability (to four decimal places) that she will pass

a. both math and physics

$P(M \cap P) = P(M) \times P(P)$
$= 0.85 \times 0.75 = 0.6375$

b. math but not physics

$P(M \cap P') = P(M) \times P(P')$
$= 0.85 \times 0.25 = 0.2125$

c. physics but not math

$P(M' \cap P) = P(M') \times P(P)$
$= 0.15 \times 0.75 = 0.1125$

d. neither math nor physics

$P(M' \cap P') = P(M') \times P(P')$
$= 0.15 \times 0.25 = 0.0375$

Try:

On Friday the probability that the Flyers win their game in Prince George is 5/9 and the probability that the Bears win their game in Smithers is 12/17. Assuming independence, what is the probability that on Friday the Flyers win their game and the Bears do not win their game?

$P(F) = 5/9$

$P(B') = 1 - 12/17 = 5/17$

$P(F \cap B') = P(F) \times P(B')$

$= 5/9 \times 5/17$

$= 25/153 \doteq 16.3\%$

Determine Probabilities of Independent Events with More Than One Case

Example 3:

All 1000 tickets for a charity raffle have been sold and placed in a drum. There will be two draws. The first draw will be for the grand prize, and the second draw will be for the consolation prize. After each draw, the winning ticket will be returned to the drum so that it might be drawn again. Max has bought five tickets. Determine the probability, to a tenth of a percent, that he will win at least one prize.

compliment: winning no prizes.

$P(W_1') = 995/1000$

$P(W_2') = 995/1000$

$P(\text{at least one}) = 1 - P(\text{not winning at all})$

$= 1 - P(W_1' \cap W_2')$

$= 1 - P(W_1') \times P(W_2')$

$= 1 - 995/1000 \times 995/1000 = 0.009975 = 0.9975\%$

Example 4:

compliment

Two dart players each throw independently one dart at a target. The probability of each player hitting the bullseye is 0.3 and 0.4 respectively. What is the probability that at least one of them will hit the bullseye?

$(\text{at least one})' = (\text{no bullseye})$

$P(A') = 0.7$

$P(B') = 0.6$

$P(\text{at least one}) = 1 - P(\text{no bullseye})$

$= 1 - P(A') \times P(B')$

$= 1 - 0.7 \times 0.6 = 1 - 0.42 = 0.58 = 58\%$

Assignment: p. 360 #1 – 15

given that

P(Rain today | Sunney yesterday)

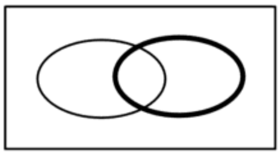
Ch 5: Conditional Probability p. 328

Probabilities of Events A and B

General Case:

given that

$P(\underset{\text{and}}{A \cap B}) = P(A) \cdot P(B|A)$



"probability of A and B"

Equivalently:

$\boxed{P(\boldsymbol{B}|A)} = \dfrac{P(A \overset{\text{and}}{\cap} B)}{P(A)}$

Prob of B given that A is true

"probability of B given A"

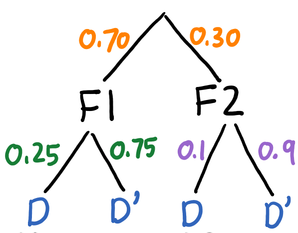
P(Head from Coin | rolled 6 on a die) = $\frac{1}{2}$

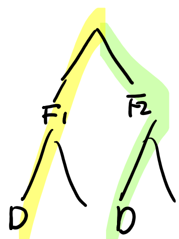
Note: Events A and B are independent if $P(B|A) = P(B)$

For independent events: $P(A \cap B) = P(A) \cdot P(B)$

Example 1: A company has two factories that make computer chips. Suppose 70% of the chips come from Factory 1 and 30% come from Factory 2. In Factory 1, 25% of the chips are defective; in Factory 2, 10% of the chips are defective.

a) Suppose it is not known from which factory a chip came. What is the probability that the chip is defective?





$P(D) = 0.7 \times 0.25 \overset{\text{or}}{+} 0.3 \times 0.1$

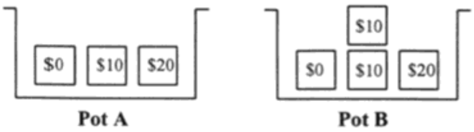
$P(D) = 0.205 = \boxed{20.5\%}$

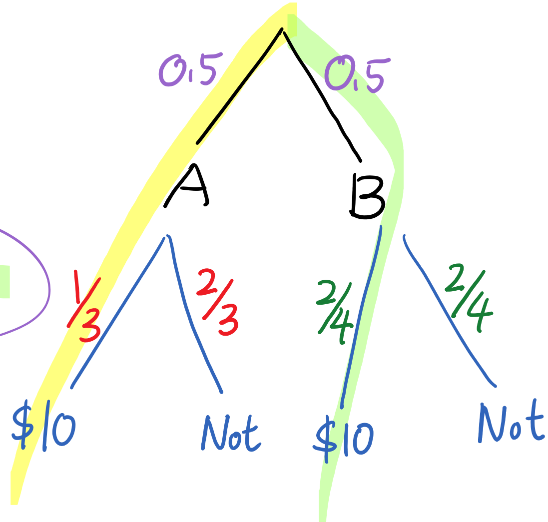
b) Suppose a defective chip is discovered. What is the probability that the chip came from Factory 1?

fact

$P(F1 | \text{Defective}) = \dfrac{P(F1 \text{ and } D)}{P(D)} = \dfrac{0.7 \times 0.25}{0.205} = \boxed{85\%}$

Example 2: A pot is randomly selected, then a bill is randomly chosen from that pot.
What is the probability that
a) a $10 bill is chosen?





$$P(\$10) = 0.5 \times \frac{1}{3} + 0.5 \times \frac{2}{4}$$

$$P(\$10) = 0.417 \Rightarrow \boxed{41.7\%}$$

b) if a $10 bill is chosen, what is the probability that it came from Pot 1? A

Fact

$$P(A \mid \$10) = \frac{P(A \text{ and } \$10)}{P(\$10)} = \frac{0.5 \times 1/3}{0.417} = \boxed{40\%}$$