6.1 – EXPLORING THE GRAPHS OF POLYNOMIAL FUNCTIONS

A polynomial function consist of one or more terms, which are separated by + or – signs.

> The degree of a polynomial function is the value of the highest exponent in the function. If a polynomial function includes a term with no variable, this term is called a constant term.

Determine the Degree and the Constant

Example 1: Determine the degree and the constant of each polynomial function.

a. $f(x) = x^2 + 4x - 5$

b. $g(x) = 3x - 7$

c. $h(x) = 8$

> A number that multiplies the variable in a polynomial is called a coefficient. The leading coefficient is the number that multiplies the term with the highest power.

Determine the Leading Coefficient

Example 2: Determine the leading coefficient of each polynomial function.

a. $f(x) = x^2 + 4x - 5$

b. $g(x) = 3x - 7$

> The terms in a polynomial function are normally written so that the powers are in descending order.
>
> For example, $f(x) = 2x^3 + 3x^2 - 2x + 5$

Example 3: Write a polynomial function in descending order that satisfies the following conditions.

a. degree 2, leading coefficient –3

b. degree 2, leading coefficient 7, two terms

c. degree 1, leading coefficient 1

d. degree 0

e. degree 3, constant term –8

f.

Domain The domain is the set of all possible ***x*-values** which will make the function "work" and will output real *y*-values.

Range The range of a function is the complete set of all possible **resulting *y*-values** of the dependent variable.

End behaviour The end behaviour of a polynomial is the description of the shape of the graph, from left to right, on the coordinate plane.

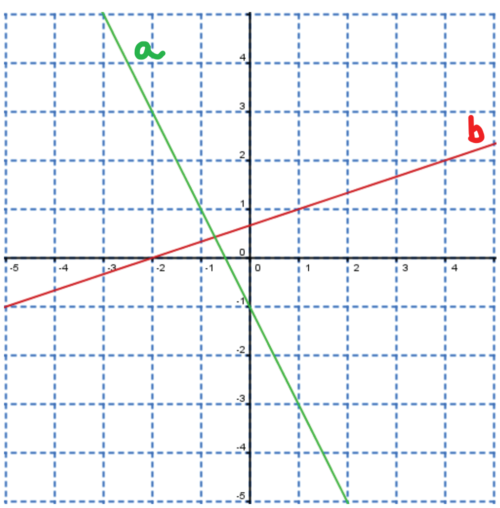
Turning Point A turning point is any point where the graph of a function changes from increasing to decreasing or from decreasing to increasing.

Polynomial functions are named according to their degree. Polynomial functions of degrees 0, 1, 2, and 3 are called constant, linear, quadratic, and cubic functions, respectively.

Characteristics of Polynomial Functions

Example 4: Determine the type of function, the degree, the x-intercepts, y-intercepts, end behaviour, range and number of turning points for each type of function.

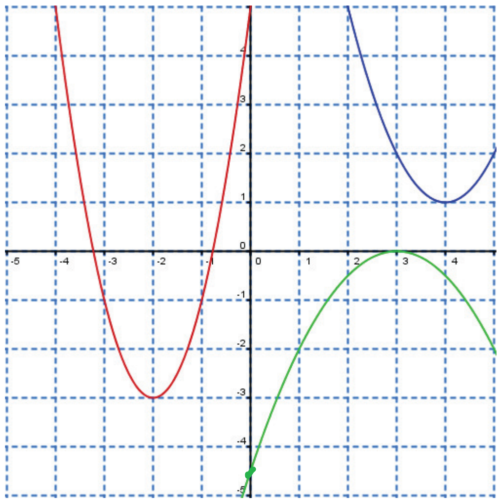
a.



Type of function		
Degree		
Number of x-ints		
Number of y-ints		
Domain		
Range		

End Behaviour		
Number of Turning Points		

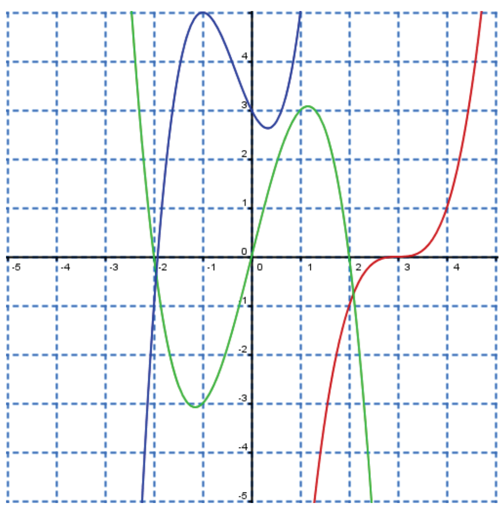
b.



Type of function	
Degree	
Number of *x*-ints	
Number of *y*-ints	
Domain	
Range	

End Behaviour		
Number of Turning Points		

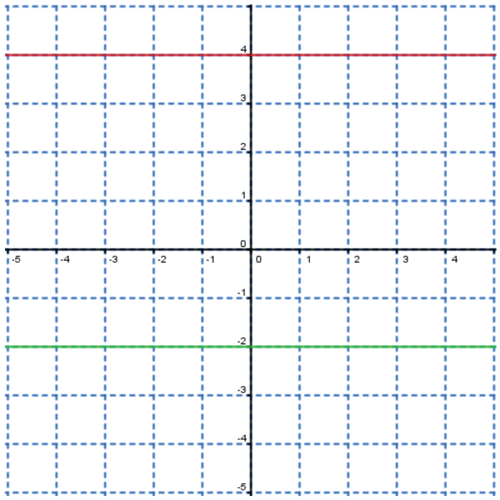
c.



Type of function	
Degree	
Number of *x*-ints	
Number of *y*-ints	
Domain	
Range	

End Behaviour		
Number of Turning Points		

d.

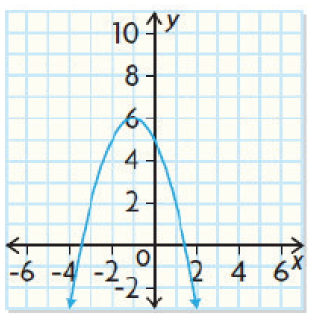


Type of function		
Degree		
Number of x-ints		
Number of y-ints		
Domain		
Range		

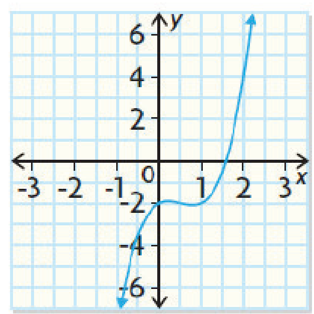
End Behaviour		
Number of Turning Points		

Example 5: Which of the following graphs might represent polynomial functions?

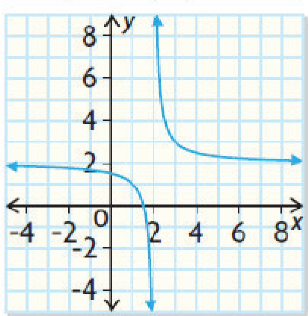
a)



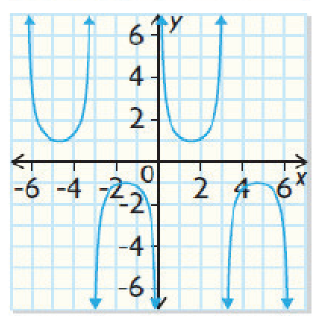
b)



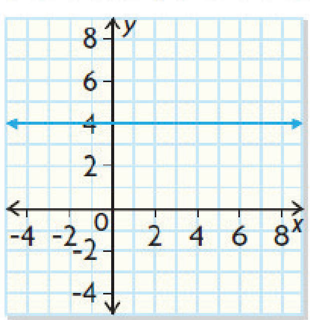
c)



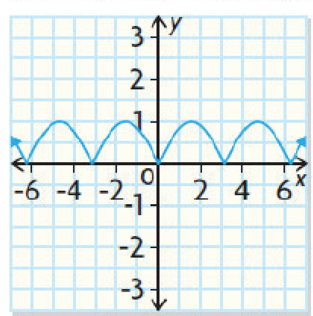
d)



e)



f)



Assignment: p. 383 #1 – 4

6.2 – CHARACTERISTICS OF THE EQUATIONS OF POLYNOMIAL FUNCTIONS

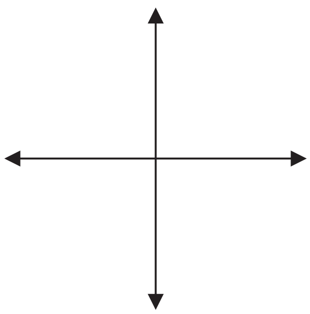
Standard Form: The standard forms for polynomial functions are:

Linear	*Quadratic*	*Cubic*
$f(x) = ax + b$, where $a \neq 0$.	$f(x) = ax^2 + bx + c$, where $a \neq 0$.	$f(x) = ax^3 + bx^2 + cx + d$, where $a \neq 0$.

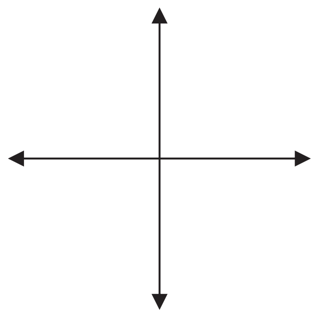
Observe the Characteristics of the Graphs of Polynomial Functions

Example 1: Sketch the graph on the grid using the graphing calculator window *x*: [–8, 8, 2] *y*: [–20, 20, 5]

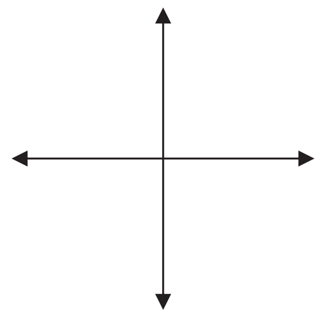
$f(x) = \frac{1}{2}x - 6$



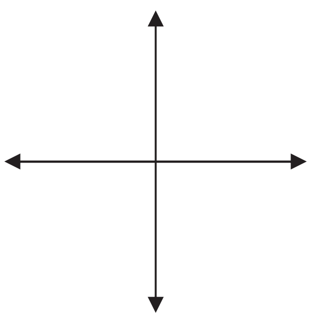
$f(x) = -5x - 2$



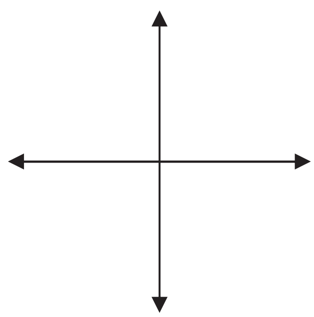
$f(x) = -2x^2 + 2x + 4$



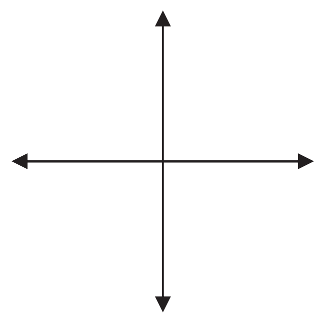
$f(x) = x^2 - 6x + 12$



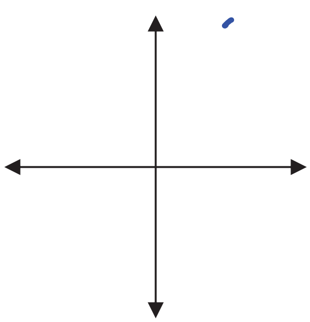
$f(x) = -2x^3 + 4x^2 - 3x + 1$



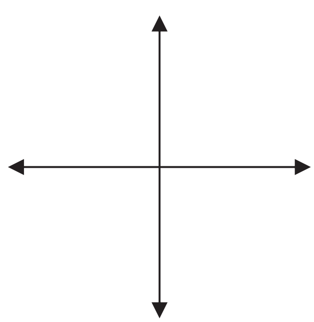
$f(x) = 2x^3 + 4x^2 - 3x + 1$



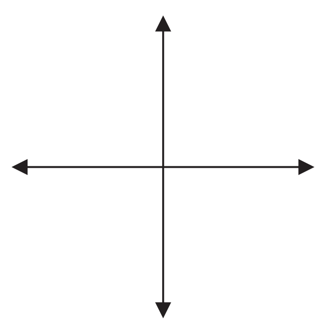
$f(x) = x^3 - 2x^2 - 15x + 36$



$f(x) = x^3 - 8$



$f(x) = -x^3 + 2x^2 + 15x - 10$



a. How is the constant term in a polynomial function related to the y-intercept of the graph of the function?

b. How does the sign of the leading coefficient affect the end behaviour of the graph of each type of polynomial function?

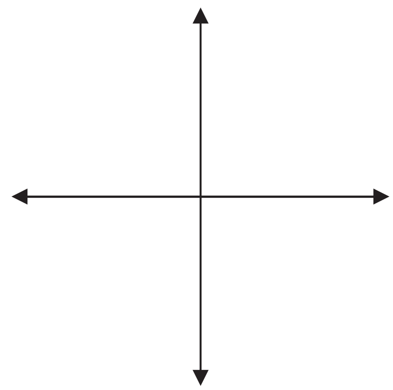
The *degree* of a polynomial function determines the *shape of the function*. The graphs of polynomial functions of the same degree have common characteristics.

Reason about the Characteristics of the Graph of a Given Polynomial Function Using Its Equation

Example 2: Predict the number of possible x-intercepts, y-intercept, domain, range, end behaviour, number of possible turning points of each function using its equation.

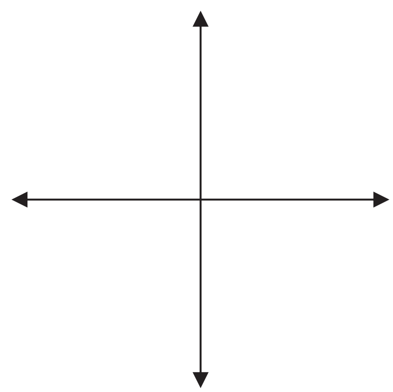
a. $f(x)=3x-5$

x-intercepts	
y-intercept	
domain	
range	
end behaviour	
number of turning pts.	



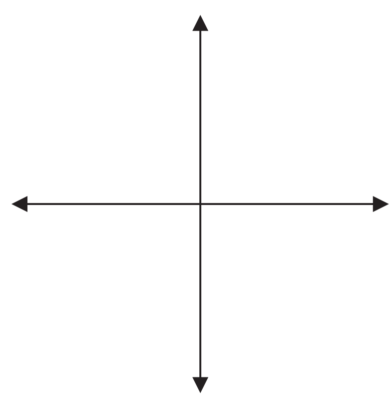
b. $f(x)=-2x^2-4x+8$

x-intercepts	
y-intercept	
domain	
range	
end behaviour	
number of turning pts.	



c. $f(x)=2x^3+10x^2-2x-10$

x-intercepts	
y-intercept	
domain	
range	
end behaviour	
number of turning pts.	

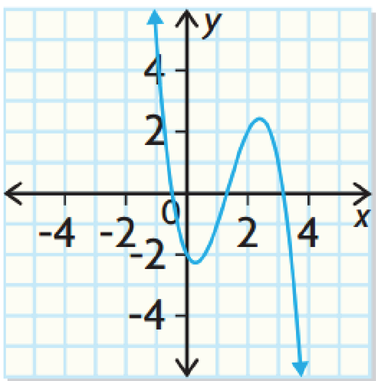


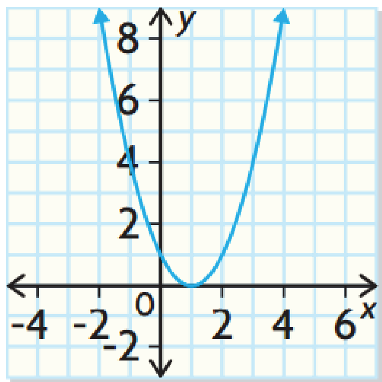
Connect Polynomial Functions to Their Graphs

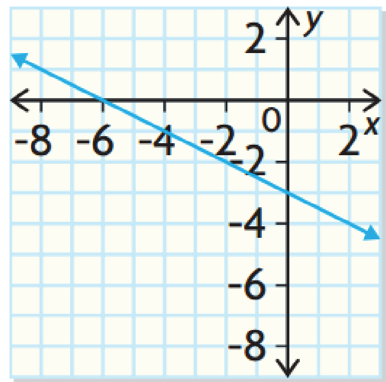
Example 3: Match each graph with the correct polynomial function. Justify your reasoning.

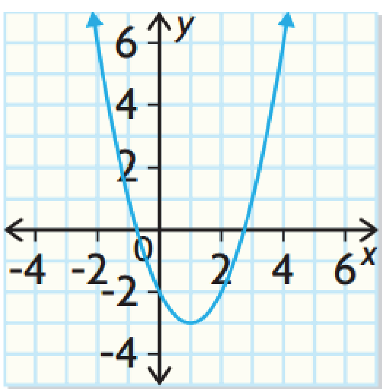
$g(x) = -x^3 + 4x^2 - 2x - 2$ $\quad j(x) = x^2 - 2x - 2$ $\quad p(x) = x^3 - 2x^2 - x - 2$

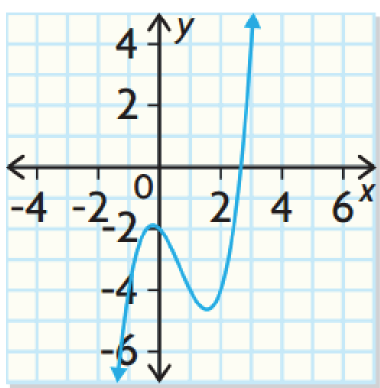
$h(x) = -\frac{1}{2}x - 3$ $\quad k(x) = x^2 - 2x + 1$ $\quad q(x) = -2x - 3$

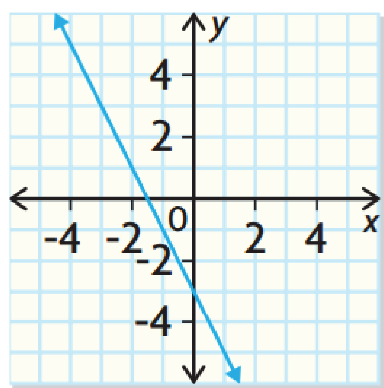








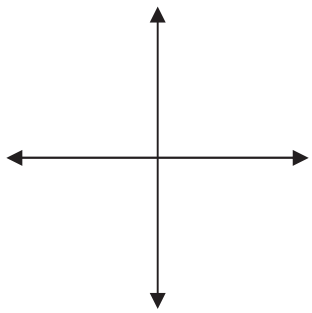




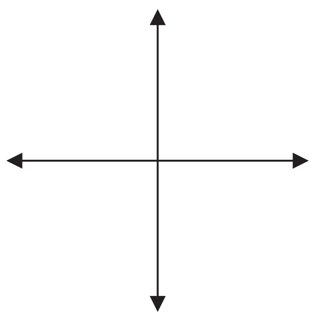
Reason about the Characteristics of the Graphs of Polynomial Functions

Example 4: Sketch the graph of a possible polynomial function for each set of characteristics below.

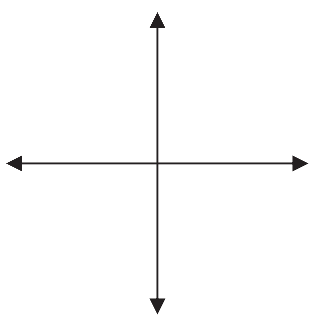
a. Range: $\{y \mid y \geq -2, y \in \mathbb{R}\}$ and y-intercept: 4



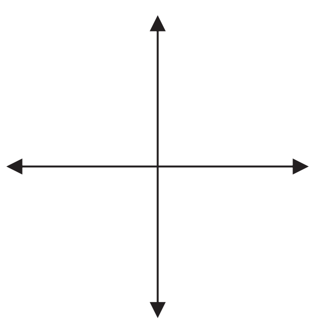
b. Range: $\{y \mid y \in \mathbb{R}\}$ and turning points: one in quadrant III and another in quadrant I



c. Extending from quadrant II to quadrant IV, degree 1, y-intercept of –3



d. Range: $\{y \mid y \leq 6, y \in \mathbb{R}\}$ and x-intercept: 2 and 6



Assignment: p. 393 #4 – 11, 13

6.3 – MODELLING DATA WITH A LINE OF BEST FIT

Interpolation

Interpolation is the process used to estimate a value within the domain of a set of data, based on a trend.

You can graph the scatter plot and interpolate using a graphing calculator.

Step 1. Enter the data

→ Press STAT key → Select EDIT → Clear any numbers that are written in L1, L2

→ Under Column L1, enter the data (x-values)

→ Under Column L2, enter the data (y-values)

Step 2. Choose window

→ Press WINDOW and adjust Xmin, Xmax, Ymin, Ymax

→ Graph

Step 3. Obtain the function

→ Press STAT key → Select CALC → Select #4 LinReg (or #5 QuadReg or #6 CubicReg)

→ Enter L1 , L2 ,

→ VARS → Select Y-VARS → Select #1 Function → Y1

Use Technology to Determine a Linear Model for Continuous Data

Example 1: The one-hour record is the farthest distance travelled by bicycle in 1 h. The table below shows the world-record distances and the dates they were accomplished.

Year	1996	1998	1999	2002	2003	2004	2007	2008	2009
Distance (km)	78.04	79.14	81.16	82.60	83.72	84.22	86.77	87.12	90.60

International Human Powered Vehicle Association

a. Use technology to create a scatter plot and to determine the equation of the line of best fit. Round to three decimal places.

b. Interpolate a possible world-record distance for the year 2006, to the nearest hundredth of a kilometre.

Linear regression results in an equation that balances the points in the scatter plot on both sides of the line. A line of best fit can be used to predict values that are not recorded or plotted. Predictions can be made by reading values from the line of best fit on a scatter plot or by using the equation of the line of best fit.

Try: Consider the data in the table. Use technology to create a scatter plot and to determine the equation of the line of best fit.

x	0	2	4.5	5.2	9.5	12
y	5.1	6.7	8.2	8.8	11.9	13.4

a. Determine, to the nearest tenth, the value of y when x is 10.6.

b. Determine, to the nearest tenth, the value of x when y is 9.8.

Extrapolation

Extrapolation is the process used to estimate a value outside the domain of a set of data, based on a trend.

Use Linear Regression to Solve a Problem that Involves Discrete Data

Example 2: Matt buys T-shirts for a company that prints art on T-shirts and then resells them. When buying the T-shirts, the price Matt must pay is related to the size of the order. Five of Matt's past orders are listed in the table below.

Number of Shirts	**Cost per Shirt ($)**
500	3.25
700	1.95
200	5.20
460	3.51
740	1.69

Matt has misplaced the information from his supplier about price discounts on bulk orders. He would like to get the price per shirt below $1.50 on his next order.

a. Use technology to create a scatter plot and determine an equation for the linear regression function that models the data. Round to three decimal places.

b. What do the slope and y-intercept of the equation of the linear regression function represent in this context?

c. Use the linear regression function to extrapolate the size of order necessary to achieve the price of $1.50 per shirt.

Assignment: p. 407 #1 – 11 (odds)

6.4 – MODELLING DATA WITH A CURVE OF BEST FIT

Curve of best fit

Curve of best fit is a curve that best approximates the trend on a scatter plot.

Use Technology to Solve a Quadratic Problem

Example 1: The concentration (in milligrams per litre) of a medication in a patient's blood is measured as time passes. Susan has collected the following data and is attempting to express the concentration as a polynomial function of time.

Time (*T hours*)	0	1.5	3	4.5	6	7.5	9
Concentration (*C* mg/L)	0	26.9	41.2	47.8	46.0	36.8	20.3

a. On a graphing calculator, enter the data in two lists. Time in L_1 and Concentration in L_2. Create a scatter plot of the data and use the quadratic regression feature to determine the polynomial function, $C = aT^2 + bT + c$, that best fits the data. Round the parameters a, b, and c to 2 decimal places.

b. The doctor has decided that the patient needs a second dose of medication when the concentration in the blood is less than 10 mg/L. If the first dose of medication was given at 9:00am, at what time should the second dose be given?

Example 2: Consider the data in the table. Use technology to create a scatter plot and to determine the equation of the line of best fit.

x	0	1.5	3.3	5.1	7.4	8.6	10.0
y	19.5	10.3	3.4	1.6	6.2	16.1	20.3

a. Determine, to the nearest tenth, the value of y when x is 10.6.

b. Determine, to the nearest tenth, the value of x when y is 9.8.

Technology uses polynomial regression to determine the curve of best fit. Polynomial regression results in an equation of a curve that balances the points on both sides of the curve. A curve of best fit can be used to predict values that are not recorded or plotted. Predictions can be made by reading values from the curve of best fit on a scatter plot or by using the equation of the curve of best fit.

Use Technology to Solve a Cubic Regression Function

Example 3: The following table shows the average retail price of gasoline, per litre, for a selection of years in a 30-year period beginning in 1979.

Years after 1979	Price of Gas (¢/L)	Years after 1979	Price of Gas (¢/L)
0	21.98	17	58.52
1	26.18	20	59.43
2	35.63	22	70.56
3	43.26	23	70.00
4	45.92	24	74.48
7	45.78	25	82.32
8	47.95	26	92.82
9	47.53	27	97.86
12	57.05	28	102.27
14	54.18	29	115.29

Statistics Canada

a. Use technology to graph the data as a scatter plot. What polynomial function could be used to model the data? Explain.

b. Determine the cubic regression equation that models the data. Use your equation to estimate the average price of gas in 1984 and 1985.

c. Estimate the year in which the average price of gas was 56.0¢/L.

5. Consider the data in the table. Create a scatter plot from the data using a graphing calculator.

x	0	5	10	15	20	25	30	35	40	45	50
y	120	102	83	74	67	64	62	54	45	31	10

a. Use the cubic regression feature of a calculator to determine a cubic function that models the data. Round to three decimal places.

b. Use the cubic regression equation to determine the value of x when y = 90.

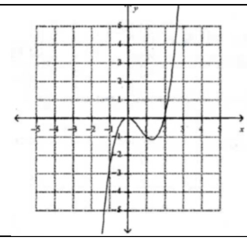
c. Use the linear regression feature of a calculator to determine a leaner function that models the data. Round to three decimal places.

d. Use the linear regression equation to determine the value of x when y = 90.

e. Which model appears to be the better for the data?

FOM 12 CHAPTER 6 **Practice Test 2021**

1 Determine the degree of this function

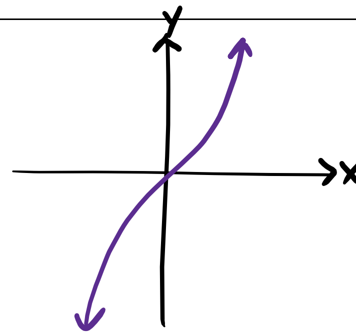


2 Determine the degree of this polynomial function

$$f(x) = \frac{3}{4} + 2x$$

3 Determine the degree of this function

A. $f(x) = -x^2 - 3x - 1$
B. $g(x) = x^2 - 2x + 1$
C. $h(x) = -x^3 - 2x^2 + 1$
D. $j(x) = x^3 + 2x$



4 Determine the following characteristics of the polynomial function $f(x) = 2(x - 1)(x^2 + 4)$. Show your work.

- number of possible x-intercepts
- y-intercept
- end behaviour
- domain
- range
- number of possible turning points

5 The distance a marathon runner covers can be modelled by the function

$$d(t) = 153.8t + 86$$

where d represents the distance **in metres** and t represents the time in minutes. Approximately how long will it take her to run the 42.2 **km** race?

6 Determine the equation of the quadratic regression function for the data.

x	–6	–3	–1	1	1	3	5	8
y	35	46	50	53	51	47	36	11

7

The path of a shot put thrown at a track and field meet is modelled by the quadratic function

$$h(d) = -0.048(d^2 - 20.7d - 26.28)$$

where h is the height in metres and d is the horizontal distance in metres.
What is the horizontal distance when the shot put is at its maximum height?

8

A farming cooperative collected data showing the effect of different amounts of fertilizer, *x*, in hundreds of kilograms per hectare (kg/ha), on the yield of beets, *y*, in tonnes (t). Use the data below and quadratic regression to compare the possible yields of beets when the amount of fertilizer used is 1.50 kg/ha and 1.75 kg/ha. Show your work.

Fertilizer (kg/ha)	0	0.25	0.50	0.75	1.00	1.25
Yield (t)	0.22	0.49	0.74	0.92	1.05	1.15

Shane tracked the depth of the water at an ocean marina one afternoon. His data is summarized in the table below.

Time of Day	12:00	13:00	14:00	15:00	16:00	18:00	19:00
Depth (ft)	12.3	12.7	13.3	13.8	14.3	14.6	14.3

a) Determine the equation for the cubic regression function that models this data. Let *x* be the number of hours since 12:00 and *y* be the depth in feet.

b) Interpolate the depth of the water at 17:00. Show your work.

c) what is the depth of water at 10:30 that morning.