F Math 12

3.1 Types of Sets and Set Notation p. 146

Name ________________

Date ________________

Goal: Understand sets and set notation.

1. **set**: A collection of distinguishable objects; for example, the set of whole numbers is *W* = {0, 1, 2, 3, ...}.
2. **element**: An object in a set; for example, 3 is an element of *D*, the set of digits.
3. **universal set**: A set of all the elements under consideration for a particular context (also called the sample space); for example, the universal set of digits is *D* = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}.
4. **subset**: A set whose elements all belong to another set; for example, the set of odd digits, *O* = {1, 3, 5, 7, 9}, is a subset of *D*, the set of digits. In set notation, this relationship is written as: $O \subset D$.
5. **complement**: All the elements of a universal set that do not belong to a subset of it; for example, O' = {0, 2, 4, 6, 8} is the complement of *O* = {1, 3, 5, 7, 9}, a subset of the universal set of digits, *D*. The complement is denoted with a prime sign, O'.
6. **empty set**: A set with no elements; for example, the set of odd numbers divisible by 2 is the empty set. The empty set is denoted by { } or ∅.
7. **disjoint**: Two or more sets having no elements in common; for example, the set of even numbers and the set of odd numbers are disjoint.
8. **finite set**: A set with a countable number of elements; for example, the set of even numbers less than 10, *E* = {2, 4, 6, 8}, is finite.
9. **infinite set**: A set with an infinite number of elements; for example, the set of natural numbers, *N* = {1, 2, 3,...}, is infinite.
10. **mutually exclusive**: Two or more events that cannot occur at the same time; for example, the Sun rising and the Sun setting are mutually exclusive events.

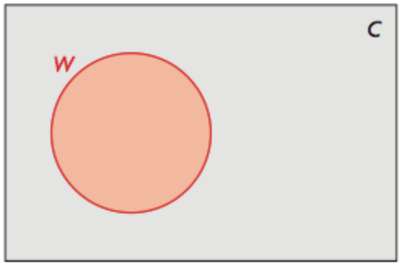
INVESTIGATE the Math

Jasmine is studying the provinces and territories of Canada. She has decided to categorize the provinces and territories using **sets** .

How can Jasmine use sets to categorize Canada's regions?

A. List the elements of the universal set of Canadian provinces and territories, *C*.

B. One subset of *C* is the set of Western provinces and territories, *W*. Write *W* in set notation.



C. The Venn diagram above represents the universal set, *C*. The circle in the Venn diagram represents the subset *W*. The **complement** of *W* is the set *W′*.

i. Describe what *W′* contains.

ii. Write *W′* in set notation.

iii. Explain what *W′* represents in the Venn diagram.

D. Jasmine wrote the set of Eastern provinces as follows: *E* = {NL, PE, NS, NB, QC, ON} Is *E* equal to *W′*? Explain.

E. List *T*, the set of territories in Canada. Is *T* a subset of *C*? Is it a subset of *W*, or a subset of *W′*? Explain using your Venn diagram.

F. Explain why you can represent the set of Canadian provinces south of Mexico by the **empty set** .

G. Consider sets *C*, *W*, *W′*, and *T*. List a pair of disjoint sets. Is there more than one pair of **disjoint** sets?

H. Complete your Venn diagram by listing the elements of each subset in the appropriate circle.

Communication | *Notation*

The following is a summary of notation introduced so far.

Sets are defined using brackets. For example, to define the universal set of the numbers 1, 2, and 3, list its elements:

$U = \{1, 2, 3\}$

To define the set *A* that has the numbers 1 and 2 as elements:

$A = \{1, 2\}$

All elements of *A* are also elements of *U*, so *A* is a subset of *U*:

$A \subset U$

The set *A′*, the complement of *A*, can be defined as:

$A' = \{3\}$

To define the set *B*, a subset of *U* that contains the number 4:

$B = \{\ \}$ or $B = \varnothing$

$B \subset U$

Communication | *Notation*

The phrase "from 1 to 5" means "from 1 to 5 inclusive."

In set notation, the number of elements of the set *X* is written as $n(X)$.

For example, if the set *X* is defined as the set of numbers from 1 to 5:

$X = \{1, 2, 3, 4, 5\}$

$n(X) = 5$

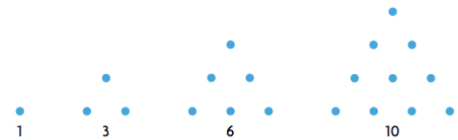
Example 1: Sorting numbers using set notation and a Venn diagram (p.148)

a) Indicate the multiples of 5 and 10, from 1 to 500, using set notation. List any subsets.

b) Represent the sets and subsets in a Venn diagram.

Example 2: Determining the number of elements in sets (p. 149)

A triangular number, such as 1, 3, 6, or 10, can be represented as a triangular array.



a) Determine a pattern you can use to determine any triangular number.

b) Determine how many natural numbers from 1 to 100 are

   i) even and triangular,
   ii) odd and triangular, and
   iii) not triangular.

c) How many numbers are triangular?

Example 3: Describing the relationships between sets (p. 151)

Alden and Connie rescue homeless animals and advertise in the local newspaper to find homes for the animals. They are setting up a web page to help them advertise the animals that are available. They currently have dogs, cats, rabbits, ferrets, parrots, lovebirds, macaws, iguanas, and snakes.

a) Design a way to organize the animals on the web page. Represent your organization using a Venn diagram.

b) Name any disjoint sets.

c) Show which sets are subsets of one another using set notation.

d) Alden said that the set of fur-bearing animals could form one subset. Name another set of animals that is equal to this subset.

Example 4: Solving a problem using a Venn diagram (p.152)

Bilyana recorded the possible sums that can occur when you roll two four-sided dice in an outcome table:

	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

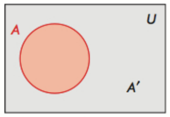
- Display the following sets in one Venn diagram:
  - rolls that produce a sum less than 5
  - rolls that produce a sum greater than 5
- Record the number of elements in each set.
- Determine a formula for the number of ways that a sum less than or greater than 5 can occur. Verify your formula.

In Summary

Key Ideas

- You can represent a set of elements by:
  - listing the elements; for example, $A = \{1, 2, 3, 4, 5\}$
  - using words or a sentence; for example, A = {all integers greater than 0 and less than 6}
  - using set notation; for example, $A = \{x \mid 0 < x < 6, x \in I\}$
- You can show how sets and their subsets are related using Venn diagrams. Venn diagrams do not usually show the relative sizes of the sets.
- You can often separate a universal set into subsets, in more than one correct way.

Need to Know



- Sets are equal if they contain exactly the same elements, even if the elements are listed in different orders.
- You may not be able to count all the elements in a very large or infinite set, such as the set of real numbers.
- The sum of the number of elements in a set and its complement is equal to the number of elements in the universal set:

$$n(A) + n(A') = n(U)$$

- When two sets A and B are disjoint,

$$n(A \text{ or } B) = n(A) + n(B)$$

HW: 3.1 p. 154-158 #4, 6, 8, 9, 11, 12, 14, 15, 16 & 19

3.2 Exploring Relationships between Sets p. 159

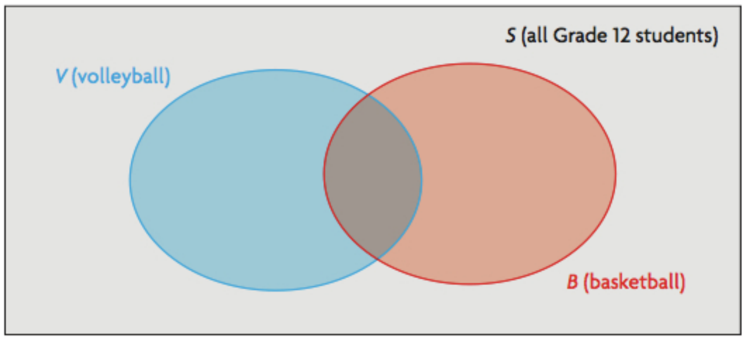
Name ________________

Date ________________

Goal: Explore what the different regions of a Venn diagram represent.

EXPLORE the Math

In an Alberta school, there are 65 Grade 12 students. Of these students, 23 play volleyball and 26 play basketball. There are 31 students who do not play either sport. The following Venn diagram represents the sets of students.



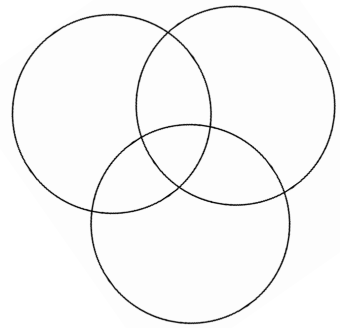
How many students play: Volleyball only? _______ Both Volleyball and Basketball? _______

Basketball only? _______ Neither sport? _______

Reflect: Consider the set of students who play volleyball and the set of students who play basketball. Are these two sets disjoint? Explain how you know.

Example 2: Each member of a sports club plays at least one of soccer, rugby, or tennis. The following information is known. 43 members play tennis, 11 play tennis and rugby, 7 play tennis and soccer, 6 play soccer and rugby, 84 play rugby or tennis, 68 play soccer or rugby, and 4 play all three sports.

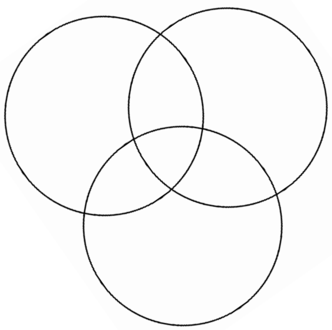
a) Display the information in a Venn diagram.



b) How many members does the club have?

Example 3: In a high school, there are 130 grade 11 students. Currently, 82 students are taking math, 27 are taking math and physics, 25 are taking math and chemistry, 20 are taking chemistry and physics, 110 are taking math or chemistry, and 87 are taking chemistry or physics. Eleven students are taking all three courses.

a) Draw a Venn diagram to display the information.

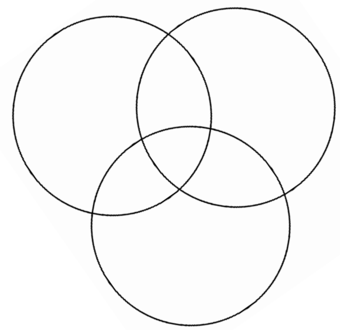


b) How many students are taking math or physics?

c) How many students are taking none of these three courses?

Example 4: Each student at a music camp plays at least one of the following instruments: violin, piano, or saxophone. It is known that 6 students play all three instruments, 163 play piano, 36 play piano and violin, 13 play piano and saxophone, 11 play saxophone and violin, 208 play violin or piano, and 98 play saxophone or violin.

a) Draw a Venn diagram to display the information.

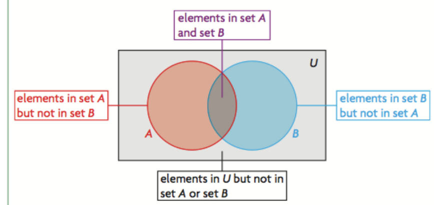


b) How many students are there at the camp?

In Summary

Key Ideas

- Sets that are not disjoint share common elements.
- Each area of a Venn diagram represents something different.
- When two non-disjoint sets are represented in a Venn diagram, you can count the elements in both sets by counting the elements in each region of the diagram just once.



Need to Know

- Each element in a universal set appears only once in a Venn diagram.
- If an element occurs in more than one set, it is placed in the area of the Venn diagram where the sets overlap.

HW: 3.2 p. 160-161 #1-5

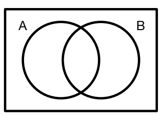
3.3 Intersection and Union of Two Sets p. 162

Goal: Understand and represent the intersection and union of two sets.

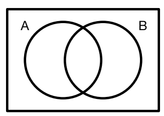
1. **intersection**: The set of elements that are common to two or more sets. In set notation, $A \cap B$ denotes the intersection of sets A and B; for example, if $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$, then $A \cap B = \{3\}$.
2. **union**: The set of all the elements in two or more sets; in set notation, $A \cup B$ denotes the union of sets A and B; for example, if $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$, then $A \cup B = \{1, 2, 3, 4, 5\}$.
3. **Principle of Inclusion and Exclusion**: The number of elements in the union of two sets is equal to the sum of the number of elements in each set, less the number of elements in both sets; using set notation, this is written as $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

Venn Diagrams & Notation

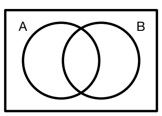
Shade the region that contains the elements that belong.



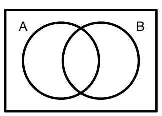
A



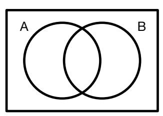
B



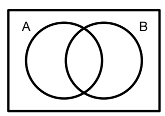
A'



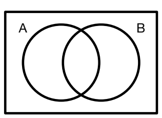
B'



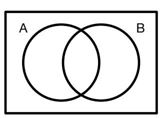
$A \cup B$



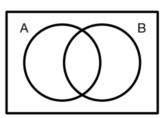
$(A \cup B)'$



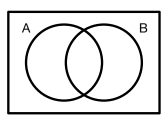
$A \cap B$



$(A \cap B)'$



$A \backslash B$



$B \backslash A$

Example 1: Determining the union and intersection of disjoint sets (p.164)

If you draw a card at random from a standard deck of cards, you will draw a card from one of four suits: clubs (C), spades (S), hearts (H), ordiamonds (D).

a) **Describe** sets C, S, H, and D, and the universal set U for this situation.

b) Determine $n(C)$, $n(S)$, $n(H)$, $n(D)$, and $n(U)$.

$U =$ ______________________________ $n(U) =$ ______

$S =$ ______________________________ $n(S) =$ ______

$H =$ ______________________________ $n(H) =$ ______

$C =$ ______________________________ $n(C) =$ ______

$D =$ ______________________________ $n(D) =$ ______

c) **Describe** the union of S and H. Determine $n(S \cup H)$.

d) **Describe** the intersection of S and H. Determine $n(S \cap H)$.

e) Determine whether the events that are described by sets S and H are mutually exclusive, and whether sets S and H are disjoint.

f) **Describe** the complement of $S \cup H$.

Example 2: Determining the number of elements in a set using a formula (p. 166)

The athletics department at a large high school offers 16 different sports:

badminton	hockey	tennis
basketball	lacrosse	ultimate
cross-country running	rugby	volleyball
curling	cross-country skiing	wrestling
football	soccer	
golf	softball	

Determine the number of sports that require the following types of equipment:

a) a ball and an implement, such as a stick, a club, or a racquet

b) only a ball

d) either a ball or an implement

c) an implement but not a ball

e) neither a ball nor an implement

Principle of Inclusion and Exclusion

The number of elements in the union of two sets is equal to the sum of the number of elements in each set, less the number of element in ___________________________

Subtract the elements in the intersection, so they are not counted twice, once in $n(A)$ and once in $n(B)$

If two sets, A and B are disjoint, they contain no common elements.

Example 3: Determining the number of elements in a set by reasoning (p. 168)

Jamaal surveyed 34 people at his gym. He learned that 16 people do weight training three times a week, 21 people do cardio training three times a week, and 6 people train fewer than three times a week. How can Jamaal interpret his results?

HW: 3.3 p. 171-175 #5, 6, 7, 9, 10, 11, 12, 16, 17 & 18

3.4 Application of Set Theory p. 179

Name ________________

Date ________________

Goal: Use sets to model and solve problems.

Example 1: Solving a puzzle using the Principle of Exclusion and Inclusion (p.180)

Use the following clues to answer the questions below:

- 28 children have a dog, a cat, or a bird.
- 13 children have a dog.
- 13 children have a cat.
- 13 children have a bird.
- 4 children have only a dog and a cat.
- 3 children have only a dog and a bird.
- 2 children have only a cat and a bird.
- No child has two of each type of pet.

a) How many children have a cat, a dog, and a bird?

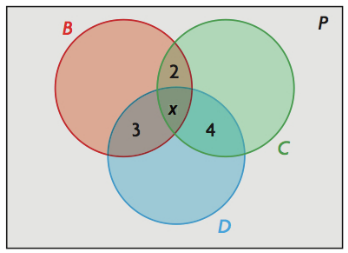
Define the sets and draw a Venn diagram.
Let *x* represent the number of children with a bird, a cat, and a dog.

P= {children with ___________}

C= {children with a __________}

B= {children with a __________}

D= {children with a __________}



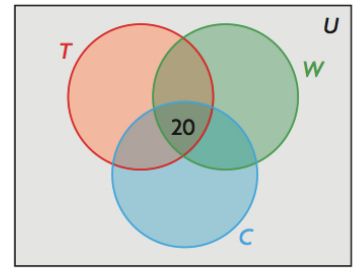
b) How many children have only one pet?

Example 3: Shannon's high school starts a campaign to encourage students to use "green" transportation for travelling to and from school. At the end of the first semester, Shannon's class surveys the 750 students in the school to see if the campaign is working. They obtain these results:

- 370 students use public transit.
- 100 students cycle and use public transit.
- 80 students walk and use public transit.
- 35 students walk and cycle.
- 20 students walk, cycle, and use public transit.
- 445 students cycle or use public transit.
- 265 students walk or cycle.

Complete the Venn Diagram to show how many students are using green transportation for travelling to and from school.

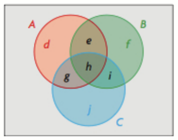
U={students who attend Shannon's school}
T= {students who use public transit}
W={students who walk}
C= {students who cycle}



In Summary

Key Ideas

- Set theory is useful for solving many types of problems, including Internet searches, database queries, data analyses, games, and puzzles.
- To represent three intersecting sets with a Venn diagram, use three intersecting circles. For example, in the following Venn diagram,



  - $A \cap B \cap C$ is represented by region h,
  - $A \cap B$ is represented by the union of regions e and h,
  - $A \cap C$ is represented by the union of regions g and h, and
  - $B \cap C$ is represented by the union of regions h and i.

  Each region of a Venn diagram contains elements that occur only in that particular region.
- You can use the Principle of Inclusion and Exclusion to determine the number of elements in the union of three sets:
  $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

Need to Know

- You can use concepts related to sets to search for websites on the Internet:
  - Put an exact phrase in quotation marks.
  - Connect words or phrases with "and" to search for sites that contain both. The word "and" represents the intersection of two or more sets.
  - Connect words or phrases with "or" to search for sites that contain either one or the other, or both. The word "or" represents the union of two or more sets.
- When solving a puzzle or problem, it is often useful to visualize the problem. First identify which sets are defined by the context. Then identify how the sets overlap. Finally, identify regions of the overlaps that are of interest in the puzzle or problem. It is often advisable to consider how much is known about each region, and use the information about the region that is most known to deduce information about regions that are less well known. A systematic approach will result in answers that are easier to verify.

HW: 3.4 p. 191-194 # 2, 4, 6, 7, 9, 11, 12 & 13

3.5 Conditional Statements and Their Converse p. 195

Name ________________

Date ________________

Goal: Understand and interpret conditional statements.

1. **conditional statement**: An "if–then" statement; for example, "If it is Monday, then it is a school day."

2. **hypothesis**: An assumption; for example, in the statement "If it is Monday, then it is a school day," the hypothesis is "It is Monday."

3. **conclusion**: The result of a hypothesis; for example, in the statement "If it is Monday, then it is a school day," the conclusion is "it is a school day."

4. **counterexample**: An example that disproves a statement; for example, "If it is Monday, then it is a school day" is disproved by the counterexample that there is no school on Thanksgiving Monday. Only one counterexample is needed to disprove a statement.

5. **converse**: A conditional statement in which the hypothesis and the conclusion are switched; for example, the converse of "If it is Monday, then it is a school day" is "If it is a school day, then it is Monday."

6. **biconditional**: A conditional statement whose converse is also true; in logic notation, a biconditional statement is written as "*p* if and only if *q*." For example, the statement "If a number is even, then it is divisible by 2" is true. The converse, "If a number is divisible by 2, then it is even," is also true. The biconditional statement is "A number is even if and only if it is divisible by 2."

Communication | *Notation*

$p \Rightarrow q$ is notation for "If p, then q."
$p \Rightarrow q$ is read as "p implies q."

Communication | *Notation*

$p \Leftrightarrow q$ is notation for "p if and only if q."
This means that both the conditional statement and its converse are true statements.

LEARN ABOUT the Math

James and Gregory like to play soccer, regardless of the weather. Their coach made this **conditional statement** about today's practice: "If it is raining outside, then we practise indoors."

When will the coach's conditional statement be true, and when will it be false?

Example 1: Verifying a conditional statement (p.195)

Verify when the coach's conditional statement is true or false.

Hypothesis: ________________________________

Conclusion: ________________________________

Each of these statements is either true or false, so to verify this conditional statement, consider four cases.

Case 1: The hypothesis is true and the conclusion is true.

When the hypothesis and conclusion are both true, a conditional statement is ________

Case 2: The hypothesis is false, and the conclusion is false.

When the hypothesis and conclusion are both false, a conditional statement is ________

Case 3: The hypothesis is false, and the conclusion is true.

When the hypothesis is false and conclusion is true, a conditional statement is ________

Case 4: The hypothesis is true, and the conclusion is false.

When the hypothesis is true and conclusion is false, a conditional statement is ________

This _________________ shows that the conditional statement is _________

Use a Truth Table to Summarize the Observations

Let p represent the hypothesis: *It is raining outside*.
Let q represent the conclusion: *We practise indoors.*

p	q	$p \Rightarrow q$

When the hypothesis is false, regardless of whether the conclusion is true or false, the conditional statement is **true**

From the truth table, I can see that the **only** time a **conditional statement** will be **false** is when the **hypothesis** is __________ and the **conclusion** is __________.

Example 2: Writing conditional statements (p. 200)

"A person who cannot distinguish between certain colours is colour blind."

a) Write this sentence as a conditional statement in "if *p*, then *q*" form.

b) Write the converse of your statement.

c) Is your statement biconditional? Explain.

The first statement is _________

The converse is ______________

The statement can be written:

"A person is colour blind ________________________ that person cannot distinguish between certain colours."

Example 5: Verifying a biconditional statement (p. 200)

Reid stated the following biconditional statement: “A quadrilateral is a square if and only if all of its sides are equal.” Is Reid’s biconditional statement true? Explain.

Conditional Statement:

Converse:

In Summary

Key Ideas

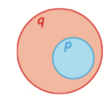
- A conditional statement consists of a hypothesis, p, and a conclusion, q. Different ways to write a conditional statement include the following:
  - If p, then q.
  - p implies q.
  - $p \Rightarrow q$
- To write the converse of a conditional statement, switch the hypothesis and the conclusion.

Need to Know

- A conditional statement is either true or false. A truth table for a conditional statement, $p \Rightarrow q$, can be set up as follows:

p	q	$p \Rightarrow q$
T	T	T
F	F	T
F	T	T
T	F	F

  A conditional statement is false only when the hypothesis is true and the conclusion is false. Otherwise, the conditional statement is true, even if the hypothesis is false.
- You can represent a conditional statement using a Venn diagram, with the inner oval representing the hypothesis and the outer oval representing the conclusion. The statement “p implies q” means that p is a subset of q.



- Only one counterexample is needed to show that a conditional statement is false.
- If a conditional statement and its converse are both true, you can combine them to create a biconditional statement using the phrase “if and only if.”

HW: 3.5 p. 203-206 #1-8 & 12

3.6 The Inverse and the Contrapositive of Conditional Statements p. 208

Name ________________

Date ________________

Goal: Understand and interpret the contrapositive and inverse of a conditional statement.

1. **inverse**: A statement that is formed by negating both the hypothesis and the conclusion of a conditional statement; for example, for the statement "If a number is even, then it is divisible by 2," the inverse is "If a number is **not** even, then it is **not** divisible by 2."

2. **contrapositive**: A statement that is formed by negating both the hypothesis and the conclusion of the **converse** of a conditional statement; for example, for the statement "If a number is even, then it is divisible by 2," the contrapositive is "If a number is **not** divisible by 2, then it is **not** even."

p	q	$p \Rightarrow q$
T	T	T
F	F	T
F	T	T
T	F	F

Communication | *Notation*

In logic notation, the inverse of "if p, then q" is written as "If $\neg p$, then $\neg q$."

Communication | *Notation*

In logic notation, the contrapositive of "if p, then q" is written as "If $\neg q$, then $\neg p$."

Example 1: Verifying the inverse and contrapositive of a conditional Statement (p. 209)
Consider the following conditional statement: "If today is February 29, then this year is a leap year."

a) Verify the statement, or disprove it with a counterexample.

Hypothesis (p):______________________ Conditional statement: if p, then q.

Conclusion (q):______________________

p	q	$p \Longrightarrow q$

b) Verify the converse, or disprove it with a counterexample.

converse:____

Hypothesis (p):______________________ Converse: if q, then p.

Conclusion (q):______________________

q	p	$q \Longrightarrow p$

c) Verify the inverse, or disprove it with a counterexample.

Inverse:____

Hypothesis ($\neg p$):______________________ Inverse: if ________, then ______.

Conclusion ($\neg q$):______________________

$\neg p$	$\neg q$	$\neg p \Longrightarrow \neg q$

d) Verify the contrapositive, or disprove it with a counterexample.

Contrapositive:____

Hypothesis ($\neg q$):______________________ Contrapositive: if __, then ______.

Conclusion ($\neg p$):______________________

$\neg q$	$\neg p$	$\neg q \Longrightarrow \neg p$

Example 2: Examining the relationship between a conditional statement and its contrapositive (p. 210)

Consider the following conditional statement: "If a number is a multiple of 10, then it is a multiple of 5."

a) Write the contrapositive of this statement.

b) Verify that the conditional and contrapositive statements are both true.

In Summary:

- You form the **inverse** of a conditional statement by ________________________ the hypothesis and the conclusion.
- You form the **converse** of a conditional statement by _______________________ the hypothesis and the conclusion
- You form the contrapositive of a conditional statement by ___________________ the hypothesis and the conclusion of it's ____________________.
- If a conditional statement is true, then it's ________________________ is true, and vice versa
- If the inverse of a conditional statement is true, then the ___________________ of the statement is also true, and vice versa.

HW: 3.6 p. 214-216 #1, 5, 6, 7, 9 & 12