FOM 12

Practice Test

Chapter 4 – Counting Methods

Name: ________________

Block: _____

Multiple Choice

Identify the choice that best completes the statement or answers the question.

B

1. Eve can choose from the following notebooks:
• lined pages come in red, green, blue, and purple
• graph paper comes in orange and black

How many different colour variations can Eve choose if she needs one lined notebook and one with graph paper?

Fundamental Counting Principle: If there are "a" ways to perform one task and "b" ways to perform another, then there are a·b ways of performing both.

A. 6
B. $\boxed{8}$
C. 12
D. 16

#variations = (# lined)(# graph)
= (4)(2)
= 8

A

2. A combination lock opens with the correct four-letter code. Each wheel rotates through the letters A to L. How many different four-letter codes are possible?

A. $\boxed{20\ 736}$
B. 48
C. 1728
D. 456 976

It doesn't say that you can't use the same letter more than once ⇒ So repetition is allowed

#different codes = (#Letters)(#Letters)(#Letters)(#Letters)

A B C D E F G H I J K L = 12 possible letters.

different codes = 12×12×12×12 = 20736

C

3. A restaurant offers 60 flavours of wings. How many ways can two people order two servings of wings, either the same flavour or different flavours?

↳ Repetition is OK.

A. 3481
B. 3540
C. $\boxed{3600}$
D. 3660

$$\# \text{possibilities} = \underbrace{60}_{\text{choices for Person 1}} \times \underbrace{60}_{\text{choices for person 2}} = 3600$$

B

4. How many possible ways can you draw a single card from a standard deck and get an even number?

A. 13
$\boxed{\textbf{B.}\ 20}$
C. 21
D. 26

Hearts : 2, 4, 6, 8, 10
Diamonds : 2, 4, 6, 8, 10
Clubs : 2, 4, 6, 8, 10
Spades : 2, 4, 6, 8, 10

$\underbrace{5}_{\text{5 even cards in each suit}} \times \underbrace{4}_{\text{4 suits}} = 20$ ways

B

5. Evaluate.

$$\frac{10!}{9!} + 3! = \frac{10 \cdot \cancel{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}{\cancel{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}} + 3 \cdot 2 \cdot 1$$

A. 13
$\boxed{\textbf{B.}\ 16}$
C. 20
D. 23

$= 10 + 3 \cdot 2 \cdot 1$

$= 10 + 6$

$= 16$

A

6. Identify the expression that is equivalent to the following:

$n(n+1)(n-1)$

$\boxed{\textbf{A.}\ \frac{(n+1)!}{(n-2)!}}$

B. $\frac{(n+2)!}{(n-1)!}$

C. n^3

D. $(n+1)!$

$\boxed{A}$

$$\frac{(n+1)!}{(n-2)!} = \frac{(n+1)(n)(n-1)\cancel{(n-2)(n-3)\cdots(2)(1)}}{\cancel{(n-2)(n-3)\cdots(2)(1)}}$$

$= (n+1)(n)(n-1)$

$= n(n+1)(n-1)$

$\boxed{B}$

$$\frac{(n+2)!}{(n-1)!} = \frac{(n+2)(n+1)(n)\cancel{(n-1)\cdots(2)(1)}}{\cancel{(n-1)(n-2)\cdots(2)(1)}}$$

$= (n+2)(n+1)$

$\boxed{C}$ $n^3 = n \times n \times n$

$\boxed{D}$ $(n+1)!$

$= (n+1)(n)(n-1)(n-2)\cdots(2)(1)$

D

7. How many different permutations can be created when 7 people line up to buy movie tickets?

A. 49
B. 128
C. 720
$\boxed{\textbf{D.}\ 5040}$

7 People in a line:

$$\underline{7}\ \ \underline{6}\ \ \underline{5}\ \ \underline{4}\ \ \underline{3}\ \ \underline{2}\ \ \underline{1}$$

7 choices for first position

1 person is already placed. 6 choices left

Now only one choice for last position.

$$\#\text{Permutations} = (\#\text{ for } 1^{st}\text{ position})(\#2nd)(\#3^{rd})\ldots = 7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1 = 7! = 5040$$

A

8. Evaluate.

$_{14}P_7$

$\boxed{\textbf{A.}\ 17\ 297\ 280}$
B. 2 162 160
C. 121 080 960
D. 105 413 504

$$_nP_r = \frac{n!}{(n-r)!}$$

$$_{14}P_7 = \frac{14!}{(14-7)!} = \frac{14!}{7!}$$

$$= \frac{14\cdot13\cdot12\cdot11\cdot10\cdot9\cdot8\cdot\cancel{7\cdot6\cdots3\cdot2\cdot1}}{\cancel{7\cdot6\cdot5\cdots3\cdot2\cdot1}}$$

$$= 14\cdot13\cdot12\cdot11\cdot10\cdot9\cdot8$$
$$= 17297280$$

A

9. Suppose a word is any string of letters. How many two-letter words can you make from the letters in LETHBRIDGE if you do not repeat any letters in the word?

$\boxed{\textbf{A.}\ 72}$
B. 100
C. 81
D. 90

If you take out the repeated letters you are left with:

LETHBRIDG (9 different letters)

$$\underline{9}\cdot\underline{8} = 72$$

9 options for first letter.

Once the first is chosen, there are 8 options left.

A **10.** How many numbers are there from 1000 to 1999 that do not have any repeated digits?

$\boxed{\textbf{A.}\ 504}$
B. 1000
C. 888
D. 776

(4 digit numbers that begin with 1)

Possible digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

$$\underline{1} \cdot \underline{9} \cdot \underline{8} \cdot \underline{7} = 504$$

the first digit must be a 1 so only 1 option

9 #s to choose from now 1 is gone

8 left

7 left

A **11.** Solve for *r*.

$${}_{15}P_{r-2} = 2730$$

A. $r = 5$
B. $r = 6$
C. $r = 1$
D. $r = 3$

* You can also just substitute in these values and see which one works.

$${}_nP_r = \frac{n!}{(n-r)!}, \quad {}_{15}P_{(r-2)} = \frac{15!}{(15-(r-2))!} = \frac{15!}{(15-r+2)!} = \frac{15!}{(17-r)!}$$

$$\frac{15!}{(17-r)!} = 2730 \Rightarrow 15! = 2730(17-r)! \Rightarrow \frac{15!}{2730} = (17-r)!$$

$$\frac{15 \cdot 14 \cdot 13 \cdot 12!}{2730} = (17-5)! \Rightarrow \frac{\cancel{2730} \cdot 12!}{\cancel{2730}} = (17-r)!$$

$$12! = (17-r)! \Rightarrow 12 = 17 - r \Rightarrow -5 = -r \Rightarrow \boxed{r = 5}$$

$-17 \quad -17$

since there is only one term in each factorial ⇒ they must be equal!

A **12.** Evaluate.

$$\frac{15!}{10! \cdot 3! \cdot 2!} = \frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot \cancel{10!}}{\cancel{10!} \cdot 3! \cdot 2!}$$

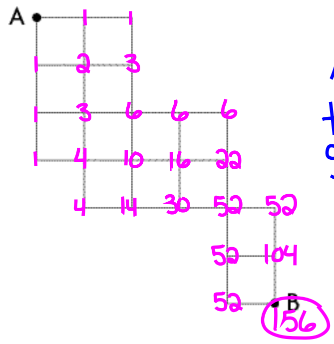
$\boxed{\textbf{A.}\ 30\,030}$
B. 30 300
C. 60 060
D. 60 600

$$= \frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11}{3 \cdot 2 \cdot 1 \cdot 2 \cdot 1}$$

$$= \frac{360360}{12}$$

$$= 30030$$

C 13. How many different routes are there from A to B, if you only travel south or east?



Add to get each value.

Numbers along top and far left stay the same.

A. 128
B. 256
C. $\boxed{156}$
D. 104

B 14. Eight quarters are flipped simultaneously. How many ways can at least six coins land heads?

A. 36
B. $\boxed{37}$
C. 44
D. 56

* Order doesn't matter so it is a Combination.

(# 6 heads) + (# 7 heads) + (# 8 heads)

$_8C_6 + {}_8C_7 + {}_8C_8$

(out of 8 coins choose 6 to be heads) (out of 8 choose 7 to be heads) (out of 8 choose all 8 to be heads)

$= 28 + 8 + 1 = 37$ ways

A 15. The numbers 10 to 16 are written on identical slips of paper and put in a hat. How many ways can 2 numbers be drawn simultaneously?

A. $\boxed{21}$
B. 15
C. 30
D. 42

No Replacement.

Possible Numbers: $\underbrace{10, 11, 12, 13, 14, 15, 16}_{7 \text{ to choose from}}$

Order doesn't matter so it is a combination.

$$_7C_2 = \frac{7!}{2!(7-2)!} = \frac{7 \cdot 6 \cdot \cancel{5!}}{2! \cdot \cancel{5!}} = \frac{7 \times 6}{2} = 21$$

B 16. Identify the term that best describes the following situation:
Determine the number of pizzas with 4 different toppings from a list of 40 toppings.

A. permutations
B. combinations
C. factorial
D. none of the above

order doesn't matter, so out of 40 toppings choose 4.

(In Permutations order is important.) $_{40}C_4$ ← Combination

Short Answer

17. The “Pita Patrol” offers these choices for each sandwich:

- white or whole wheat pitas
- 3 types of cheese
- 5 types of filling
- 12 different toppings
- 4 types of sauce

How many different pitas can be made with 1 cheese, 1 filling, 1 topping, and no sauce?

Pita	Cheese	Filling	Toppings	Sauce
2 options ×	3 options ×	5 options ×	12 options ×	1 option (the only option is "No Sauce")

$2 \times 3 \times 5 \times 12 \times 1 = \boxed{360 \text{ possible pitas}}$

18. Solve for n, where $n \in$ I.

$$\frac{(n+1)!}{2(n-1)!} = 6$$

$n \in I$ means that n is an Integer.

$$\frac{(n+1)(n)\cancel{(n-1)(n-2)\cdots(3)(2)(1)}}{2\cancel{(n-1)(n-2)\cdots(3)(2)(1)}} = 6$$

$$\cancel{2} \times \frac{(n+1)(n)}{\cancel{2}} = 6 \times 2$$

$$(n+1)(n) = 12$$

$$n^2 + n = 12$$

$$n^2 + n - 12 = 0$$

FACTOR!

$n^2 + n - 12$

$4 \times -3 = -12$

$4 + -3 = +1$

	n	$+4$
n	n^2	$4n$
-3	$-3n$	-12

$(n-3)(n+4)$

$$(n-3)(n+4) = 0$$

$\boxed{n = 3}$ OR $n = -4$ X

The only way to get zero when multiplying is if either $(n-3) = 0$ OR $(n+4) = 0$

Check your answers:

$$\frac{(3+1)!}{2(3-1)!} = \frac{4!}{2(2!)} = 6 \checkmark$$

$$\frac{(-4+1)!}{2(-4-1)!} = \frac{(-3)!}{2(-5)!} \times$$

Negative Factorials are undefined so $n \neq -4$

19. How many different arrangements can be made using all the letters in YELLOWKNIFE, if the first letter must be L and the last letter must be Y?

If you take out the L & Y (they MUST be first and last) we are left with the letters: ELOWKNIFE

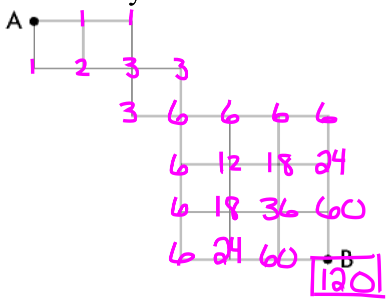
9 letters (with 2 E's)

$$\# \text{arrangements} = \frac{(\text{total letters})!}{\text{factorials of each repeated letter}} = \frac{9!}{2!} \boxed{= 181440}$$

9! → 9 letters

2! → 2 E's

20. How many different routes are there from A to B, if you only travel south or east?



To get each number add the numbers from top and left.

$\boxed{\text{There are 120 possible paths}}$

21. How many 4-person committees can be formed from a group of 8 teachers and 5 students if there must be either 1 or 2 teachers on the committee?

*Order doesn't matter ⇒ Combinations

One Teacher:

$${}_8C_1 \cdot {}_5C_3$$

${}_8C_1$: out of 8 teachers choose 1

${}_5C_3$: out of 5 students choose 3

adds to 4

\+

two teachers:

$${}_8C_2 \cdot {}_5C_2$$

${}_8C_2$: out of 8 teachers choose 2

${}_5C_2$: out of 5 students choose 2

adds to 4

$$= 8 \cdot 10 + 28 \cdot 10$$

$$= 80 + 280 \quad \boxed{= 360 \text{ committees}}$$

*Order doesn't matter ⇒ Combinations

22. From a standard deck of 52 cards, how many different four-card hands are there with at most two diamonds?

No Diamonds + One Diamond + Two Diamonds

$$\underbrace{\underset{\substack{\text{Don't choose}\\\text{any of the}\\\text{13 diamonds}}}{{}_{13}C_0} \cdot \underset{\substack{\text{out of the}\\\text{39 Non-Diamond}\\\text{Cards choose 4}}}{{}_{39}C_4}}_{\text{4 cards in total}} + \underbrace{\underset{\substack{\text{13 diamond}\\\text{cards.}\\\text{Choose 1}}}{{}_{13}C_1} \cdot \underset{\substack{\text{39 Non-}\\\text{Diamond.}\\\text{Choose 3}}}{{}_{39}C_3}}_{\text{4 cards in hand.}} + \underbrace{\underset{\substack{\text{choose 2}\\\text{diamonds}}}{{}_{13}C_2} \cdot \underset{\substack{\text{choose 2}\\\text{Non-Diamonds}}}{{}_{39}C_2}}_{\text{4 cards}}$$

$$= 1 \cdot 82251 + 13 \cdot 9139 + 78 \cdot 741$$

$$= 82251 + 118807 + 57798$$

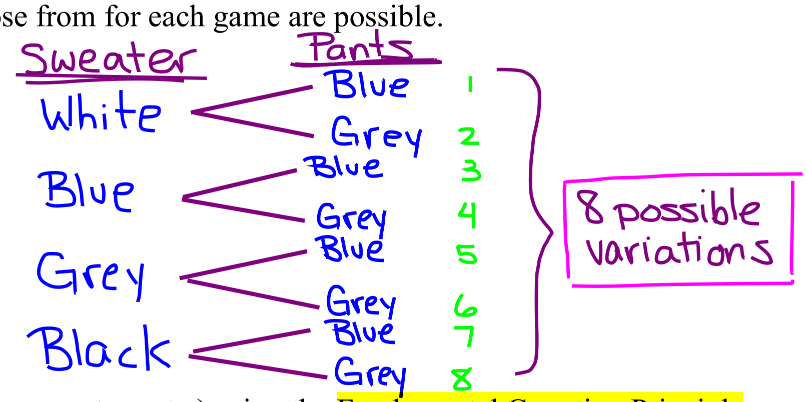
$$\boxed{= 258856 \text{ possible 4-card hands}}$$

Problem

23. Hannah plays on a local hockey team. The hockey uniform has:

- four different sweaters: white, blue, grey, and black, and
- two different pants: blue and grey.

a) Draw a tree diagram to determine how many different variations of the uniform the coach can choose from for each game are possible.



b) Confirm your answer to part a) using the Fundamental Counting Principle.

$$\text{Variations} = (\#\text{possible sweaters})(\#\text{different pants})$$

$$= 4 \cdot 2$$

$$= 8$$

$\boxed{\text{There are 8 different variations}}$

No car is the exact same

24. At a used car lot, $\boxed{\text{8 different car models}}$ are to be parked close to the street for easy viewing, but there is only space for 6 cars. How many ways can 6 of the 8 cars be parked in a row? Show your work.

Order does matter $\Rightarrow$ Permutations

Out of 8 cars Permute 6.

$$_8P_6 = \frac{8!}{(8-6)!} = \frac{8!}{2!}$$

$$= 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3$$

$$\boxed{= 20160 \text{ ways}}$$

* Or you can think about how many cars you have to choose from to place in each position

$$\underbrace{8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3}_{6 \text{ positions}}$$

25. An isogram is a word or phrase without a repeating letter. (No repeated letters in the original word) Vito and Kira are playing a guessing game involving isograms. Kira thinks of a word with no repeating letters. She tells Vito that her word can be used to make 100 one – or two – letter phrases, without repetition. She gives A, EJ, and JE as examples.

her word must have an A, an E, and a J.

a) How many letters are in Kira's word? Show your work

Order matters since EJ and JE are different examples $\Rightarrow$ Permutations

different 1 or 2 letter combinations $= {}_nP_1 + {}_nP_2$

$$100 = \frac{n!}{(n-1)!} + \frac{n!}{(n-2)!} = \frac{n\cancel{(n-1)(n-2)\cdots(3)(2)(1)}}{\cancel{(n-1)(n-2)\cdots(3)(2)(1)}} + \frac{n(n-1)\cancel{(n-2)(n-3)\cdots(2)(1)}}{\cancel{(n-2)(n-3)\cdots}}$$

$$100 = n + n(n-1) = n + n^2 - n$$

$$100 = n^2 \Rightarrow \boxed{n = 10} \text{ or } -10$$

But that would mean the original word has -10 letters. That doesn't make sense. So $n \neq -10$

$\boxed{\text{Kira's original word has 10 letters}}$

b) Which of the following could be Kira's word? Explain your answer.

Switzerland atmospheric lumberjack duplicate trapezoid juxtaposes

Switzerland, atmospheric: This word has 11 letters, so it can't be Kira's word

lumberjack: This word has 10 letters with no repeats. This word contains A, E & J. Kira's word must be Lumberjack

duplicate, trapezoid: only 9 letters

juxtaposes: 10 letters, but there are 2 s's so it can't be this one.

26. There are 18 boys and 13 girls in an English classroom. A group of 6 students is needed to read from a play. If there are 2 roles for boys, 3 roles for girls, and a narrator who could be a boy or a girl, how many different groups of 6 students are possible? Show your work.

You must be very careful when reading these types of questions. The question asks for the number of different groups. So how the parts are divided within the groups doesn't matter ⟹ Combinations.

Narrator is a boy | Narrator is a Girl

$${}_{18}C_3 \cdot {}_{13}C_3 + {}_{18}C_2 \cdot {}_{13}C_4 = (816)(286) + (153)(715)$$

$$= 233376 + 109395$$

${}_{18}C_3$: Narrator is a boy so there are 3 boy parts

${}_{13}C_3$: 3 girl parts

${}_{18}C_2$: 2 boy parts

${}_{13}C_4$: Narrator is a girl so there are 4 girl parts

$\boxed{= 342771}$ different groups

27. Fifteen camp counselors are signing up for training courses that have only a limited number of spaces. Only 5 people can take the water safety course, 4 people can take the first aid course, 3 people can take the conflict management course, and 3 people can take the astronomy course. How many ways can the 15 counselors be placed in the four courses? Show your work.

Within each course order doesn't matter ⟹ Combination

$${}_{15}C_5 \times {}_{10}C_4 \times {}_{6}C_3 \times {}_{3}C_3$$

${}_{15}C_5$: Choose 5 for water safety

${}_{10}C_4$: Now only 10 counselors left to choose from. Choose 4 for first aid.

${}_{6}C_3$: Now only 6 left to choose from. Choose 3 for conflict management

${}_{3}C_3$: the last 3 must be chosen for astronomy.

$$= 3003 \times 210 \times 20 \times 1$$

$\boxed{= 12612600}$ different ways to place the 15 counselors into 4 different courses.