7.1 – EXPLORING THE CHARACTERISTICS OF EXPONENTIAL FUNCTIONS

Exponential Function (Increasing)

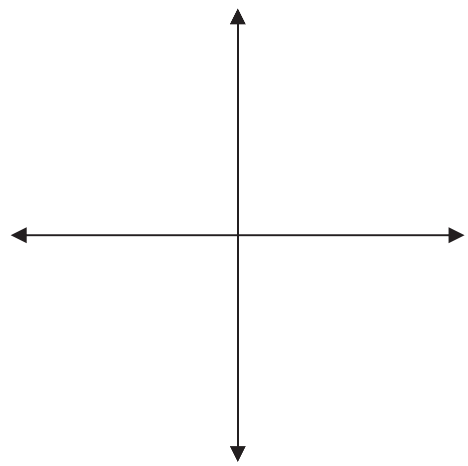
An exponential function is a function of the form $y = a \cdot b^x$, where $a \neq 0$ and $b > 1$.

Investigate the Characteristics of the Graphs of Exponential Functions (Increasing)

Example 1: Graph each exponential function. Determine the number of x-intercepts, the y-intercept, the end behaviour, the domain, and the range.

a. $f(x) = 10^x$

X	*f(x)*
–3	
–2	
–1	
0	
1	
2	
3	

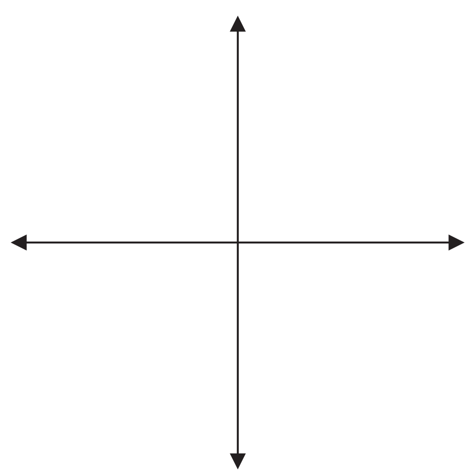


number of *x*-intercepts: *y*-intercept: domain:

range: end behaviour:

b. $g(x) = 2(5)^x$

x	*f(x)*
–3	
–2	
–1	
0	
1	
2	
3	



number of *x*-intercepts: *y*-intercept: domain:

range: end behaviour:

Exponential Function (Decreasing)

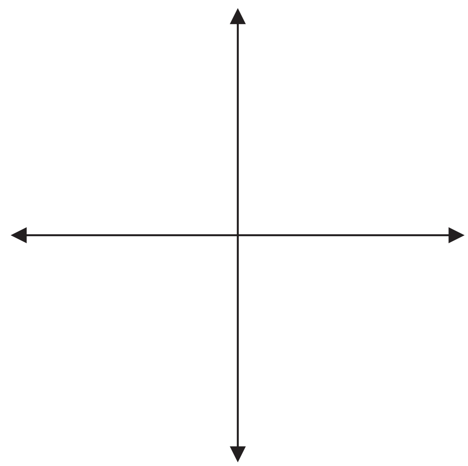
An exponential function is a function of the form $y = a \cdot b^x$, where $a \neq 0$ and $b < 1$.

Investigate the Characteristics of the Graphs of Exponential Functions (Decreasing)

Example 2: Graph each exponential function. Determine the number of x-intercepts, the y-intercept, the end behaviour, the domain, and the range.

a. $h(x) = \left(\frac{1}{2}\right)^x$

x	*f(x)*
–3	
–2	
–1	
0	
1	
2	
3	

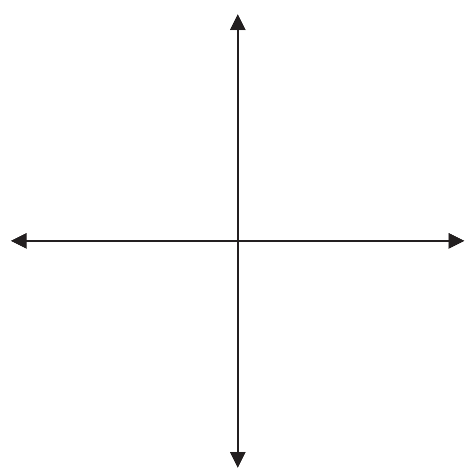


number of *x*-intercepts: *y*-intercept: domain:

range: end behaviour:

b. $j(x) = 8\left(\frac{1}{4}\right)^x$

X	*f(x)*
–3	
–2	
–1	
0	
1	
2	
3	



number of *x*-intercepts: *y*-intercept: domain:

range: end behaviour:

Assignment: p. 439 #1 – 3

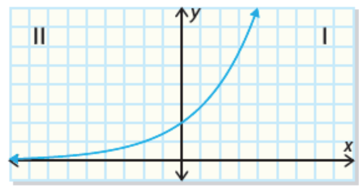
7.2 – RELATING THE CHARACTERISTICS OF AN EXPONENTIAL FUNCTION TO ITS EQUATION

- There are two different shapes of the graphs of an exponential function of the form $f(x) = a(b)^x$, where $a > 0$, $b > 0$, and $b \neq 1$:

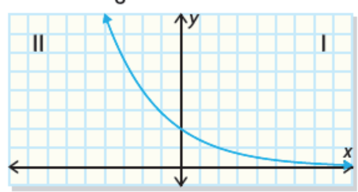
  - Case 1: An increasing function; the curve extends from quadrant II to quadrant I.

  - Case 2: A decreasing function; the curve extends from quadrant II to quadrant I.

Increasing



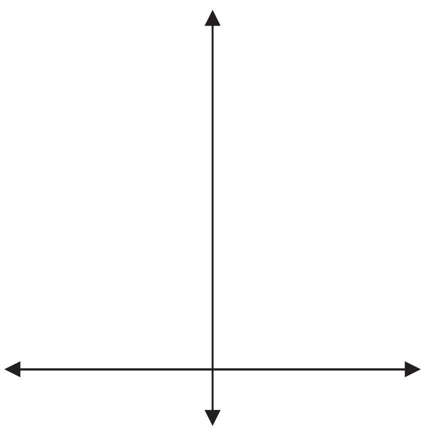
Decreasing



Connect the Characteristics of an Increasing Exponential Function to Its Equation and Graph

Example 1: State the number of x-intercepts, the y-intercept, end behaviour, domain, and range for each function, without graphing the function. Predict whether the function is increasing or decreasing. Verify your answers by graphing.

a. $f(x) = 2(5)^x$



number of x-int =

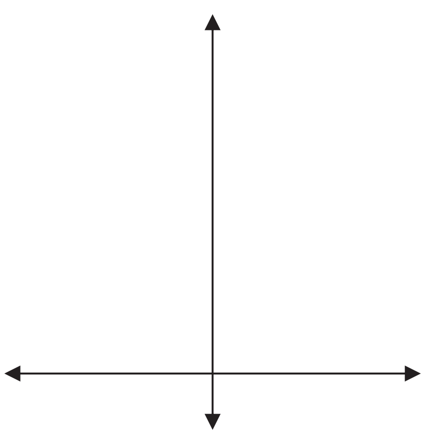
y-int =

end behaviour :

domain :

range :

b. $f(x) = e^x$



a = ; b =

number of x-int =

y-int = end behaviour :

domain : range :

Connect the Characteristics of a Decreasing Exponential Function to Its Equation and Graph

Example 2: State the number of x-intercepts, the y-intercept, end behaviour, domain, and range for each function, without graphing the function. Predict whether the function is increasing or decreasing.

Verify your answers by graphing.

	number of x-intercepts	y-intercept	end behaviour	domain	range	increasing or decreasing
$f(x)=125(0.78)^x$						
$f(x)=0.12(0.85)^x$						
$f(x)=3^x$						
$f(x)=0.85(5)^x$						

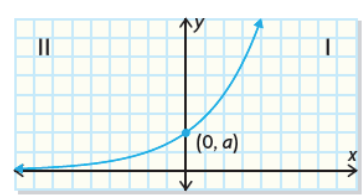
Try: State the number of x-intercepts, the y-intercept, end behaviour, domain, and range for each function, without graphing the function. Predict whether the function is increasing or decreasing.

Verify your answers by graphing.

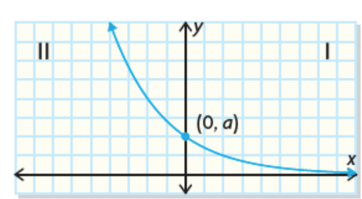
	number of x-intercepts	y-intercept	end behaviour	domain	range	increasing or decreasing
$f(x)=9\left(\frac{2}{3}\right)^x$						
$f(x)=8\left(\frac{3}{4}\right)^x$						

Need to Know

- An exponential function is an increasing function if $a > 0$ and $b > 1$.
- An exponential function is a decreasing function if $a > 0$ and $0 < b < 1$.
- Changing the parameters a and b in exponential functions of the form $y = a(b)^x$, where $a > 0$, $b > 0$, and $b \neq 1$, does not change the number of x-intercepts, the end behaviour, the domain, or the range of the function. These characteristics are identical in all exponential functions of this form.



$a > 0, b > 1$

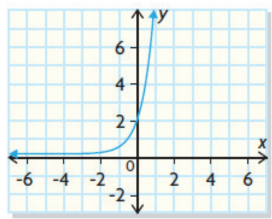


$a > 0, 0 < b < 1$

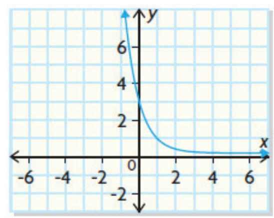
Which exponential function matches each graph below? Provide your reasoning.

i) $y = 3(0.2)^x$ **ii)** $y = 4(3)^x$ **iii)** $y = 4(0.5)^x$ **iv)** $y = 2(4)^x$

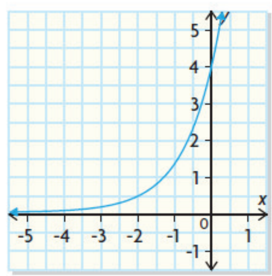
a)



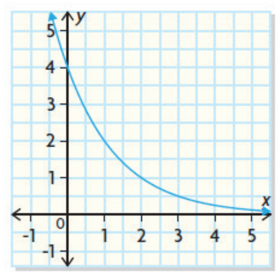
c)



b)



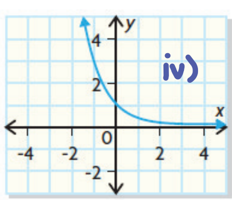
d)



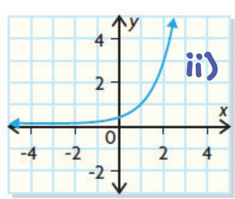
Match each function with the corresponding graph below. Provide your reasoning.

i) $y = (3)^x$ **ii)** $y = \frac{1}{3}(3)^x$ **iii)** $y = 3\left(\frac{1}{3}\right)^x$ **iv)** $y = \left(\frac{1}{3}\right)^x$

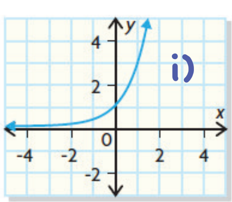
a)



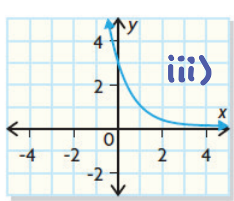
c)



b)



d)



Assignment: p. 448 #1, 2, 5 – 7, 9, 13

7.3 – MODELLING DATA USING EXPONENTIAL FUNCTIONS

You can graph the scatter plot and interpolate using Technology (TI 83).

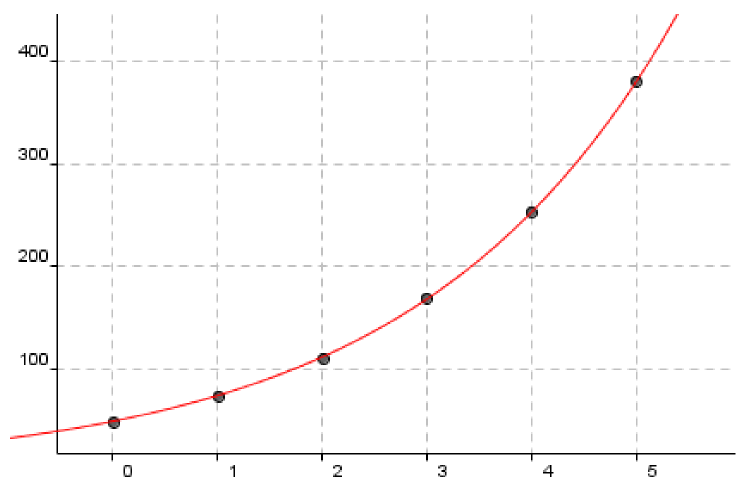
Step 1. Enter the data
→ Press STAT key → Select EDIT → Clear any numbers that are written in L1, L2
→ Under Column L1, enter the data (x-values)
→ Under Column L2, enter the data (y-values)
Step 2. Choose window
→ Press WINDOW and adjust Xmin, Xmax, Ymin, Ymax
→ Graph
Step 3. Obtain the function
→ Press STAT key → Select CALC → Select #0 ExpReg
→ Enter L1 , L2 ,
→ VARS → Select Y-VARS → Select #1 Function → Y1

Make sure Plot1 is highlighted to see the scatter plot

Create Graphical and Algebraic Models of Given Data

Example 1: Simon, a biologist, is investigating a new bacteria culture which could help strengthen a person's immune system. He isolates fifty cells and records the growth in the number of cells over a period of five hours. His results are shown in the table and graph below.

Number of Hours (x)	Number of Bacteria (y)
0	50
1	75
2	112
3	169
4	253
5	380



a. Determine if the data can be represented by an exponential model.

b. Use regression to determine the exponential function that best models the data. Round a and b to three decimal places.

c. Determine the numbers of bacteria, to the nearest whole number, when x = 8.

Example 2: Angela invests $2000 in GIC that increases in value every 3 months. The table below shows the value of the investment during the first 18 month.

Month (x)	0	3	6	9	12	15	18
Value in Dollars (y)	$2000	$2012	$2024.07	$2036.22	$2048.43	$2060.72	$2073.09

a. Use regression to determine the exponential function that best models that data. Give a to the nearest whole number, and b to the nearest thousandth.

b. Determine the value of the investment after two years.

Example 3: The following data represents the winning times, to the nearest minute, for the men's Olympic Marathon in some of the Olympics in the twentieth century.

Year (x)	1900	1912	1928	1936	1960	1972	1984
Time in Minutes (y)	180	157	153	149	135	132	129

a. Use regression to determine the exponential function that best models that data. Give a to the nearest whole number, and b to the nearest thousandth.

b. Estimate the winning time by the Finnish Athlete in the 1924 Olympics.

c. Estimate the winning time by the Czech Athlete in the 1952 Olympics.

Try: The following data gives the population in a town over a period of fifty years.

Year (x)	1	11	21	31	41	51
Time in Minutes (y)	7045	22043	42812	54096	125032	206825

a. Use regression to determine the exponential function that best models that data. Give *a* to the nearest whole number, and b to the nearest thousandth.

b. Estimate population after 35 years.

Assignment: p. 461 #1 – 11 (odds)

7.4 – CHARACTERISTICS OF LOGARITHMIC FUNCTIONS WITH BASE 10 AND BASE *e*

Logarithmic Function

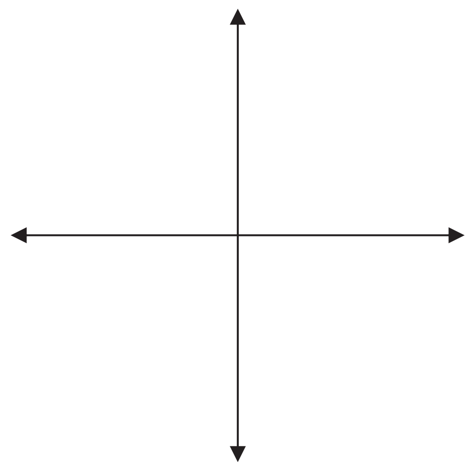
A logarithmic function is a function of the form $y = a\log_b x$, where $a \neq 0$, $b > 0$ and $b \neq 1$.

Investigate the Characteristics of the Graphs of Logarithmic Functions

Example 1: Graph each logarithmic function. Determine the number of x-intercepts, the y-intercept, the end behaviour, the domain, and the range.

a. $f(x) = \log_{10} x$

x	*f(x)*
–1	
0	
1	
2	
3	
4	
5	

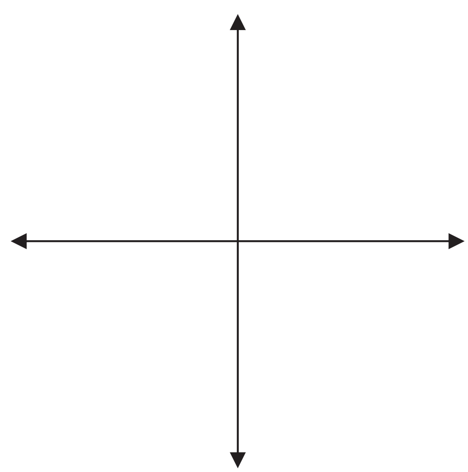


number of *y*-intercepts: *x*-intercept: domain:

range: end behaviour:

b. $g(x) = 2\log_{10} x$

x	*f(x)*
–1	
0	
1	
2	
3	
4	
5	

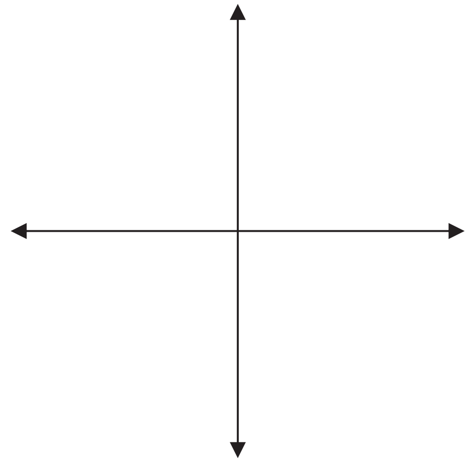


number of *y*-intercepts: *x*-intercept: domain:

range: end behaviour:

c. $h(x) = 5\log_{10} x$

x	$f(x)$
–1	
0	
1	
2	
3	
4	
5	

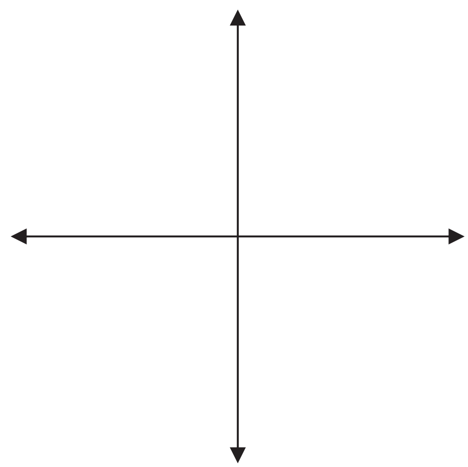


number of y-intercepts: x-intercept: domain:

range: end behaviour:

d. $i(x) = -5\log_{10} x$

x	$f(x)$
–1	
0	
1	
2	
3	
4	
5	



number of y-intercepts: x-intercept: domain:

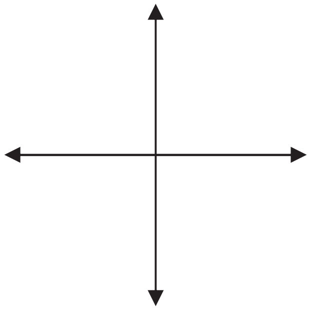
range: end behaviour:

All logarithmic functions of the form $f(x) = a \log x$ or $f(x) = a \ln x$ have these unique characteristics:

- If $a > 0$, the function increases.
- If $a < 0$, the function decreases.

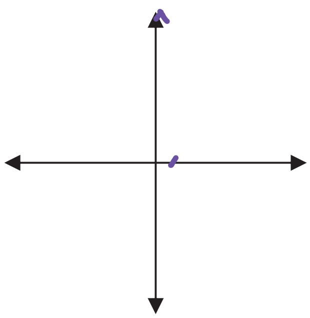
Connect the Characteristics of an Increasing Log Function to Its Equation and Graph

Example 2: Predict the x-intercept, the number of y-intercepts, the end behaviour, the domain, and the range of the function $y = 15\log x$.

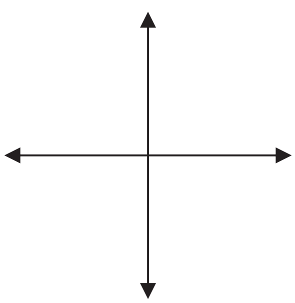


Connect the Characteristics of a Decreasing Natural Log Function to Its Equation and Graph

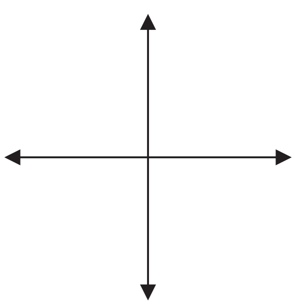
Example 3: Predict the x-intercept, the number of y-intercepts, the end behaviour, the domain, and the range of the function $y = -4\ln x$.



Try: Predict the x-intercept, the number of y-intercepts, the end behaviour, the domain, and the range of this function $y = 12\ln x$.



Try: Predict the x-intercept, the number of y-intercepts, the end behaviour, the domain, and the range of this function $y = -5\log x$.

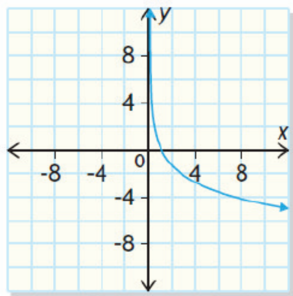


Match Equations of Exponential and Log Functions with Their Graphs

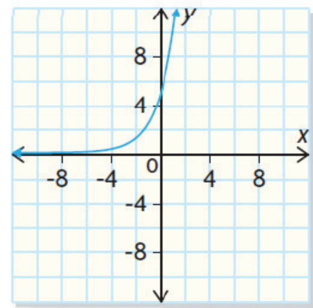
Example 4: Which exponential function matches each graph below? Provide your reasoning.

a. $y = 5(2)^x$ b. $y = 2(0.1)^2$ c. $y = 6\log x$ d. $y = -2\ln x$

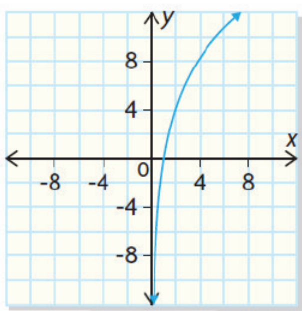
a)



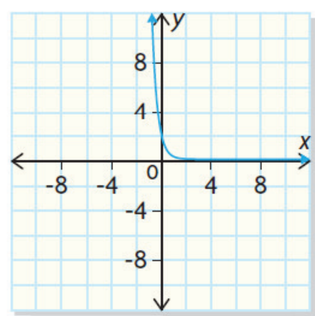
b)



c)

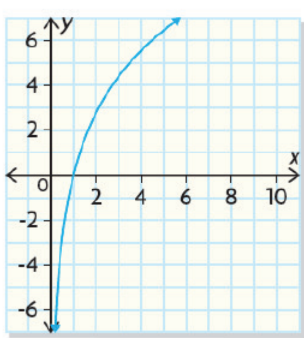


d)

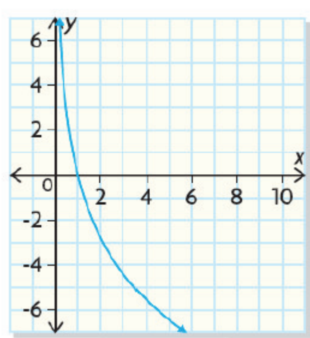


The graph of a logarithmic function of the form
$f(x) = a \log x$ or $f(x) = a \ln x$ will look like one of the following cases:

Case 1: an increasing function, where $a > 0$



Case 2: a decreasing function, where $a < 0$



7.5 – MODELLING DATA USING LOGARITHMIC FUNCTIONS

Step 1. Enter the data

→ Press STAT key → Select EDIT → Clear any numbers that are written in L1, L2

→ Under Column L1, enter the data (*x*-values)

→ Under Column L2, enter the data (*y*-values)

Step 2. Choose window

→ Press WINDOW and adjust Xmin, Xmax, Ymin, Ymax

→ Graph

Make sure Plot1 is highlighted to see the scatter plot

Step 3. Obtain the function

→ Press STAT key → Select CALC → Select #9 LnReg

→ Enter L1 , L2 ,

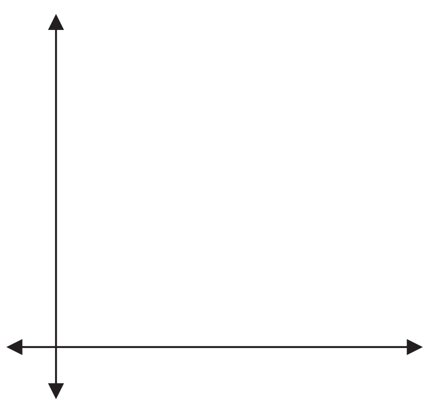
→ VARS → Select Y-VARS → Select #1 Function → Y1

Use Log Regression to Solve a Problem Graphically and Algebraically

Example 1: The decay of radioactive elements can sometimes be used to date events from the earth's past. In a living organism, the ratio of radioactive carbon, carbon-14, to ordinary carbon remains fairly constant. However, when the organism dies, no new carbon is ingested and the proportion of carbon -14 decreases as it decays. The table below shows data for five recently discovered fossils.

% carbon-14 (*x*)	95	79	68	38	27
Age in years (*y*)	425	1950	3191	8000	10824

a. Determine if the data can be represented by a log model.



b. Use the natural log regression feature of a calculator (LnReg) to determine a function that models the data. Use integer values for *a* and *b*.

c. A bone fragment was discovered. If the carbon dating test indicated that approximately 20.3% of carbon-14 was left, estimate the age of the bone fragment to the nearest 1000 years.

Example 2: The number of years, y, that it takes for an investment of $1000 to increase in value to x dollars can be modelled by a log function. The table shows the value of Scott's investment over a period of 10 years.

Value (x)	1082	1170	1265	1369	1480
Years (y)	2	4	6	8	10

a. Use the natural log regression feature of a calculator (LnReg) to determine a function that models the data. Round a and b to three decimal places.

b. Estimate the number of years if the value of investment is $1800.37.

Try: Martin is a fruit grower. He has planted and tracked the growth of a new variety of cherry tree he is considering planting on 10 acres of his farm.

Age of Tree (years)	Height (feet)	Age of Tree (years)	Height (feet)
1	5.0	7	18.8
2	9.2	8	19.0
3	13.1	9	19.3
4	15.0	10	19.7
5	16.8	11	20.0
6	17.1	12	20.8

a. Determine the equation of the log regression function that models the tree's growth.

b. Determine the height of a tree of this variety when it is 15 years old.

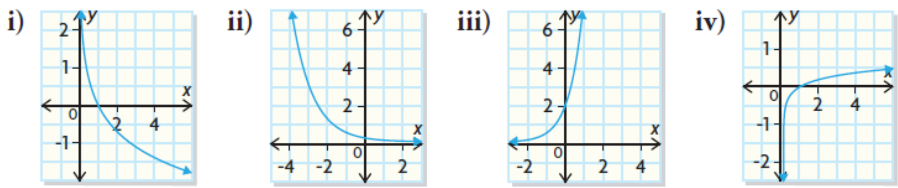
c. Determine the age of a tree of this variety when it is 12 feet tall.

CHAPTER 7 REVIEW

1. Match each function with the corresponding graph below. Provide your reasoning.

a. $y = 0.2(0.4)^x$ b. $y = 2(4)^x$

c. $y = 0.5\log x$ d. $y = -2\log x$



2. The table to the right shows the Canadian government's net debt, in billions of dollars.
   a. Create graphical and algebraic exponential models for the data.
   b. What was the approximate net federal debt in 1988, to the nearest hundredth of a billion dollars?
   c. Assuming the same growth rate, when did the net federal debt reach \$600 billion?

years since '55	Year	Net Federal Debt (\$ billions)
0	1955	17.56
5	1960	20.40
10	1965	26.84
15	1970	35.82
20	1975	55.13
25	1980	110.61
30	1985	250.52
35	1990	406.61
40	1995	550.69
45	2000	561.73

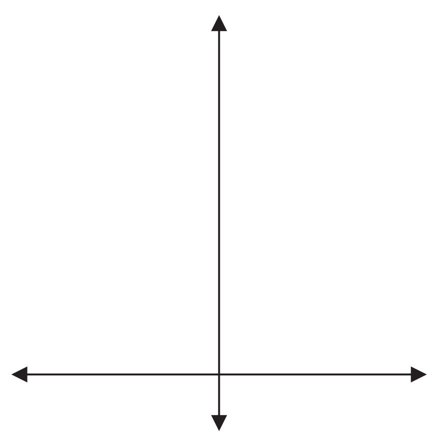
Statistics Canada

3. Predict the number of x-intercepts, the y-intercept, the end behaviour, the domain, and the range of the function

$$f(x) = 6\left(\frac{1}{4}\right)^x$$

Use the equation of the function to make your predictions. Verify your predictions using graphing technology.

x-intercept	
y-intercept	
end behaviour	
domain	
range	

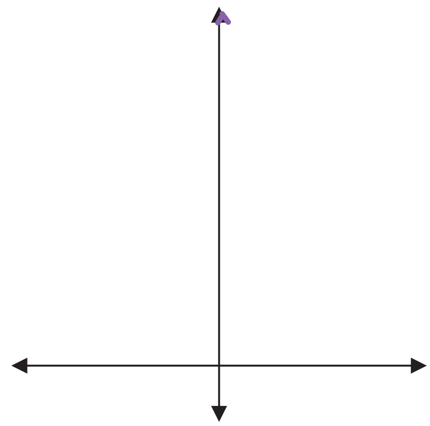


4. Use the characteristics below to describe the graph of this function:

$$y = -7 \ln x$$

- the location of any intercepts
- the end behaviour
- the domain and range
- whether the function is increasing or decreasing

x-intercept	
y-intercept	
end behaviour	
domain	
range	



5. The table to the right shows the approximate energy, in kilojoules (kJ), that is released by earthquakes of different magnitudes. In 1960, the Valdivia earthquake in Chile released approximately 1.1×10^{16} kJ of energy.

Energy Released (kJ)	Magnitude of Earthquake
63	0
2 000	1
63 000	2
2 000 000	3
63 000 000	4
2 000 000 000	5

a. Determine the equation of the logarithmic regression function for the given data.

b. Use the equation of the logarithmic regression function to determine the magnitude of this earthquake to the nearest tenth.

Assignment: p. 504 #1 – 11