

Circular Motion and Gravitation Notes 1 – Centripetal Acceleration and Force

This unit we will investigate the special case of kinematics and dynamics of objects in **uniform circular motion**.

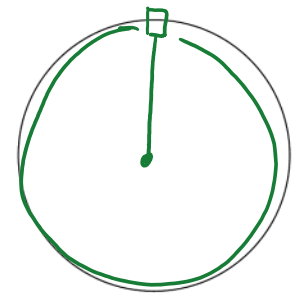
First let's consider a mass on a string being twirled in a horizontal circle at a constant speed.

Let's determine the speed of the object.

Remember that speed is defined as: $v = d/t$

We define the period of motion (T) as the time it takes to complete one rotation.
How far does it travel in one rotation? *Circumference*

We can find the circumference of the circular path (distance traveled) by: $C = 2\pi r$



Therefore the speed of an object in uniform circular motion is:

$$V = \frac{2\pi r \rightarrow (m)}{T \leftarrow \text{period (s)}}$$

Example:

A plane makes a complete circle with a radius of 3622 m in 2.10 min. What is the speed of the plane?

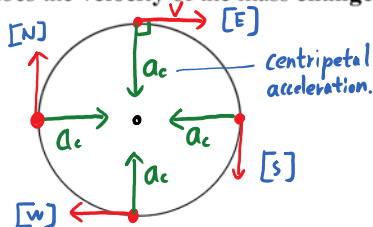
$$V = \frac{2\pi(3622)}{126 s} = \boxed{181 \text{ m/s}}$$

$\frac{\Delta \vec{v}}{\Delta t}$ ←

Ok so we've figured out its speed, but is the mass accelerating?

Remember that the mass is traveling at a constant speed. However, *acceleration* is defined as:

So how does the **velocity** of the mass **change** with respect to **time**?



Notice that the direction of the velocity at any time is ...

tangent to the circle!!

So even though it may be traveling at a constant speed anything traveling in a circular path is accelerating because the direction of its velocity is always changing.

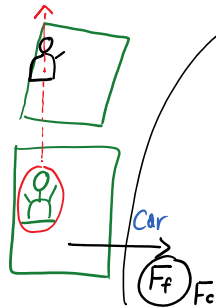
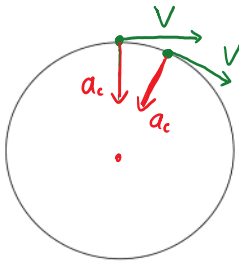
The acceleration of an object in uniform circular motion is:

$$v = \frac{2\pi r}{T} \quad v^2 = \frac{4\pi^2 r}{T^2}$$

Formula sheet.

$$a_c = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$$

Let's do a quick derivation of this formula:



It is worth noting from the above derivation that the direction of the change in velocity is always...

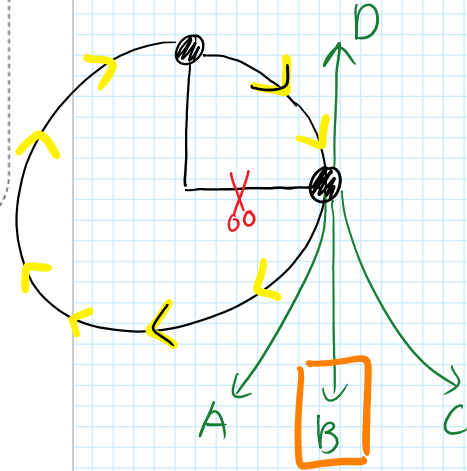
toward the centre of the circle.

Therefore the acceleration of an object in circular motion is always towards the...

toward the centre !!

This is the definition of centripetal, which means: center-seeking.

horizontal circle



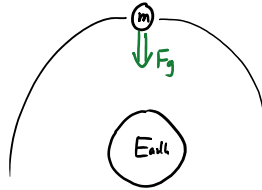
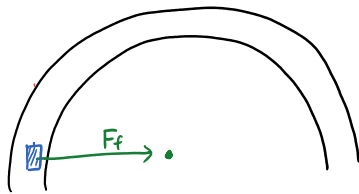
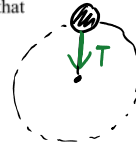
Whenever an object is accelerated there must be a...

net Force $F_c = ma_c$

This force is known as centripetal force, F_c . This is not a new force, it is simply the net force that accelerates an object towards the center of its circular path.

Examples:

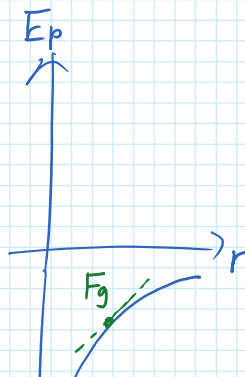
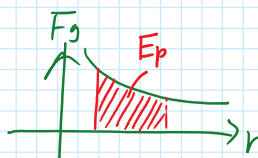
- 1) A mass is twirled in a circle at the end of a string, the centripetal force is provided by Tension
- 2) When a car rounds a corner on a highway, the centripetal force is provided by Friction
- 3) When the Moon orbits the Earth, the centripetal force is provided by Gravity.

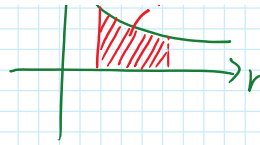


$$E_p = -\frac{GM_1 m_2}{r} = -GMm r^{-1}$$

$$\frac{dE_p}{dr} = -GMm (-1) r^{-2} = \frac{GMm}{r^2} = F_g$$

$$F_g = \frac{dE_p}{dr}$$





$F_{net} = ma$ $a_c = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$ // F_{net} that point toward the centre!!
 Newton's Second Law we can help us to determine a formula for centripetal force:

$$F_c = \frac{m v^2}{r} = \frac{m 4\pi^2 r}{T^2}$$

not on formula sheet.

Example:
 a. A skater travels at 2.0 m/s in a circle of radius 4.0 m . What is her centripetal acceleration?

$$a_c = \frac{v^2}{r} = \frac{2^2}{4} = 1 \text{ m/s}^2$$

$T = 2.36 \times 10^6 \text{ s}$ $r = 3.84 \times 10^8$

b. What is the centripetal acceleration of the Moon towards Earth?

$$a_c = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2 (3.84 \times 10^8)}{(2.36 \times 10^6)^2}$$

$$= 0.00272 \text{ m/s}^2$$

2.7×10^{-3}

Example: Level 1
 A car traveling at 14 m/s goes around an unbanked curve in the road that has a radius of 96 m . What is its centripetal acceleration?

$$a_c = \frac{v^2}{r} = \frac{14^2}{96} = \frac{196}{96} = 2.04 \text{ m/s}^2$$

What is the minimum coefficient of friction between the road and the car's tires?

$$F_{net} = F_f$$

$$F_c = F_f$$

$$\frac{m v^2}{r} = \mu F_N \Rightarrow \frac{m v^2}{r} = \mu m g$$

$$\frac{14^2}{96} = \mu (9.8)$$

$\mu = 0.208$

One last note on a little thing called centrifugal force. While centripetal means center-seeking centrifugal means center-fleeing.

An inertial frame of reference is a one where Newton's Law's are true. In an inertial frame of reference, centrifugal force is actually an apparent force - it does not exist. It is simply the apparent force that causes a revolving or rotating object to move in a straight line.

However, Newton's First Law tells us that we do not need a force to keep an object moving in a straight line, you only need a force to deflect an object from moving in a straight line.

Example:
 When riding in the backseat of a car that is turning a corner, you slide across the seat, seeming to accelerate outwards, away from the center of the turning circle. Explain why the force in this case is actually working towards the center of the turn and not outwards.

$$F_{net} = F_T$$

$$F_c = F_T \Rightarrow \frac{m_1 4\pi^2 r}{T^2} = m_2 g$$

$$\frac{m_1 v^2}{r} = F_{g2}$$

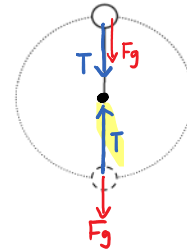
$$m_1 \frac{v^2}{r} = m_2 g$$

Circular Motion and Gravitation Notes

2 – More Centripetal Problems

We have already seen the forces acting on a mass moving in a horizontal circle, now let's see how this differs from a mass moving in a vertical circle.

Draw the forces acting on a mass on a string being spun in a vertical circle at the top and bottom of its path.



As with any object moving in a circle there is a net force acting on it. toward the center of circle.

This net force is a centripetal force.

Notice that at the top of its arc the centripetal force (or net force) is:

$$F_c = T + F_g \Rightarrow T = F_c - F_g$$

Also at the bottom of the arc the centripetal force is:

$$F_c = T - F_g \Rightarrow T = F_c + F_g$$

Example:

A 1.7 kg object is swung from the end of a 0.60 m string in a vertical circle. If the time of one revolution is 1.1 s, what is the tension in the string:

a) at the top?

$$F_c = F_g + F_T$$

$$F_T = F_c - mg$$

$$F_T = \frac{m4\pi^2r}{T^2} - mg$$

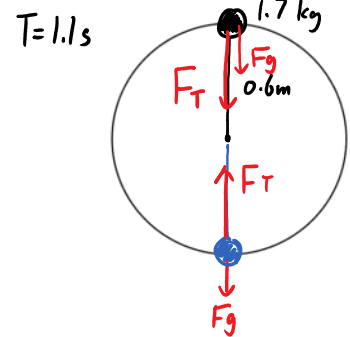
$$F_T = \frac{(1.7)4\pi^2(0.6)}{(1.1)^2} - (1.7)(9.8)$$

$$F_T = 33.3 - 16.66 = 16.6 \text{ N (Top Tension (min))}$$

b) at the bottom?

$$F_c = F_T - F_g$$

$$F_T = F_c + mg = 33.3 + 16.66 = 50 \text{ N (bottom Tension)}$$



Now suppose the mass is spun with just enough speed to keep it moving in a circular path. What is the tension in the string at the top?

$$T = 0$$

We say that the mass at the peak of the arc is weightless, because the net force working on it is only gravity. This is the same as an object in total free fall.

Example:

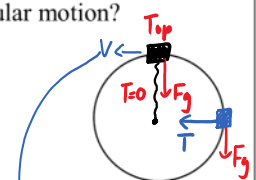
An object is swung in a vertical circle with a radius of 0.75 m. What is the minimum speed of the object at the top of the motion for the object to remain in circular motion?

$$F_c = F_g + T \leftarrow T=0$$

$$\frac{mv^2}{r} = mg$$

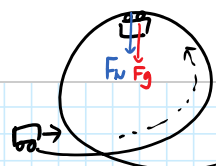
$$v = \sqrt{rg} = \sqrt{0.75(9.8)} = 2.7 \text{ m/s}$$

min speed at the top.



$$F_c = T$$

$$\frac{mv^2}{r} = T$$



Top $F_c = F_g + F_N = 0 \leftarrow$ min speed Same result $v_{min} = \sqrt{rg}$

If Notice that the velocity of the object... constant (horizontal) $V = \frac{2\pi r}{T}$
 it depends only on... radius and period.

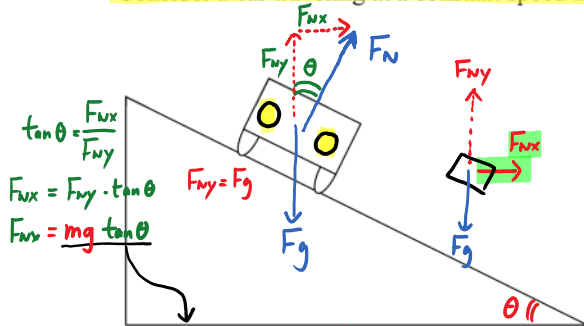
For the special case of finding the minimum speed of an object at the top of its circular arc we can use the equation:

$$F_c = F_g \rightarrow V = \sqrt{rg} \text{ (min @ top)}$$

(horizontal) Banked Curves (and other 2-D Problems):

When cars travel at high speeds on highways, they do not rely solely on friction to keep the cars from sliding off the road. A greater centripetal force can exist if the turn is banked.

Consider a car traveling at a constant speed around a frictionless banked corner. (Level 2)



On a frictionless corner only F_g and F_N act on the car.
 Note that in this case F_N is larger because it both:
 (1) matches F_g
 (2) accelerates inwards

$\tan \theta = \frac{F_{Nx}}{F_{Ny}}$
 $F_{Nx} = F_{Ny} \cdot \tan \theta$
 $F_{Nx} = mg \tan \theta$
 $F_{ny} = F_g$

$\therefore F_c = F_{Nx}$
 $\frac{mv^2}{r} = mg \tan \theta \therefore V = \sqrt{rg \tan \theta}$ (The only speed allowed!!)

The sum of F_N and F_g must equal F_c

Example
 Calculate the angle at which a frictionless curve must be banked if a car is to round it safely at a speed of 22 m/s if its radius is 475 m.

$$\frac{v^2}{r} = g \tan \theta \quad \tan \theta = \frac{v^2}{rg} = \frac{(22)^2}{475 \cdot 9.8}$$

$\theta = 6^\circ$

Example
 A 0.25 kg toy plane is attached to a string so that it flies in a horizontal circle with a radius of 0.80 m. The string makes a 28° angle to the vertical. What is its period of rotation?

$F_c = F_{Tx}$
 $\frac{m4\pi^2 r}{T^2} = 1.3 \text{ N}$
 $\frac{0.25(4\pi^2 \cdot 0.8)}{1.3} = T^2$
 $T = 2.46 \text{ second}$

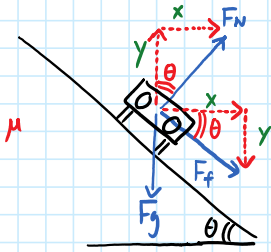
$F_g = 2.45 \text{ N} = F_{Ty}$
 $\tan(28^\circ) = \frac{F_{Tx}}{2.45}$
 $F_{Tx} = 1.3 \text{ N}$

friction but no angle (Level 1) $V = \sqrt{rg}$
 Banked no friction (Level 2) $V = \sqrt{rg \tan \theta}$

Banked Curve (Level 3) With Friction

Banked Curve (Level 3) With Friction

A) max speed



$$x \quad F_c = F_{Nx} + F_{fx}$$

$$\frac{mv^2}{r} = F_N \sin \theta + F_f \cos \theta$$

$$\frac{mv^2}{r} = F_N [\sin \theta + \mu \cos \theta]$$

$$\frac{mv^2}{r} = mg \frac{[\sin \theta + \mu \cos \theta]}{\cos \theta - \mu \sin \theta}$$

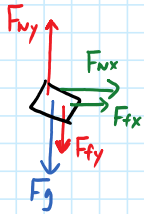
$$y \quad F_{Ny} = F_g + F_{fy}$$

$$F_N \cos \theta = mg + F_f \sin \theta$$

$$F_N \cos \theta - \mu F_N \sin \theta = mg$$

$$F_N [\cos \theta - \mu \sin \theta] = mg$$

$$F_N = \frac{mg}{\cos \theta - \mu \sin \theta}$$



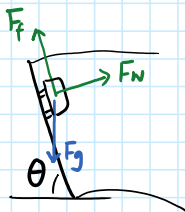
$$\therefore V_{\max} = \sqrt{rg \frac{\sin \theta + \mu \cos \theta}{\cos \theta - \mu \sin \theta}}$$

no friction $\mu = 0$
 $V = \sqrt{rg \tan \theta}$

friction, $\theta = 0$
 $V = \sqrt{rg \frac{\sin \theta + \mu \cos \theta}{\cos \theta - \mu \sin \theta}} = \sqrt{\mu rg}$

Min speed

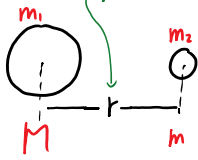
$$V_{\min} = \sqrt{rg \frac{\sin \theta - \mu \cos \theta}{\cos \theta + \mu \sin \theta}}$$



Quiz on Note 1 + 2
next day

Circular Motion and Gravitation Notes
3 - Gravitation

Newton discovered that gravity attracts any two objects depending on their masses and their distant apart.



F_g is proportional to the two masses

$F_g \propto m_1 m_2$ ex, $m_1 \rightarrow 4m_1$ then $F_g \rightarrow 4F_g$

F_g is inversely proportional to the square of the distance between their centers of mass

$F_g \propto \frac{1}{r^2}$ ex, $r \rightarrow 3r$ then $F_g \rightarrow \frac{1}{9}F_g$ (9 times weaker)

or $r \rightarrow \frac{1}{2}r$ then $F_g \rightarrow 4F_g$

$$F_g = \frac{G m_1 m_2}{r^2}$$

Where:
 $G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$
 $m_1, m_2 = \text{masses (kg)}$
 $r = \text{dist. center to center (m)}$

Example E1 x10⁷ E1
Calculate the force of gravity between two 75 kg students if their centers of mass are 0.95 m apart.

$$F_g = \frac{G m_1 m_2}{r^2} = \frac{6.67 \times 10^{-11} (75 \text{ kg})(75)}{(0.95)^2}$$

$$F_g = 4.2 \times 10^{-7} \text{ N}$$

That is why you can't feel it !!

Common Question Alert!!!

You will often see problems that ask something like this...

A satellite weighs 9000 N on Earth's surface. How much does it weigh if its mass is tripled and its orbital radius is doubled?

$F_g = 9000 \text{ N}$

$F_{g \text{ old}} = \frac{GMm}{r^2} = 9000 \text{ N}$

$F_{g \text{ new}} = \frac{GM(3m)}{(2r)^2} = \frac{3}{4} \frac{GMm}{r^2}$

$F_{g \text{ new}} = \frac{3}{4} F_{g \text{ old}}$

$= \frac{3}{4} [9000 \text{ N}] = 6750 \text{ N}$

Ex2) $m \rightarrow 2m$
 $r \rightarrow \frac{1}{5}r$

$$F_{g \text{ new}} = \frac{GM(2m)}{(\frac{1}{5}r)^2} = \frac{2}{\frac{1}{25}} \frac{GMm}{r^2} = 50 \frac{GMm}{r^2} = 50 F_{g \text{ old}}$$

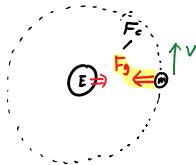
While we're at it, let's make sure we clear up another common misconception: mass vs. weight.

- Mass:**
- Amount of matter
 - Constant everywhere.

- Weight:**
- gravitational attraction (F_g)
 - Changes depending on location.

Satellites in Orbit

A satellite of the Earth, such as the moon, is constantly falling. But it does not fall towards the Earth, rather it falls around the Earth. Just as if you were in an elevator that was falling towards the Earth you would feel weightless if you were on an artificial satellite falling around the Earth. Consider the Moon:



Example: A 4500 kg Earth satellite has an orbital radius of $8.50 \times 10^7 \text{ m}$. At what speed does it travel?

Orbital Speed



$F_c = F_g$

$\frac{mv^2}{r} = \frac{GMm}{r^2}$

$v = \sqrt{\frac{GM}{r}}$

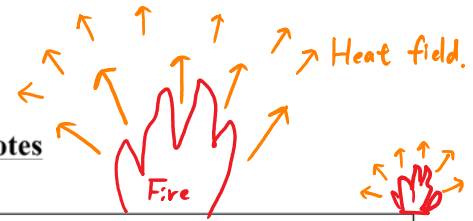
Orbital Speed

Speed of the satellite does not depend on its own mass.

mass of the center by object !!

$v = \sqrt{\frac{6.67 \times 10^{-11} (5.98 \times 10^{24} \text{ kg})}{8.5 \times 10^7 \text{ m}}} = 2166 \text{ m/s}$

Circular Motion and Gravitation Notes
4 – Gravitational Fields



Scientists had difficulty explaining how two objects that are not in contact can exert a force on one another. In order to help conceptualize how this can occur, we had invented the idea of FIELDS.

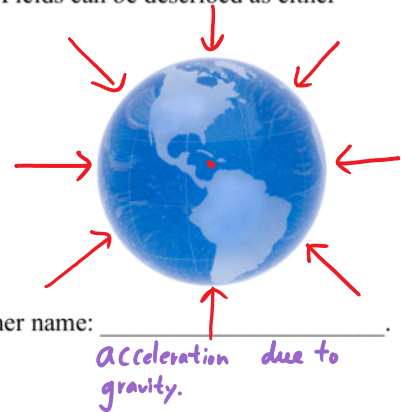
To help imagine how these fields work, consider a **campfire**. It seems as though the fire is emitting a *heat field*.

As you approach the fire the ...
the field strength increases.
As you increase the size of the fire the ...
the field strength increases.

A field is defined as ...
An area of influence.

Just like this so-called heat field, gravitational fields surround any mass. Fields can be described as either **vector** or **scalar**.

While heat is measured by temperature (a scalar) its field is also scalar. **Gravitational fields** are **force fields** and as such are vector.



Vector fields, like vector quantities, are represented by arrows. In this case, the density of the arrows represents the magnitude of the field strength...

We are already quite familiar with gravitational field strength by its other name: acceleration due to gravity.

Recall that: $F_g = mg$

Therefore

$$F_g = mg$$

$$g = \frac{F_g}{m}$$

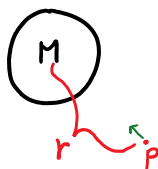
Where **g** = acceleration due to gravity
= gravitational field strength
= 9.80 m/s² near Earth's surface

This formula works fine if we stay put on Earth, but it falls way short once we leave Terra Firma because ...
"g" varies with distance.

However, we can derive a more useful formula:


$$F_g = \cancel{m}g = \frac{GM\cancel{m}}{r^2}$$

$$g = \frac{GM}{r^2}$$



Example: g
 What is the gravitational field strength on the surface of the Moon?

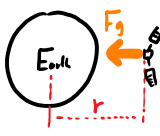
$m_{\text{moon}} = 7.35 \times 10^{22} \text{ kg}$
 $r_{\text{moon}} = 1.74 \times 10^6 \text{ m}$



$$g = \frac{GM}{r^2} = \frac{(6.67 \times 10^{-11}) (7.35 \times 10^{22} \text{ kg})}{(1.74 \times 10^6)^2}$$

$$g = 1.62 \text{ [m/s}^2\text{]} \text{ or } \left[\frac{\text{N}}{\text{kg}} \right] \quad g = \frac{F_g \text{ (N)}}{m \text{ (kg)}}$$

Example:
 A satellite orbits the Earth at a radius of $2.20 \times 10^7 \text{ m}$.
 What is its orbital period?



$$F_c = F_g$$

$$\frac{4\pi^2 r}{T^2} = \frac{GM}{r^2}$$

$$\frac{4\pi^2 r^3}{GM} = T^2$$

$$T = \sqrt{\frac{4\pi^2 (2.2 \times 10^7)^3}{6.67 \times 10^{-11} (5.98 \times 10^{24})}} = 32464 \text{ sec} \approx 9 \text{ hrs}$$

$$= 3.25 \times 10^4 \text{ sec}$$

Geosynchronous Orbit

The orbital speed of a satellite will depend on the strength of gravitational field at the orbital radius.

Consider the following situations. Which identical satellite will be travelling faster in each case? Why?

- a) Satellite A orbits the Earth at twice the orbital radius of Satellite B.

 \therefore Satellite B b/c it is closer $\Rightarrow F_g$ is stronger. \rightarrow must move faster to stay in orbit.

- b) Satellite A orbits the Sun at the same orbital radius that Satellite B orbits the Earth.

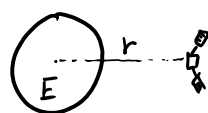
 b/c F_g is stronger for A

The orbital period of the satellite depends only on the mass of the planet and the orbital radius of the satellite. It stands to reason therefore that at a certain orbital distance the orbital period will match the rotational period of the planet. Such a satellite is said to be in geosynchronous (or geostationary) orbit.

Example:

Find the orbital radius of a satellite that is geostationary above Earth's equator.

$T_{\text{Earth}} = 8.61 \times 10^4 \text{ sec}$ $24 \text{ hr} \Rightarrow 23 \text{ hr } 56 \text{ min } 4 \text{ sec}$



$$\frac{4\pi^2 r}{T^2} = \frac{GM}{r^2} \quad r^3 = \frac{GMT^2}{4\pi^2} \quad r = \sqrt[3]{\frac{6.67 \times 10^{-11} (5.98 \times 10^{24}) (8.61 \times 10^4)^2}{4\pi^2}}$$

$$r = 4.2 \times 10^7 \text{ m}$$

What is the speed of this satellite?

$$V = \frac{2\pi r}{T} = \frac{2\pi (4.2 \times 10^7)}{(8.61 \times 10^4)} = 3.1 \times 10^3 \text{ m/s}$$

WS # 4

$$F_g = \frac{dE_p}{dr}$$

Circular Motion and Gravitation Notes
5 – Potential Energy, Satellites and Escape Velocity

Gravitational Potential Energy

We have already discussed gravitational potential energy.

$$E_p = mgh$$

However, as we have seen, g is not a constant but rather depends on mass of planet and distance.

Let's Derive!!!

$$E_p = mgh$$

$$E_p = m \left(\frac{GM}{r^2} \right) h$$

$$= \frac{GMm}{r}$$

$$g = \frac{GM}{r^2} \text{ and } E_p = mgh$$

$$E_p = -\frac{GMm}{r}$$



If you look at your formula sheet you will notice that this equation has a negative sign. What's the deal?

Whenever we talk about gravitational potential energy, we have to use a reference point. At this reference point we assign a gravitational potential energy of zero.

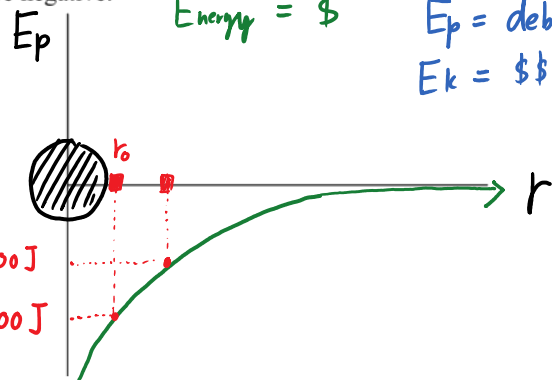
When determining the potential energy on a mass provided by the gravitational force generated by a second mass, we assign the ZERO reference point when the distance between the objects is infinite.

This means whenever the objects get closer together the potential energy between them gets less. Compared to infinity the potential energy of the object will always be negative.

Energy = \$
 $E_p = \text{debt } (-)$
 $E_k = \$\$ \text{ you have } (+)$

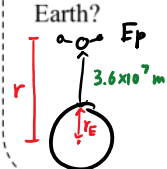
Let's sketch a graph showing the relationship between gravitational potential energy of one object relative to another and the distances between their centers.

$$E_p = -\frac{GMm}{r} \quad E_p \propto -\frac{1}{r}$$



Example:

A 2500 kg satellite is in orbit 3.60×10^7 m above the Earth's surface. What is the gravitational potential energy of the satellite due to the gravitational force due to the Earth?



$$r = 3.6 \times 10^7 + 6.38 \times 10^6$$

$$r = 4.238 \times 10^7 \text{ m}$$

$$E_p = -\frac{GMm}{r}$$

$$E_p = -\frac{6.67 \times 10^{-11} (5.98 \times 10^{24}) (2500)}{4.238 \times 10^7}$$

$$E_p = -2.35 \times 10^{10} \text{ J}$$

Note: The potential energy of this satellite relative to some infinite position is... negative.

Example cont: E_k E_p ✓
 What is the total energy of the satellite in the last question?
 $E_p = -2.35 \times 10^{10} \text{ J}$
 5.98×10^{24}
 6.67×10^{-11}
 4.238×10^7
 9.41×10^6
 $F_c = F_g$
 $\frac{mv^2}{r} = \frac{GMm}{r^2}$
 $v^2 = \frac{GM}{r}$
 $E_k = \frac{1}{2} m v^2$
 $E_k = \frac{1}{2} (2500 \text{ kg}) [9.41 \times 10^6] = 1.176 \times 10^{10} \text{ J}$
 $E_T = E_k + E_p = -1.17 \times 10^{10} \text{ J}$

The funny thing about satellites...
 $E_k = \frac{1}{2} m v^2 = -E_p$
 $E_k = \frac{1}{2} m \frac{GM}{r} = \frac{1}{2} \frac{GMm}{r}$
 $E_k = -\frac{1}{2} E_p$
 $E_T = E_p + E_k = E_p - \frac{1}{2} E_p = \frac{1}{2} E_p$
 $E_T = \frac{1}{2} E_p$ for any satellites !!

Change in Potential Energy

A change in potential energy can be found by using:

$$\Delta E_p = E_{pf} - E_{pi}$$

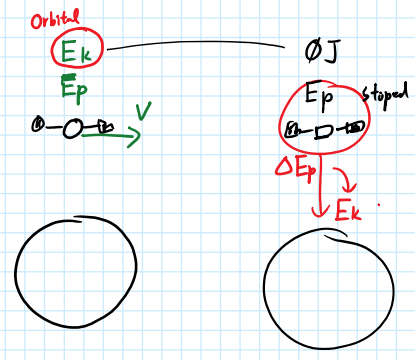
$$= -\frac{GMm}{r_f} - \left[-\frac{GMm}{r_i} \right] = \frac{GMm}{r_i} - \frac{GMm}{r_f} = GMm \left[\frac{1}{r_i} - \frac{1}{r_f} \right]$$

Example:
 How much work is required to move a 4500 kg Earth satellite from an orbital radius of $1.8 \times 10^7 \text{ m}$ to a radius of $4.2 \times 10^7 \text{ m}$? (Ignore E_k)


$W = \Delta E_p = GMm \left[\frac{1}{r_i} - \frac{1}{r_f} \right]$
 $W = (6.67 \times 10^{-11}) (5.98 \times 10^{24}) (4500) \left[\frac{1}{1.8 \times 10^7} - \frac{1}{4.2 \times 10^7} \right]$
 $W = 5.7 \times 10^{10} \text{ J}$

Example:
 The International Space Station drops a 250 kg waste shuttle from an altitude of $3.50 \times 10^5 \text{ m}$. At what speed would it impact Earth if there were no air friction? (Assume it starts at rest)

$r_f = r_E = 6.38 \times 10^6 \text{ m}$
 $r_i = r_E + \text{alt} = 6.73 \times 10^6 \text{ m}$
 $\Delta E_p + \Delta E_k = 0$
 $\Delta E_k = -\Delta E_p$
 $\frac{1}{2} m v^2 = -GMm \left[\frac{1}{r_i} - \frac{1}{r_f} \right]$
 $v^2 = -2G(5.98 \times 10^{24}) \left[\frac{1}{6.73 \times 10^6} - \frac{1}{6.38 \times 10^6} \right]$
 $v = 2550 \text{ m/s}$



Example: A 2.35×10^{16} kg asteroid falls towards the Earth from a really, really, REALLY far way away. How much energy is released when it impacts with the Earth?



$E_{p_i} = 0$
 $r_i = \infty$

$$\Delta E_k = -\Delta E_p$$

$$E_k = -[E_{p_f} - E_{p_i}]$$

$$E_k = -\frac{GMm}{r} = \frac{6.67 \times 10^{-11} (5.98 \times 10^{24}) (2.35 \times 10^{16})}{6.38 \times 10^6}$$

$$E_k = 1.5 \times 10^{24} \text{ J}$$

Launching (Escape Velocity)

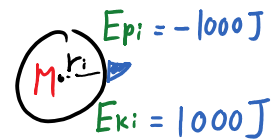
What goes up must come down, unless we throw it really, REALLY hard.

Escape velocity is the minimum speed an object requires in order to break free from Earth's pull and achieve orbit. It should stand to reason that if an object is going to be completely freed from the Earth gravitational pull that we need to supply it with enough Kinetic Energy to match its potential Energy at infinite.

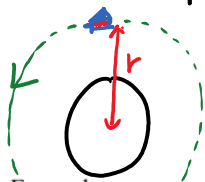
In terms of equations this means that:

$$V_{\text{orbit}} = \sqrt{\frac{GM}{r}}$$

$$\frac{1}{2}mv^2 = \frac{GMm}{r}$$



$E_{p_f} = 0$
 $E_{k_f} = 0$



$$V_{\text{Escape}} = \sqrt{\frac{2GM}{r_i}}$$

Example:

At what speed do you need to throw a 1.0 kg rock in order for it to leave the Earth's gravitational pull?

$$V_{\text{escap}} = \sqrt{\frac{2(6.67 \times 10^{-11})(5.98 \times 10^{24} \text{ kg})}{6.38 \times 10^6 \text{ m}}}$$

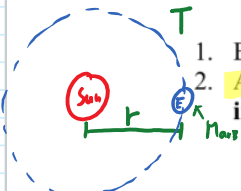
$$V_{\text{escap}} = 1.1 \times 10^4 \text{ m/s} \rightsquigarrow 11 \text{ km/s}$$

Does the mass of the rock matter?

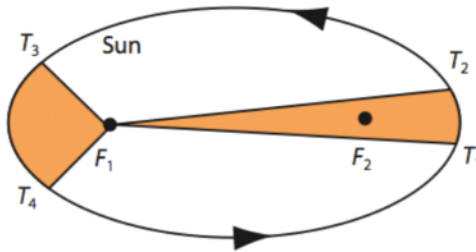
No, not for the velocity.

Circular Motion and Gravitation Notes 6 – Kepler's Laws

Johannes Kepler was a German born mathematician famous for taking the precise measurements of Danish astronomer Tycho Brahe and developing 3 laws of planetary motion around the sun. Amazingly Brahe collected ALL his precise data without the aid of a telescope!



1. Each planet orbits the Sun in an **elliptical path**, with the Sun as one of the two foci in the ellipse.
2. **Areas** carved out by the movement of the **planet around the Sun are equal** over **equal time intervals** (T_1 to T_2)



$$F_c = F_g$$

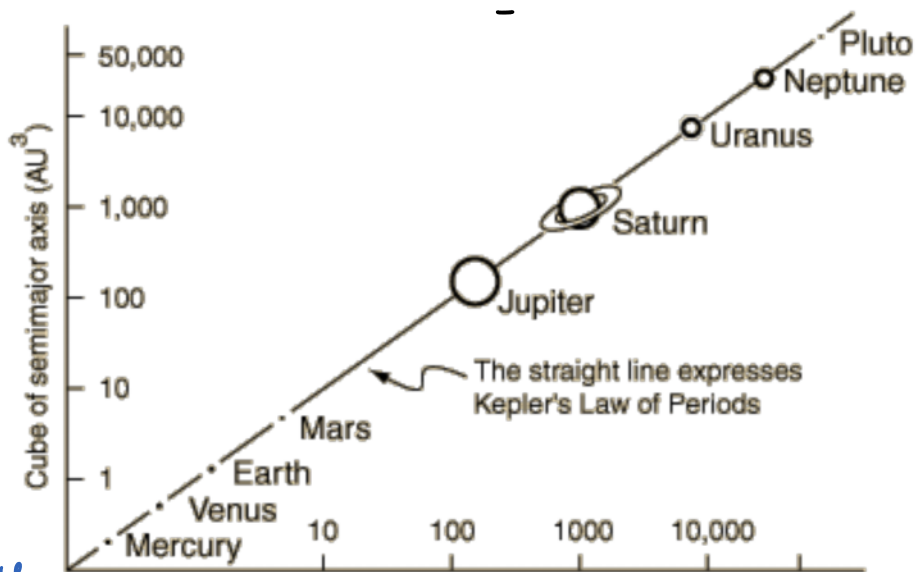
$$\frac{m 4\pi^2 r}{T^2} = \frac{GMm}{r^2}$$

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$$

3. Derive Kepler's 3rd Law from the graph below!

$$\frac{\text{orbital } r^3}{\text{orbital } T^2} = \text{constant !!} \quad \text{or} \quad \frac{r_E^3}{T_E^2} = \frac{r_{Mars}^3}{T_{Mars}^2} = \frac{r_{Jup}^3}{T_{Jup}^2} \dots \text{etc.}$$

Goal: Determine Kepler's Third Law from the Graph below!



Ex) Find ^{Orbital} Period of Jupiter!
 if $r_J = 7.785 \times 10^{11} \text{ m}$
 $T_J = ?$

$$\frac{r_E^3}{T_E^2} = \frac{r_J^3}{T_J^2}$$

$$(1.5 \times 10^{11} \text{ m})^3 = (7.785 \times 10^{11} \text{ m})^3$$

$$\frac{(1.5 \times 10^{11} \text{ m})^3}{(3.16 \times 10^7 \text{ s})^2} = \frac{(7.785 \times 10^{11} \text{ m})^3}{T_J^2}$$

$$\text{orbital } T_J = 3.74 \times 10^8 \text{ s} \rightarrow \text{Earth days} \\ = 4324 \text{ Earth days}$$

Worksheet 5.1

- 1) Calculate the centripetal force acting on a 925 kg car as it rounds an unbanked curve with a radius of 75 m at a speed of 22 m/s.
(6.0x10³ N)
- 2) A small plane makes a complete circle with a radius of 3282 m in 2.0 min. What is the centripetal acceleration of the plane?
(9.0 m/s²)
- 3) A car with a mass of 833 kg rounds an unbanked curve in the road at a speed of 28.0 m/s. If the radius of the curve is 105 m, what is the average centripetal force exerted on the car?
(6.2x10³ N)
- 4) An amusement park ride has a radius of 2.8 m. If the time of one revolution of a rider is 0.98 s, what is the speed of the rider? (18 m/s)
- 5) An electron ($m=9.11 \times 10^{-31}$ kg) moves in a circle whose radius is 2.00×10^{-2} m. If the force acting on the electron is 4.60×10^{-14} N, what is its speed?
(3.18x10⁷ m/s)
- 6) A 925 kg car rounds an unbanked curve at a speed of 25 m/s. If the radius of the curve is 72 m, what is the minimum coefficient of friction between the car and the road required so that the car does not skid?
(0.89)
- 7) A 2.7×10^3 kg satellite orbits the Earth at a distance of 1.8×10^7 m from the Earth's centre at a speed of 4.7×10^3 m/s. What force does the Earth exert on the satellite?
(3.3x10³ N)
- 8) A string can withstand a force of 135 N before breaking. A 2.0 kg mass is tied to the string and whirled in a horizontal circle with a radius of 1.10 m. What is the maximum speed that the mass can be whirled at before the string breaks?
(8.62 m/s)
- 9) A 932 kg car is traveling around an unbanked turn with a radius of 82 m. What is the maximum speed that this car can round this curve before skidding:
a) if the coefficient of friction is 0.95? (28 m/s)
b) if the coefficient of friction is 0.40? (18 m/s)

Worksheet 5.2

- 1) You are riding your bike on a track that forms a vertical circular loop. If the diameter of the loop is 10.0 m, what is the minimum speed required for you to make it around the loop? (7.00 m/s)
- 2) You are swinging a bucket of water in a vertical circle. Assuming that the radius of the rotation of the water is 0.95 m, what is the minimum velocity of the bucket at the top of its swing if the water is not to spill? (3.1 m/s)
- 3) A student has a weight of 655 N. While riding a roller coaster they seem to weigh 1.96×10^3 N at the bottom of a dip that has a radius of 18.0 m. What is the speed of the roller coaster at this point? (18.7 m/s)
- 4) A string requires 186 N of force to break. A 1.50 kg mass is tied to the string and whirled in a vertical circle with a radius of 1.90 m. What is the maximum speed that this mass can be whirled at without breaking the string? (14.7 m/s)
- 5) A 2.2 kg object is whirled in a vertical circle whose radius is 1.0 m. If the time of one revolution is 0.97 s, what is the tension in the string (assume uniform speed)
 - a) at the top? (71 N)
 - b) at the bottom? (114 N)
- 6) A 915 kg car goes over a hill of circular arc. If the radius of the curve is 43 m, how fast can the car travel without leaving the road at the top of the arc? (21 m/s)
- 7) What is the maximum speed for a car rounding a 125 m curve on a highway under very icy (no friction) conditions if the banking angle is 20.0° . (21.1 m/s)

Yowers!) An airplane traveling at a speed of 115 m/s makes a complete horizontal turn in 2 minutes. What is the banking angle?

$L_y = F_g = mg$
 $\tan \theta = \frac{L_y}{L_x}$
 $F_c = L_x$
 $\frac{mv^2}{r} = \frac{mg}{\tan \theta}$


$V = \frac{2\pi r}{T}$
 $115 = \frac{2\pi r}{120s} \Rightarrow r = 2196m$

$\rightarrow \tan \theta = \frac{rg}{V^2}$
 $\tan \theta = \frac{2196(9.8)}{115^2}$
 $\theta = 58.4^\circ$ to horizontal
 $\theta = 31.6^\circ$ to vertical.

(31.6° to the horizontal))

$$m_{\text{Earth}} = 5.98 \times 10^{24} \text{ kg} \quad m_{\text{Moon}} = 7.35 \times 10^{22} \text{ kg}$$

Worksheet 5.3

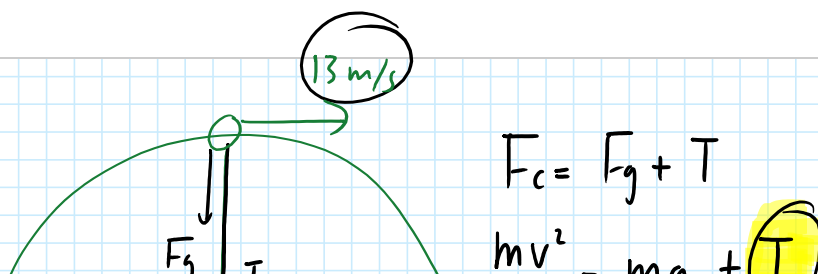
- 1) Two students are sitting 1.50 m apart. One student has a mass of 70.0 kg and the other has a mass of 52.0 kg. What is the gravitational force between them? (1.08x10⁻⁷ N)
 - 2) What gravitational force does the moon produce on the Earth if their centers are 3.84x10⁸ m apart? (1.99x10²⁰ N)
 - 3) If the gravitational force between two objects of equal mass is 2.30x10⁻⁸ N when the objects are 10.0 m apart what is the mass of each object? (186 kg)
 - 4) Calculate the gravitational force on a 6.50x10² kg spacecraft that is 4.15x10⁶ m above the surface of the Earth. (2.34x10³ N)
 - 5) The gravitational force between two objects that are 2.1x10⁻¹ m apart is 3.2x10⁻⁶ N. If the mass of one object is 55 kg, what is the mass of the other object? (38 kg)
 - 6) If two objects, each with a mass of 200 kg, produce a gravitational force of 3.7x10⁻⁶ N, what is the distance between them? (0.85 m)
 - 7) What is the gravitational force on a 70.0 kg object standing on the Earth's surface? (686 N)
 - 8) Three 10.0 kg objects are placed in a straight line 5.00x10⁻¹ m apart. What is the net gravitational force on the center object due to the other two objects? (0N)
 - 9) Three objects A, B, and C are placed 0.50 m apart along a straight line. A and B have masses of 10.0 kg and C has a mass of 15.0 kg, what is the net gravitational force on B due to A and C? (1.33x10⁻⁸ N)
- 
- 10) The force of gravity between two small masses A and B when placed very near each other is 3.24x10⁻⁷ N. What will the force between these objects be if both of their masses are doubled and the distance between them is tripled? (1.44x10⁻⁷ N)

Worksheet 5.4

- 1) Calculate the gravitational field strength on the surface of Mars. Mars has a radius of 3.43×10^6 m and a mass of 6.37×10^{23} kg. (3.61 N/kg)
- 2) At what distance from Earth's surface is the acceleration due to gravity 7.33 m/s^2 ? (9.97×10^5 m)
- 3) On the surface of Planet X an object has a mass of 22.5 kg and a weighs 63.5 N. What is the gravitational field strength on the surface? (2.82 N/kg)
- 4) On the surface of Planet Y, which has a mass of 4.83×10^{24} kg, a 30.0 kg object weighs 50.0 N. What is the radius of the planet? (1.39×10^7 m)

Worksheet 5.5

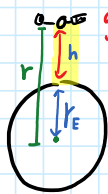
- 5) What is the gravitational potential energy (relative to infinite) of a 5.00×10^3 kg satellite that is in orbit with a radius of 9.90×10^6 m around the Earth? (-2.0×10^{11} J)
- 6) How much work is done against gravity in lifting the satellite in problem #5 to its orbital height? (1.11×10^{11} J)
- 7) A 1750 kg meteorite is 15000 m above the surface of the moon, heading directly towards the moon at 1.00×10^3 m/s. What is its speed on impact? (1.02×10^3 m/s)
- 8) What is the gravitational potential energy of a 10.0 kg object when it is sitting on Earth's surface? (-6.25×10^8 J)
- 9) What is the escape velocity of a 1300 kg shuttle taking off from the moon? (2.37×10^3 m/s)
- 10) What is the mass of a planet that has an escape speed of 9.0×10^3 m/s and a radius of 7.2×10^6 m? (4.37×10^{24} kg)
- 11) A 12500 kg satellite is in orbit at an altitude of 3.60×10^6 m. What is its **total** energy?
HINT: Total Energy = $E_p + E_k$ (-2.50×10^{11} J)





$$a_c = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$$

5.4
#2



$$g = 7.33 \text{ m/s}^2$$

$$g = \frac{GM}{r^2}$$

$$r = \sqrt{\frac{GM}{g}} = 7.38 \times 10^6 \text{ m}$$

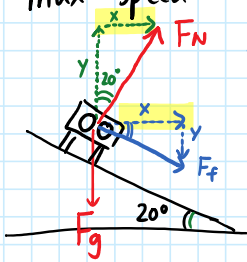
6.67×10^{-11}
 5.98×10^{24}
 7.33

$$\text{height above surface} = r - r_E = 7.38 \times 10^6 - 6.38 \times 10^6 = 9.97 \times 10^5 \text{ m}$$

Review

Banked Curve with Friction

* Max speed $m = 500 \text{ kg}$ $\mu = 0.5$ $r = 20 \text{ m}$



$$\sum F_y + F_{fy} = F_{Ny}$$

$$mg + F_f \sin(20^\circ) = F_N \cos(20^\circ)$$

$$4900 \text{ N} + \mu F_N \sin(20^\circ) = F_N \cos(20^\circ)$$

$$4900 + 0.171 F_N = 0.94 F_N$$

$$4900 = 0.769 F_N$$

$$6374.5 \text{ N} = F_N$$

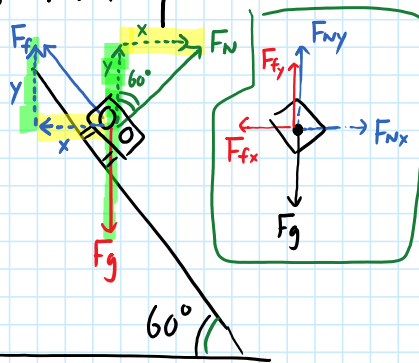
$$\sum F_c = F_{Nx} + F_{fx}$$

$$\frac{mv^2}{r} = F_N \sin(20^\circ) + F_f \cos(20^\circ)$$

$$\frac{(500) v^2}{20} = [6374.5] \sin(20^\circ) + \mu F_N \cos(20^\circ)$$

$$V_{\text{max}} = 14.4 \text{ m/s}$$

* Min Speed



$$\sum F_y = F_{Ny} + F_{fy}$$

$$F_N = ?$$

$$\sum F_c = F_{Nx} - F_{fx}$$

$$\frac{mv^2}{r}$$

Ex) 1 Pluto year = 248 Earth year. How far is Pluto from the Sun.
orbital Period.

$$\frac{\text{orbital } r_E^3}{\text{orbital } T_E^2} = \frac{r_P^3}{T_P^2}$$

$$\frac{(1.5 \times 10^{11})^3}{(1 \text{ yr})^2} = \frac{(r_P)^3}{(248 \text{ yr})^2}$$

$$r_P = 5.9 \times 10^{12} \text{ m}$$

Same unit ok!!
 $\times 365 \times 24 \times 60 = 60$
 $\times 365 \times 24 \times 60 > 60$