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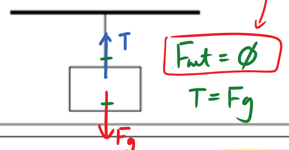
Equilibrium Notes

1 – Translational Equilibrium

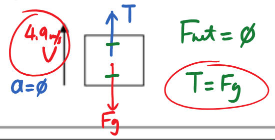
Ex.
A 20.0 kg object is suspended by a rope as shown. What is the net force acting on it?



$F_{net} = \emptyset$

$T = F_g$

Ex.
Ok that was easy, now that same 20.0 kg object is lifted at a velocity of 4.9 m/s. What is the net force acting on it?



$F_{net} = \emptyset$

$T = F_g$

Because in both case the net force on the objects is zero they are said to be in Equilibrium. If the object is stationary it is said to be in Static Equilibrium, while an object moving at a constant velocity is in dynamic Equilibrium.

These are both case where the object is in traslational Equilibrium.

Translational motion refers to motion along a line, therefore:

The condition of equilibrium:

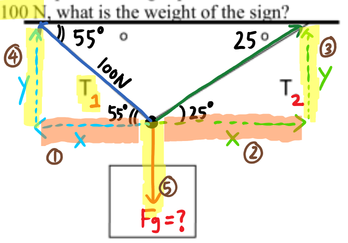
$\Sigma F = \emptyset$

And so,

$\Sigma F_x = \emptyset$

$\Sigma F_y = \emptyset$

Ex.
A sign is suspended using ropes as shown in the diagram. If T_1 is 100 N, what is the weight of the sign?



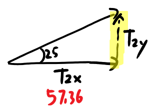
Strategy 1: Components

1. Choose a point in the system that is in equilibrium, with all forces acting on it. In this case use the intersection.
2. Draw an F B D !
3. Break these forces... into components.
4. Use… Equilibrium Conditions.

① $T_{1x} = T_1 \cos 55° = 57.36\ N$ ($T_1 = 100$)

$\Sigma F_x = \emptyset \quad \therefore T_{2x} = T_{1x} = 57.36\ N$ ②

③



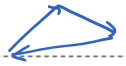
$\tan 25 = \frac{T_{2y}}{T_{2x}}$

$T_{2y} = 26.75\ N \uparrow$

④ $T_{1y} = T_1 \sin 55° = 81.92\ N \uparrow$ ($T_1 = 100$)

⑤ $\Sigma F_y = \emptyset \quad F_g = T_{1y} + T_{2y} = 109\ N$ ($F_g \downarrow$, $T_{1y} \uparrow$, $T_{2y} \uparrow$)

$\Sigma F_x = \emptyset$ $\Sigma F_y = \emptyset$



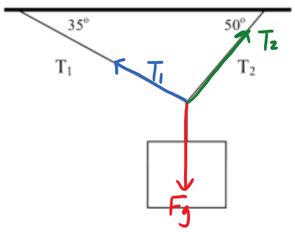
Strategy 2: Create a closed vector diagram

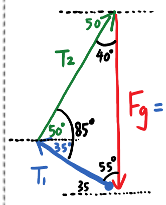
1. Since we know that $F_{net} = 0$ at any point in equilibrium, what would happen if we added if we add up all of the force vectors?

   They add to $\emptyset$

2. Use Sine Law, Cosine Law, or whatever means necessary to solve the triangle…
3. NEVER assume that it is a __right angle__ unless you can prove it geometrically.

Ex.
A 64 N object is suspended using ropes as shown in the diagram. Calculate tensions T_1 and T_2 in the ropes.



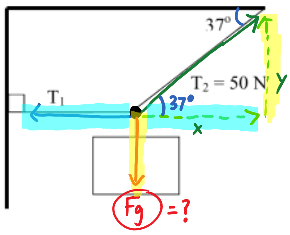


$$\Sigma F = T_1 + T_2 + F_g = \emptyset$$

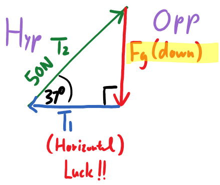
a) $\frac{T_1}{\sin 40°} = \frac{64}{\sin 85°}$ $T_1 = 41.3\text{ N}$

b) $\frac{T_2}{\sin 55°} = \frac{64}{\sin 85°}$ $T_2 = 52.6\text{ N}$

Ex.
An object is suspended as shown. If the tension in one of the ropes is 50 N as shown, what is the weight of the object?



Trig Method



$\sin 37° = \frac{F_g}{50}$

$F_g = 30.1\text{ N}$

Component method

$\Sigma F_y = \emptyset$

$\therefore F_g = T_{2y}$

$F_g = 50 \cdot \sin 37°$

$F_g = 30.1\text{ N}$

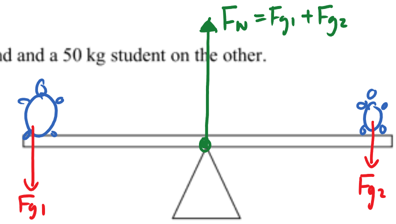
You can use **Strategy 1** or **Strategy 2**, just be sure you know both ways. You're bound to hit a brick wall eventually and it's nice to be able to try it from a different angle, no pun intended...

Equilibrium Notes

2 – Torque at 90°

A body in translational equilibrium will have no acceleration in the x or y directions. However it still could be rotating.

Consider a teeter-totter, with a 100 kg student on one end and a 50 kg student on the other.



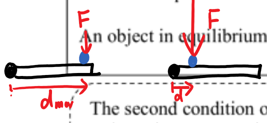
What are the net translational forces in:

The x-direction? $\Sigma F_x = 0$

The y-direction? $\Sigma F_y = 0$

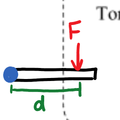
Although the net translational forces are zero, the system has a net Torque - so it is not in equilibrium.

An object in equilibrium must have both translational ($\Sigma F = 0$) and rotational ($\Sigma \tau = 0$) equilibrium. τ



The second condition of equilibrium is that in order to have no rotation, there must be no net torque.

Torque is defined as: **force x distance to pivot**



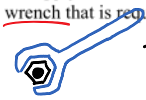
$$\tau = Fd$$

Unit of torque: N·m

Imagine trying to loosen the lug nuts to remove a tire from your car. The longer the wrench you use, the easier it will be.

Ex:

A torque of 24.0 Nm is needed to tighten a nut. If a person can apply a force of 100 N what is the minimum length of wrench that is required?



$\tau = Fd \Rightarrow 24 = 100\,d$

$0.24\text{ m} = d$

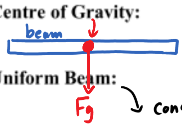
Torque is a vector quantity, which must work in either the clockwise (c) or (cw) counterclockwise (cc) (ccw) directions.

If an object is in rotational equilibrium then:

$$\Sigma \tau = 0 \quad \text{or} \quad \tau_{cw} = \tau_{ccw}$$

A few more terms we need to learn before we go on…

Centre of Gravity: where the average mass acts. where we draw F_g !!



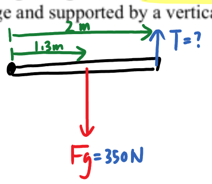
Uniform Beam: constant shape and constant density.

Arbitrary Position of Rotation:

You choose the location of pivot !!!

Ex:

A 350 N store sign hangs from a pole of negligible mass. The pole is attached to a wall by a hinge and supported by a vertical rope. What is the tension in the rope?

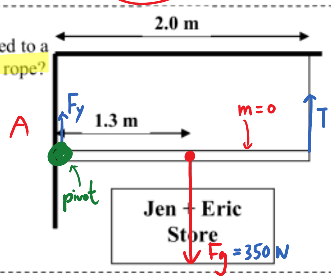


$\tau_{cw} = \tau_{ccw}$

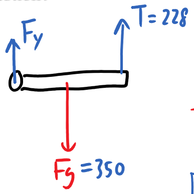
$F_g \times d_1 = T \times d_2$

$350\,(1.3\text{m}) = T \times (2\text{m})$

$T = 228\text{ N}$



Extension:
What are the vertical and horizontal components of the supporting force provided by the hinge in the last question?



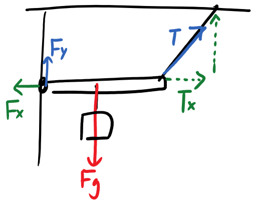
$\Sigma F_y = \emptyset$

$F_y + T = F_g$

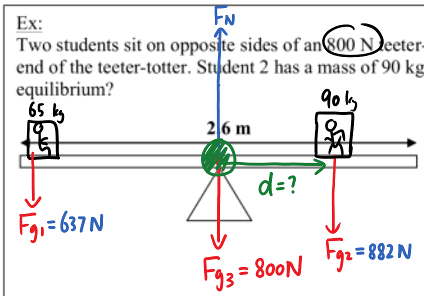
$F_y + 228 = 350$

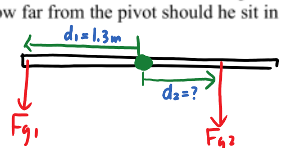
$F_y = 122\ N$

$F_x = \emptyset$



Ex:
Two students sit on opposite sides of an 800 N teeter-totter. Student 1 has a mass of 65 kg and sits at the very end of the teeter-totter. Student 2 has a mass of 90 kg. How far from the pivot should he sit in order to achieve equilibrium?





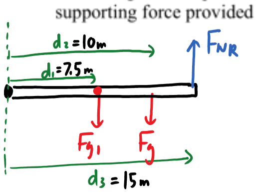
$\tau_{cw} = \tau_{ccw}$

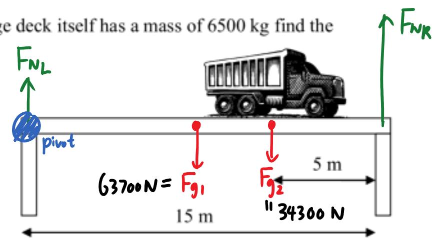
$F_{g2} \cdot d_2 = F_{g1} \times d_1$

$882\ d_2 = 637 \times 1.3$

$d_2 = 0.94\ m$

Ex:
A 3500 kg truck is parked on a bridge as shown. If the bridge deck itself has a mass of 6500 kg find the supporting force provided by each of the two support posts.





$\tau_{cw} = \tau_{ccw}$

$F_{g1} \times d_1 + F_{g2} \times d_2 = F_{NR} \times d_3$

$63700 \times (7.5) + 34300 \times (10) = F_{NR} \times (15)$

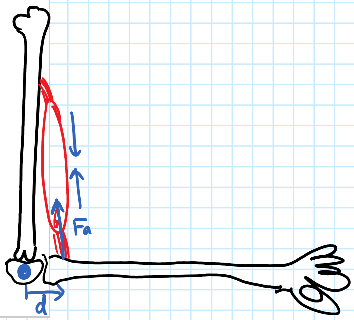
$F_{NR} = 54700\ N$

$\Sigma F_y = \emptyset$

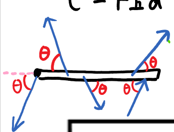
$F_{NL} + F_{NR} = F_{g1} + F_{g2}$

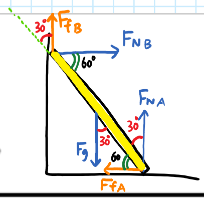
$54700 \quad 63700 \quad 34300$

$F_{NL} = 43300\ N$



$\tau = F_{\perp} d \Rightarrow \tau = F d \sin\theta$





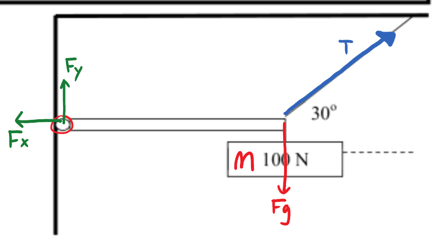
Equilibrium Notes

3 – Torque Not at 90°

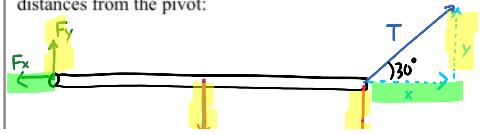
Although we've already learned about torque, we don't quite have the whole story. So far we have only seen torque provided by forces acting perpendicular to the body in equilibrium. What happens if a force acts in a direction other than perpendicular to the body?

Ex
A 2.2 m long 50.0 N uniform beam is attached to a wall by means of a hinge. Attached to the other end of the beam is a 100 N weight. A rope also helps support the beam as shown.

a) What is the tension in the rope?
b) What are the vertical and horizontal components of the supporting force provided by the hinge?

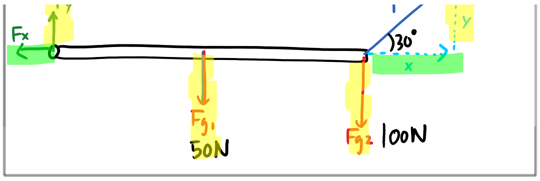


First we draw the beam with the forces acting on it and their distances from the pivot:



Notice that if we break the tension in the rope into component forces, the parallel (x) component does not contribute to the torque in either the clockwise or counterclockwise direction

instead of Tx and Ty



rope into component forces, the parallel (x) component does not contribute to the torque in either the clockwise or counterclockwise direction

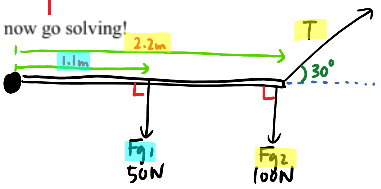
instead of T_x and T_y

use $T_\perp$ and $T_\parallel$

So, whenever we are calculating the torque on a body we must ALWAYS use the perpendicular component. of the force.

Ok now go solving!

a)



$\tau_{cw} = \tau_{ccw}$

$\tau = Fd\sin\theta$

b) $F_{g1}(1.1m) + F_{g2}(2.2m) = T(2.2m)\sin(30°)$

50 100

$T = 250\,N$

$\Sigma F_x = \emptyset$

$F_x = T_x = 250\cos(30) = 216.5\,N$

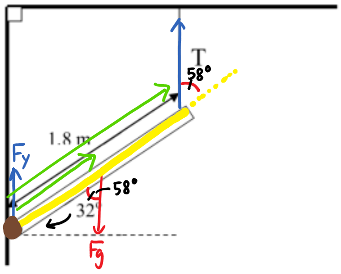
$\Sigma F_y = \emptyset$

$F_y + T_y = F_{g1} + F_{g2}$

$F_y + 250\sin(30) = 50 + 100$ $F_y = 25N$

RULE NOT TO BREAK:
When we find the torque acting on a body we MUST ALWAYS use the component of the force that is _perpendicular_ to _the_ _beam_!!!

Ex
A 1.8 m long 12.0 kg bar is attached to a wall by a hinge and supported by a rope as shown. Find the tension in the rope.



$\tau_{cw} = \tau_{ccw}$

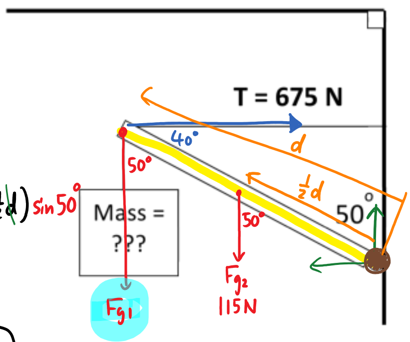
* $F_g (0.9\text{m}) \sin 58° = T(1.8\text{m}) \sin 58°$

$(12)(9.8)(0.9) = T(1.8)$

$T = 58.8\text{ N}$

Let $d = 10\text{ m}$

Ex
Find the mass of the object given the information in the diagram and that the weight of the uniform beam is 115 N.



$\tau_c = \tau_{ccw}$

* $T(d)\sin 40° = F_{g1}(d)\sin 50° + F_{g2}(\tfrac{1}{2}d)\sin 50°$
675 115

$F_{g1} = 508.9\text{ N}$

$F_{g1} = mg \qquad m = \frac{508.9}{9.8} = 52\text{ kg}$

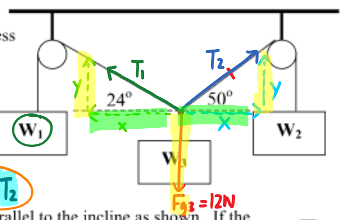
WS # 3

WS 3.1

Name: ____________________

Worksheet 3.1 - *Translational* Equilibrium – Remember to include a FBD and resolve the Forces into their components!

1. W_1, W_2 and W_3 are the weights of three objects suspended by pulleys as shown. Assuming the pulleys in this system are frictionless and weightless and that W_3 = 12 N, what are the values of W_1 and W_2? (Simpler than you think!)

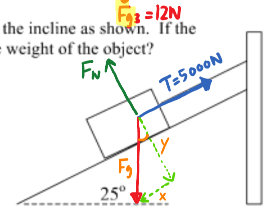


$\Sigma F_y = \emptyset \quad T_{1y} + T_{2y} = F_{g3}$

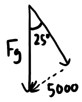
$T_1 \sin 24 + T_2 \sin 50 = 12$

$\Sigma F_x = \emptyset \quad T_{1x} = T_{2x} \Rightarrow T_1 \cos 24 = T_2 \cos 50 \Rightarrow T_1 = 0.7\, T_2$

2. An object is suspended on a frictionless inclined plane by a rope parallel to the incline as shown. If the angle of the incline is 25° and the tension in the rope is 5000. N, what is the weight of the object?



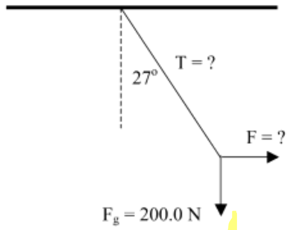
$\Sigma F_x = \emptyset \quad F_{gx} = T = 5000\,N$



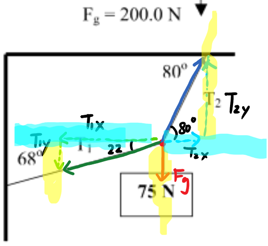
$\sin 25° = \frac{5000}{F_g}$

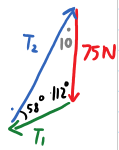
$F_g = 11831\ N$

3. A 200.0 N child sitting on a playground swing is being pushed by her father. When the rope makes an angle of 27° to the vertical what is the force exerted by her father? What is the tension in the rope, T?



4. Find the tensions T_1 and T_2 in the ropes shown in the diagram.





$\Sigma F_x = \emptyset \quad T_{1x} = T_{2x}$

$T_1 \cos 22° = T_2 \cos 80$

$T_1 = 0.187\, T_2$

$\Sigma F_y = \emptyset \quad T_{2y} = T_{1y} + F_g$

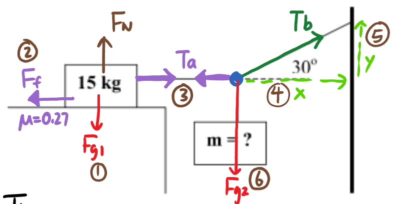
$T_2 \sin 80° = T_1 \sin 22° + 75\,N$

$0.985\, T_2 = 0.07\, T_2 + 75$

$0.915\, T_2 = 75$

$T_2 = 82\,N$

5. A 15 kg object rests on a table. A cord is attached to this object and also to a wall. Another object is hung from this cord as shown. If the coefficient of friction between the 15 kg object and the table is 0.27, what is the maximum mass that can be hung, without movement?



$\Sigma F_y = \emptyset$ ① $F_{g1} = 15 \times 9.8 = 147 = F_N$

② $F_f = \mu F_N = 39.7\,N$ (μ = 0.27)

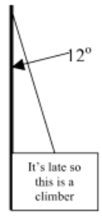
$\Sigma F_x = \emptyset$ ④ $F_f = T_a = T_{bx}$ $\Rightarrow$ ⑤ $\tan 30° = \frac{T_{by}}{T_{bx}}$

$T_{by} = 22.9\,N = F_{g2}$

$\Sigma F_y = \emptyset$

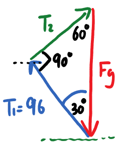
$m = 2.3\ kg$

6. A 735 N mountain climber is rappelling down the face of a vertical cliff as shown in the diagram. If the rope makes an angle of 12° with the cliff face, what is the tension in the rope? Assume that the climber pushes horizontally off of the cliff.



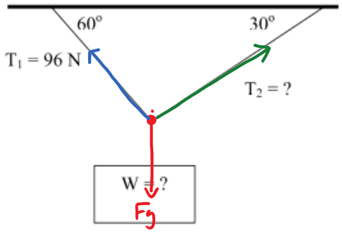
7. Given the following diagram, find W and T_2.

$\Sigma F = \emptyset$



$\cos 30° = \frac{96}{F_g}$

$\tan 30° = \frac{T_2}{96}$

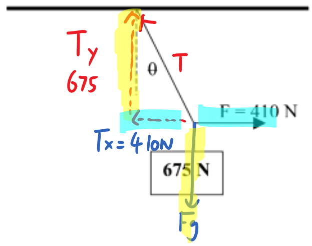


8. A 675 N object is pulled horizontally by a force of 410 N as shown. What is the angle, θ, between the rope and the vertical?

$\Sigma F_y = \emptyset \quad T_y = F_g = 675$

$\Sigma F_x = \emptyset \quad T_x = F = 410$

$\tan \theta = \frac{410}{675}$

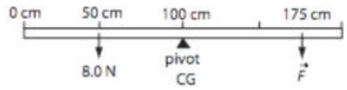


Answers:

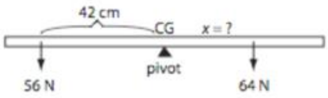
1. $W_1 = 8.0$ N, $W_2 = 11.4$ N
2. 1.183×10^4 N
3. F = 101.9 N, T = 224.5 N
4. $T_1 = 15$ N $T_2 = 82.$ N
5. 2.3 kg
6. 751 N
7. W = 111 N, $T_2 = 55$ N
8. 31.3°

Worksheet 3.2 - Torque

1. What force F_1 is needed to balance the beam in the diagram below?

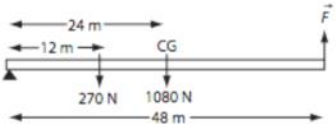


2. How far from the pivot must the 64 N object be placed to balance the beam in the diagram below?

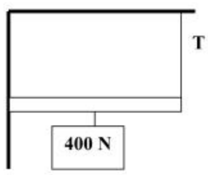


3. ~~What upward force F_1 is needed to chaieve rotational equilibrium in the diagram below?~~

4. If the torque needed to loosen a lug nut is 45 Nm and you are using a 35 cm wheel wrench, what force do you need to exert perpendicular to the end of the wrench?

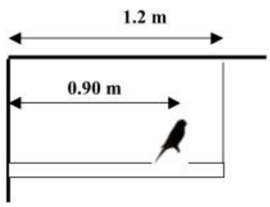


5. A beam of negligible mass is attached to a wall by a hinge. Attached to the center of the beam is a 400 N weight. A rope supports the beam as shown in the diagram. What is the tension in the rope?



6. Two students sit on either end of a uniform teeter-totter. Student 1 sits 1.10 m from the pivot while Student 2 sits 0.85 m from the pivot. If Student 1 has a mass of 72 kg, what is the mass of Student 2?

7. A 0.75 kg bird stands on a uniform 1.0 kg stick as shown. The stick is attached to a wall with a hinge and to the ceiling with a rope of negligible mass. What is the tension in the rope?



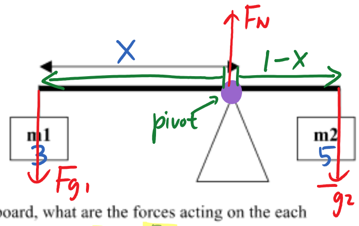
$\tau_{cw} = \tau_{ccw}$

$F_{g2}(1-x) = F_{g1}(x)$

$x = \frac{5}{8}$

$x = 0.625\,m$

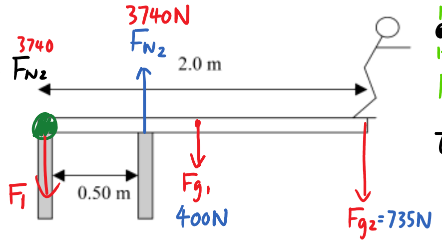
8. Two masses (m_1 = 3.00 kg, m_2 = 5.00 kg) hang from the ends of a metre stick as shown. If the mass of the metre stick is negligible, at what distance from the left of the metre stick should a pivot be placed so that the system will be balanced?

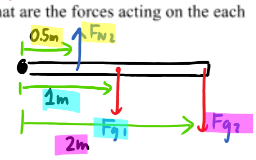


9. A uniform 400 N diving board is supported at two points as shown in the diagram. If a 75 kg diver stands at the end of the board, what are the forces acting on the each support?

$\Sigma F_y = 0$

$F_1 + F_{g1} + F_{g2} = F_{N2}$

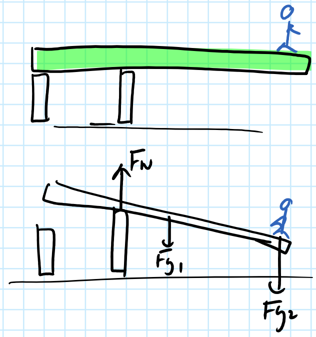




$\tau_{cw} = \tau_{ccw}$

$F_{g1}(1m) + F_{g2}(2m) = F_{N2}(0.5m)$

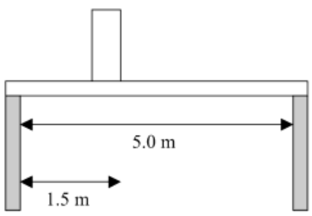
$F_{N2} = 3740\,N$



10. A 650 N student stands on a 250 N uniform beam that is supported by two supports as shown in the diagram. If the supports are 5.0 m apart and the student stands 1.5 m from the left support:

a) What is the force that the right support exerts on the beam?

b) What is the force that the left support exerts on the beam?



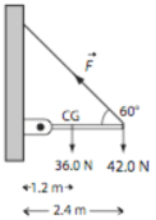
Answers:

1. (5.3 N)
2. (37 cm OR 0.37 m)
3. 6.1×10^2 N
4. (130 N)
5. (200 N)
6. (93 kg)
7. (10. N)
8. (0.625 m)
9. (left support = 2.61×10^3 N down , right support = 3.74×10^3 N up)
10. F_{right} = 320 N , F_{left} = 580N

Worksheet 3.3 – Torque not at 90° –

A lot of this work can be done DIRECTLY on the diagram provided. Please remember to resolve ALL forces into their dimensions

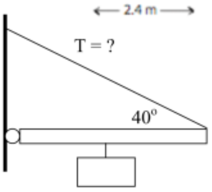
1. A small 42.0 N sign is suspended from the end of a hinged rod, which is 2.40 m long and uniform shape as shown in the diagram below and to the right. What tension force exists in the rope holding up both the rod and the sign? The rope makes an angle of 60° with the 36.0 N rod.



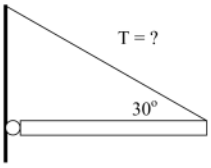
2. A beam of negligible mass is attached to a wall by means of a hinge. Attached to the centre of the beam is a 400. N weight. A rope also helps to support this beam as shown in the diagram.

a) What is the tension in the rope?

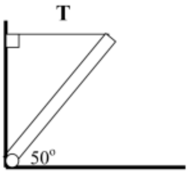
b) What are the vertical and horizontal forces that the wall exerts on the beam?



3. Find the tension in the rope supporting the 200. N hinged uniform beam as shown in the diagram.



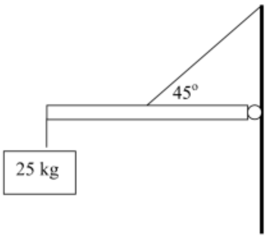
4. Find the tension in the rope supporting the 200. N hinged uniform beam as shown in the diagram.



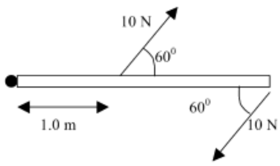
5. A uniform beam (mass = 22 kg) is supported by a cable that is attached to the centre of the beam a shown in the diagram.

a) find the tension in the cable.

b) find the horizontal and vertical forces acting on the hinge.



6. The diagram below shows the top view of a door that is 2.0 m wide. Two forces are applied to the door as indicated in the diagram. What is the net torque on the door with respect to the hinge?



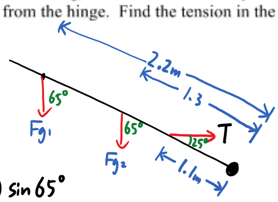
7. A 2.6 m uniform beam (mass of 9.0 kg) is attached to a wall by a hinge and supported by a rope. A 13 kg mass hangs from the beam 2.2 m from the hinge. Find the tension in the rope that is attached to the beam 1.1 m from the wall.

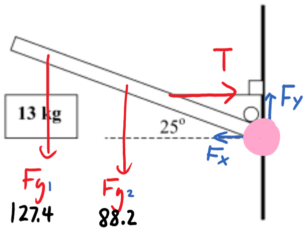
$\tau = Fd \sin\theta$

$\tau_{cw} = \tau_{ccw}$

$T(1.1m)\sin 25^\circ = F_{g1}(2.2m)\sin 65^\circ + F_{g2}(1.3m)\sin 65^\circ$

(F_{g1} = 127.4 N, F_{g2} = 88.2 N)





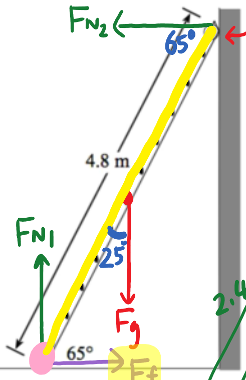
Answers:

1. (69 N)
2. a. (311 N) b. (V: 200 N, H: 238 N)
3. (200. N)
4. (83.9 N)
5. a. (1000 N) b. (F_x = 710 N, F_y = 250 N)
6. (8.66 Nm clockwise)
7. (770 N)

$\tau = F d_{\perp}$ $\tau = F d \sin\theta$

Equilibrium and Torque Extra Notes: Challenging Questions

1. A uniform 4.8 m long ladder of mass 16 kg leans against a frictionless vertical wall as shown in the diagram below.



What minimum force of friction is needed at the base of the ladder to keep it from sliding?

$\tau_{cw} = \tau_{ccw}$

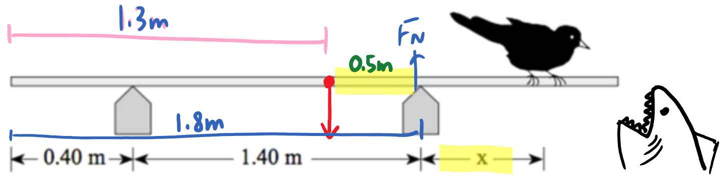
$F_g (2.4m) \sin 25° = F_{N2} (4.8m) \sin 65°$

$F_g = 16kg(9.8)$

$\therefore F_{N2} = 36.6 N$

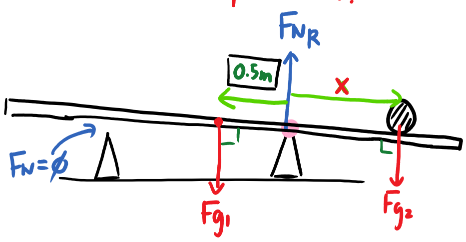
$\Sigma F_x = \emptyset \quad F_f = F_{N2} = \boxed{37 N}$

2. A 0.75 kg board of length 2.60 m initially rests on two supports as shown.



What maximum distance, x, from the right-hand support can a 1.2 kg bird walk before the board begins to leave the left-hand support?

* Just about to tip over!!!



$\tau_{cw} = \tau_{ccw}$

$F_{g2}(x) = F_{g1}(0.5m)$

$1.2(9.8) \qquad (0.75)(9.8)$

$\therefore x = 0.3125 m$

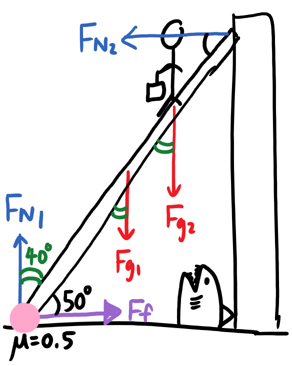
$\therefore$ The bird can walk 31.25 cm to the right

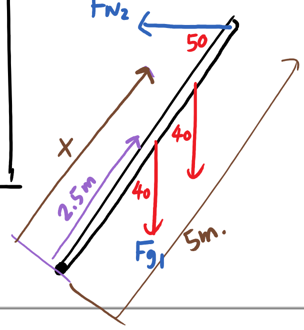
3 A 20 kg ladder leans against a frictionless vertical wall at 50° to the ground. If the ladder is 5 m long and μ=0.5 on the ground How far up the ladder can a 80 kg person climb before the ladder starts sliding?

$\Sigma F_y = \emptyset$

$F_{g1} = 20(9.8) = 196 N \quad F_{g2} = 80(9.8) = 784 N \quad F_{N1} = F_{g1} + F_{g2} = 980 N$

$F_f = \mu F_{N1} = 0.5(980) = 490 N \quad \Sigma F_x = \emptyset \quad F_{N2} = 490 N$



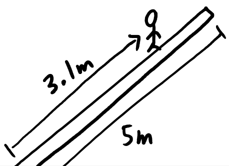


$\tau_{cw} = \tau_{ccw}$

$F_{g1}(2.5m)\sin 40° + F_{g2}(x)\sin 40° = F_{N2}(5m)\sin 50°$

$F_{g1} = 196 N \qquad F_{g2} = 784 N$

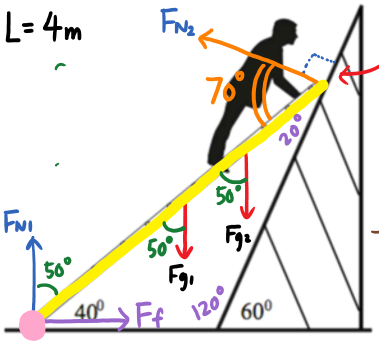
$\therefore x = 3.1 m$ max height to climb.



4 A 65 kg person is ¾ of the way up a 25 kg uniform ladder as shown in the diagram below. The ladder is leaning against a frictionless surface inclined at 60° to the horizontal. What is the minimum coefficient of friction between the ladder and the floor necessary to maintain equilibrium?

$\mu \Rightarrow F_N, F_f$

$L = 4m$



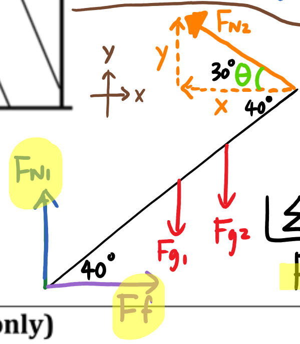
For Torque : $\theta = ?$

$\tau_{cw} = \tau_{ccw}$

$F_{g1}(2m)\sin 50° + F_{g2}(3m)\sin 50° = F_{N2}(4m)\sin 70°$

$F_{g1} = 25(9.8)$, $F_{g2} = 65(9.8)$

$\therefore F_{N2} = 489.3\,N$



$\Sigma F_y = \varnothing$

$F_{N1} + F_{N2y} = F_{g1} + F_{g2}$

$F_{N2y} = 489.3\sin(30°)$, $F_{g1} = 25(9.8)$, $F_{g2} = 65(9.8)$

$\therefore F_{N1} = 637.4\,N$

$\Sigma F_x = \varnothing$

$F_f = F_{N2x} = 489.3\cos 30° = 423.7\,N$

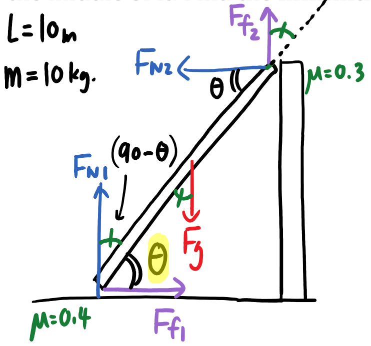
$\mu = \frac{F_f}{F_N} = 0.66$

Super challenging (for fun only)

A ladder is leant against the wall. The coefficient of the static friction μ_1 between the ladder and the wall is 0.3 and the coefficient of the static friction μ_2 between the ladder and the floor is 0.4. The center of mass of the ladder is in the middle of it. Find the **minimum angle** θ that the ladder can form with the floor not to slip down.

$L = 10m$

$m = 10kg$



Hint 1: mass is not given on purpose because you don't need it

Hint 2: this trig identity is necessary Sin(90-θ)= Cos(θ)

Hint 3: with the magic of Physics you will end up with three main equations and a few side ones; only the magic of Maths/algebra can get you out of the mess that comes after.

Hint 4: Other than θ, μ_1 and μ_2, all other unknown can be cancelled out somehow.

Hint 5: the 2nd last step should be an equation that contains only three variables θ, μ_1 and μ_2 and some trig function.