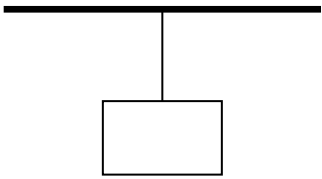


# Equilibrium Notes

## 1 – Translational Equilibrium

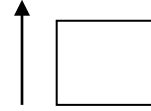
Ex.

A 20.0 kg object is suspended by a rope as shown. What is the net force acting on it?



Ex.

Ok that was easy, now that same 20.0 kg object is lifted at a velocity of 4.9 m/s. What is the net force acting on it?



Because in both case the net force on the objects is zero they are said to be in \_\_\_\_\_. If the object is stationary it is said to be in \_\_\_\_\_, while an object moving at a constant velocity is in \_\_\_\_\_.

These are both case where the object is in \_\_\_\_\_.

**Translational** motion refers to motion along a line, therefore:

The condition of equilibrium:

$$\underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

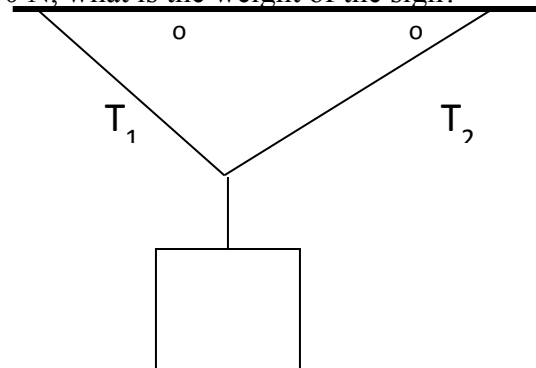
And so,

$$\underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

$$\underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

Ex.

A sign is suspended using ropes as shown in the diagram. If  $T_1$  is 100 N, what is the weight of the sign?



### Strategy 1: Components

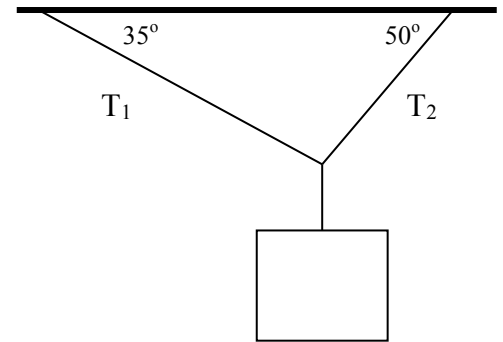
1. Choose a point in the system that is in equilibrium, with all forces acting on it. In this case use \_\_\_\_\_.
2. Draw an \_\_\_\_\_!
3. Break these forces...
4. Use...

**Strategy 2: Create a closed vector diagram**

1. Since we know that  $F_{\text{net}} = 0$  at any point in equilibrium, what would happen if we added if we add up all of the force vectors?
2. Use Sine Law, Cosine Law, or whatever means necessary to solve the triangle...
3. NEVER assume that it is a \_\_\_\_\_ unless you can prove it geometrically.

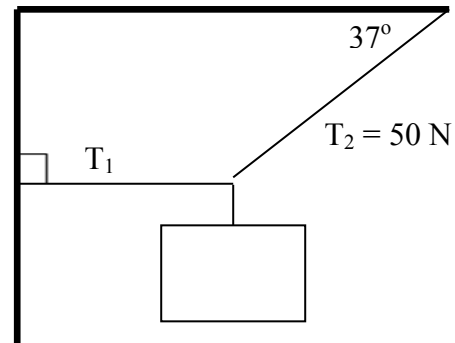
Ex.

A 64 N object is suspended using ropes as shown in the diagram. Calculate tensions  $T_1$  and  $T_2$  in the ropes.



Ex.

An object is suspended as shown. If the tension in one of the ropes is 50 N as shown, what is the weight of the object?



You can use **Strategy 1** or **Strategy 2**, just be sure you know both ways. You're bound to hit a brick wall eventually and it's nice to be able to try it from a different angle, no pun intended...

# Equilibrium Notes

## 2 – Torque at $90^\circ$

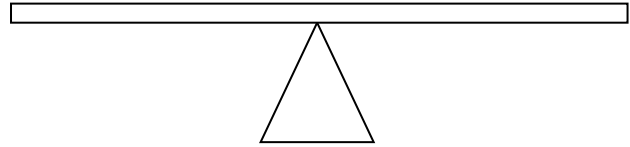
A body in translational equilibrium will have no acceleration in the x or y directions. However it still could be \_\_\_\_\_.

Consider a teeter-totter, with a 100 kg student on one end and a 50 kg student on the other.

What are the net translational forces in:

The x-direction? \_\_\_\_\_

The y-direction? \_\_\_\_\_



Although the net translational forces are zero, the system has a \_\_\_\_\_ - so it is not in equilibrium.

An object in equilibrium must have both translational and \_\_\_\_\_ equilibrium.

The second condition of equilibrium is that in order to have no rotation, there must be no net torque.

Torque is defined as: **force x distance to pivot**

Unit of torque: \_\_\_\_\_

Imagine trying to loosen the lug nuts to remove a tire from your car. The longer the wrench you use, the easier it will be.

Ex:

A torque of 24.0 Nm is needed to tighten a nut. If a person can apply a force of 100 N, what is the minimum length of wrench that is required?

Torque is a \_\_\_\_\_ quantity, which must work in either the clockwise (c) or counterclockwise (cc) directions.

A few more terms we need to learn before we go on...

**Centre of Gravity:**

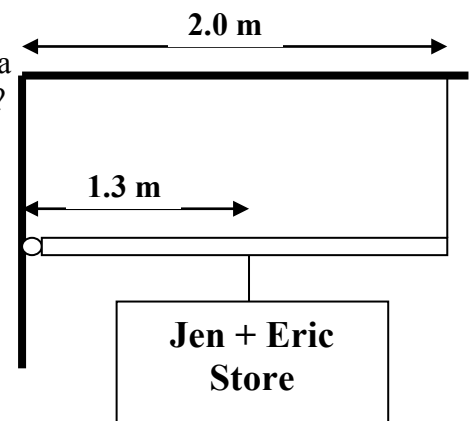
If an object is in rotational equilibrium then:

**Uniform Beam:**

**Arbitrary Position of Rotation:**

Ex:

A 350 N store sign hangs from a pole of negligible mass. The pole is attached to a wall by a hinge and supported by a vertical rope. What is the tension in the rope?

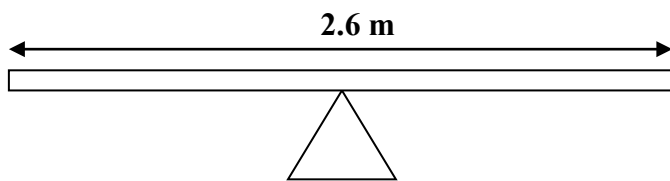


Extension:

What are the vertical and horizontal components of the supporting force provided by the hinge in the last question?

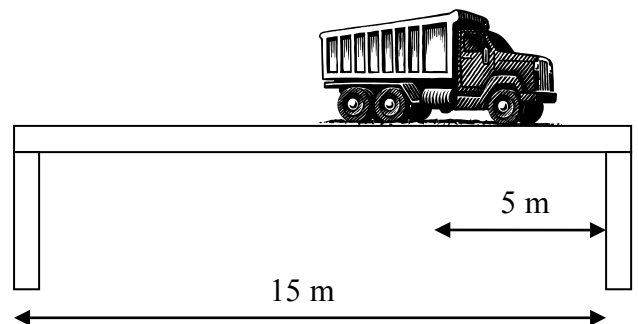
Ex:

Two students sit on opposite sides of an 800 N teeter-totter. Student 1 has a mass of 65 kg and sits at the very end of the teeter-totter. Student 2 has a mass of 90 kg. How far from the pivot should he sit in order to achieve equilibrium?



Ex:

A 3500 kg truck is parked on a bridge as shown. If the bridge deck itself has a mass of 6500 kg find the supporting force provided by each of the two support posts.



## Equilibrium Notes

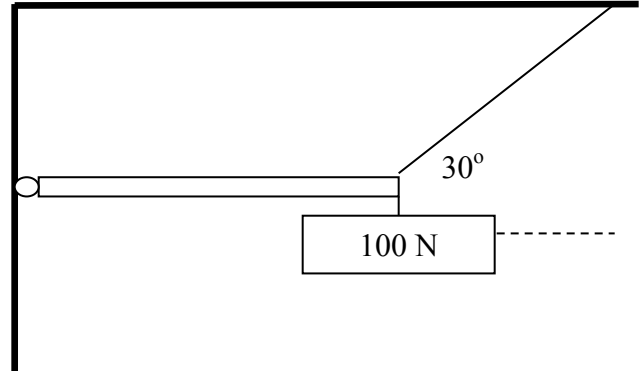
### 3 – Torque Not at $90^\circ$

Although we've already learned about torque, we don't quite have the whole story. So far we have only seen torque provided by forces acting perpendicular to the body in equilibrium. **What happens if a force acts in a direction other than perpendicular to the body?**

Ex

A 2.2 m long 50.0 N uniform beam is attached to a wall by means of a hinge. Attached to the other end of the beam is a 100 N weight. A rope also helps support the beam as shown.

- a) What is the tension in the rope?
- b) What are the vertical and horizontal components of the supporting force provided by the hinge?



First we draw the beam with the forces acting on it and their distances from the pivot:

Notice that if we break the tension in the rope into component forces, the \_\_\_\_\_ component does not contribute to the torque in either the clockwise or counterclockwise direction

So, whenever we are calculating the torque on a body we must ALWAYS use the \_\_\_\_\_ of the force.

Ok now go solving!

a)

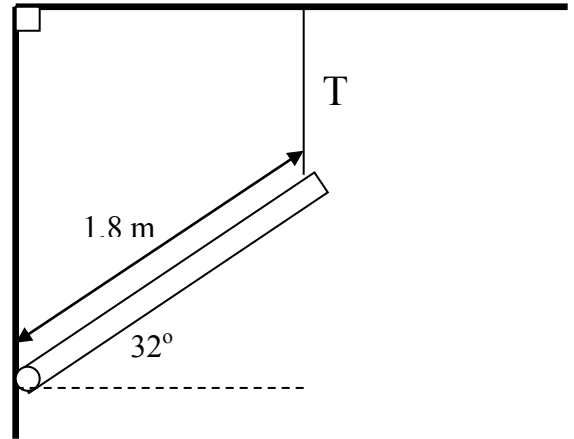
b)

**RULE NOT TO BREAK:**

When we find the torque acting on a body we **MUST ALWAYS** use the component of the force that is \_\_\_\_\_ to \_\_\_\_\_!!!

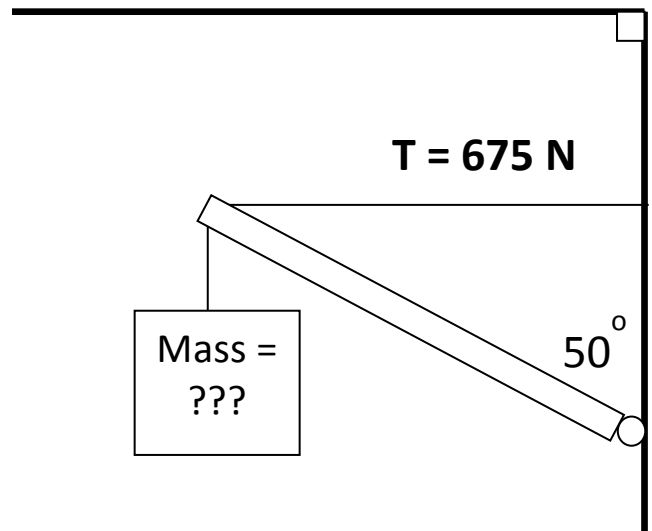
Ex

A 1.8 m long 12.0 kg bar is attached to a wall by a hinge and supported by a rope as shown. Find the tension in the rope.



Ex

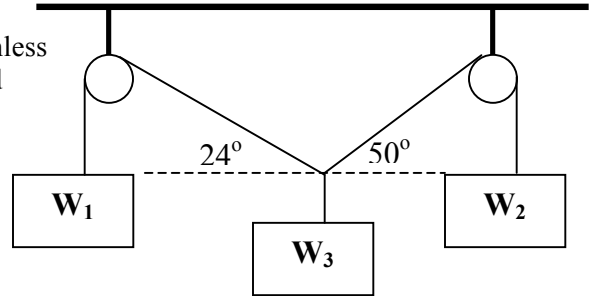
Find the mass of the object given the information in the diagram and that the weight of the uniform beam is 115 N.



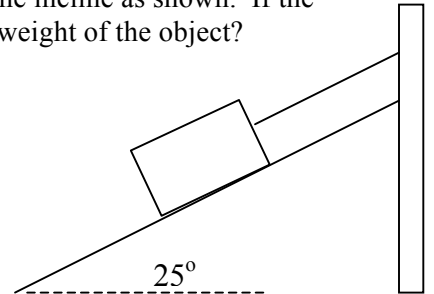
Name: \_\_\_\_\_

**Worksheet 3.1 - Translational Equilibrium** – Remember to include a FBD and resolve the Forces into their components!

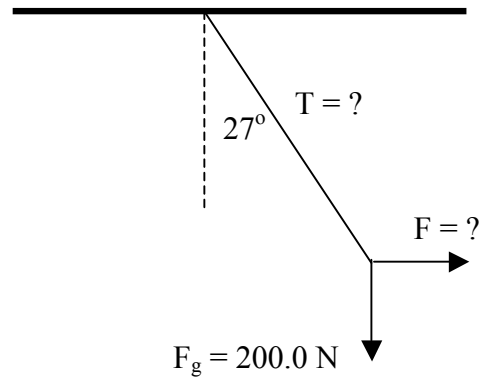
1.  $W_1$ ,  $W_2$  and  $W_3$  are the weights of three objects suspended by pulleys as shown. Assuming the pulleys in this system are frictionless and weightless and that  $W_3 = 12\text{ N}$ , what are the values of  $W_1$  and  $W_2$ ? (Simpler than you think!)



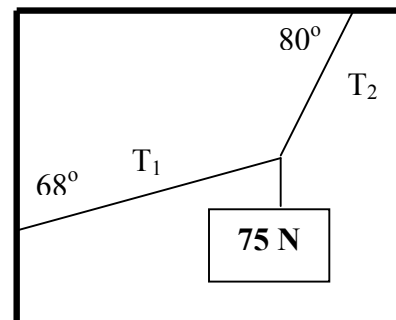
2. An object is suspended on a frictionless inclined plane by a rope parallel to the incline as shown. If the angle of the incline is  $25^\circ$  and the tension in the rope is  $5000\text{ N}$ , what is the weight of the object?



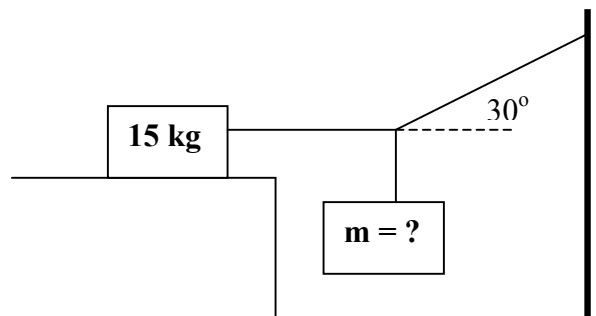
3. A  $200.0\text{ N}$  child sitting on a playground swing is being pushed by her father. When the rope makes an angle of  $27^\circ$  to the vertical what is the force exerted by her father? What is the tension in the rope,  $T$ ?



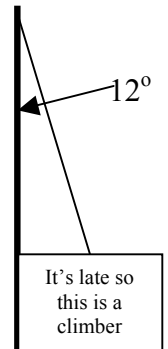
4. Find the tensions  $T_1$  and  $T_2$  in the ropes shown in the diagram.



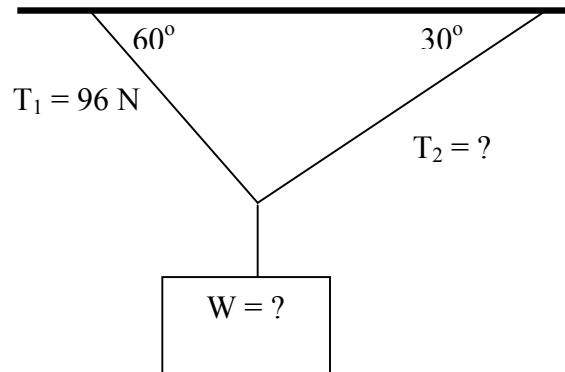
5. A  $15\text{ kg}$  object rests on a table. A cord is attached to this object and also to a wall. Another object is hung from this cord as shown. If the coefficient of friction between the  $15\text{ kg}$  object and the table is  $0.27$ , what is the maximum mass that can be hung, without movement?



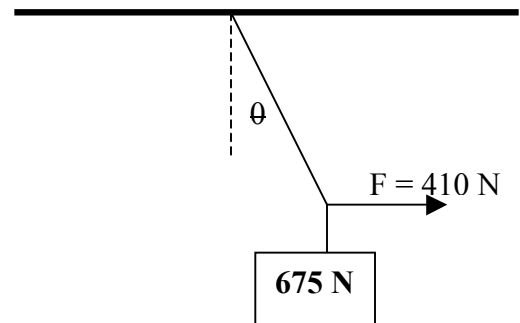
6. A 735 N mountain climber is rappelling down the face of a vertical cliff as shown in the diagram. If the rope makes an angle of  $12^\circ$  with the cliff face, what is the tension in the rope? Assume that the climber pushes horizontally off of the cliff.



7. Given the following diagram, find  $W$  and  $T_2$ .



8. A 675 N object is pulled horizontally by a force of 410 N as shown. What is the angle,  $\theta$ , between the rope and the vertical?



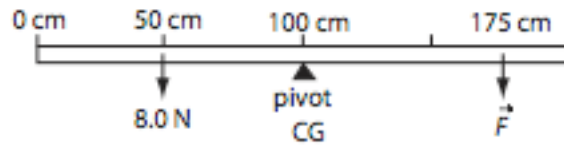
Answers:

1.  $W_1 = 8.0 \text{ N}$ ,  $W_2 = 11.4 \text{ N}$
2.  $1.183 \times 10^4 \text{ N}$
3.  $F = 101.9 \text{ N}$ ,  $T = 224.5 \text{ N}$
4.  $T_1 = 15 \text{ N}$   $T_2 = 82. \text{ N}$
5.  $2.3 \text{ kg}$
6.  $751 \text{ N}$
7.  $W = 111 \text{ N}$ ,  $T_2 = 55 \text{ N}$
8.  $31.3^\circ$

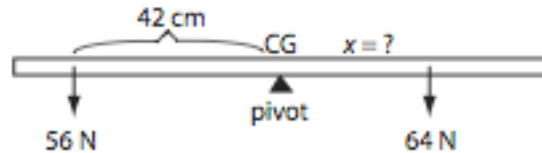


### Worksheet 3.2 - Torque

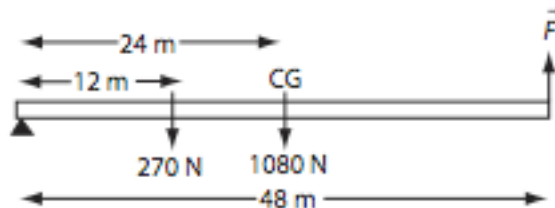
1. What force  $F_1$  is needed to balance the beam in the diagram below?



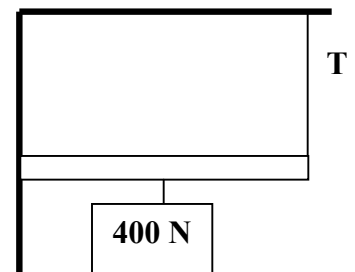
2. How far from the pivot must the 64 N object be placed to balance the beam in the diagram below?



3. What upward force  $F_1$  is needed to achieve rotational equilibrium in the diagram below?
4. If the torque needed to loosen a lug nut is 45 Nm and you are using a 35 cm wheel wrench, what force do you need to exert perpendicular to the end of the wrench?

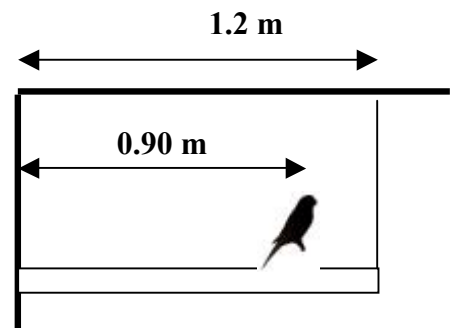


5. A beam of negligible mass is attached to a wall by a hinge. Attached to the center of the beam is a 400 N weight. A rope supports the beam as shown in the diagram. What is the tension in the rope?

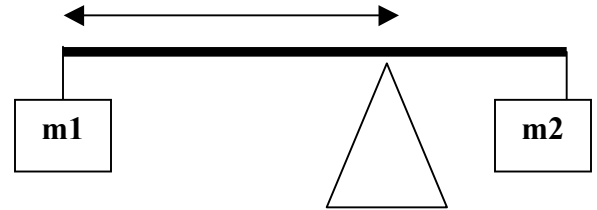


6. Two students sit on either end of a uniform teeter-totter. Student 1 sits 1.10 m from the pivot while Student 2 sits 0.85 m from the pivot. If Student 1 has a mass of 72 kg, what is the mass of Student 2?

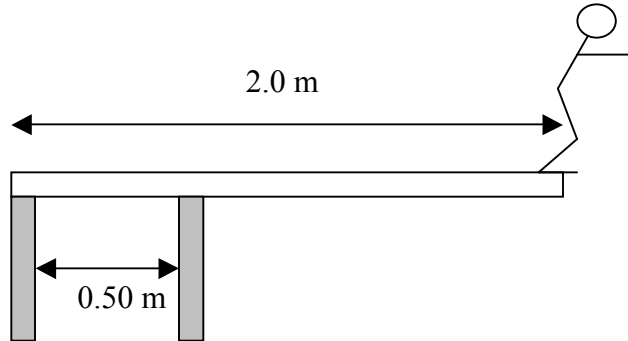
7. A 0.75 kg bird stands on a uniform 1.0 kg stick as shown. The stick is attached to a wall with a hinge and to the ceiling with a rope of negligible mass. What is the tension in the rope?



8. Two masses ( $m_1 = 3.00 \text{ kg}$ ,  $m_2 = 5.00 \text{ kg}$ ) hang from the ends of a metre stick as shown. If the mass of the metre stick is negligible, at what distance from the left of the metre stick should a pivot be placed so that the system will be balanced?

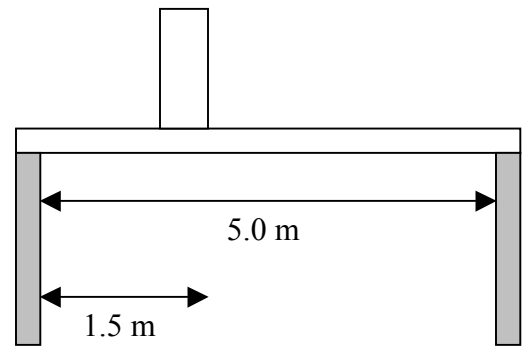


9. A uniform 400 N diving board is supported at two points as shown in the diagram. If a 75 kg diver stands at the end of the board, what are the forces acting on the each support?



10. A 650 N student stands on a 250 N uniform beam that is supported by two supports as shown in the diagram. If the supports are 5.0 m apart and the student stands 1.5 m from the left support:

- What is the force that the right support exerts on the beam?
- What is the force that the left support exerts on the beam?



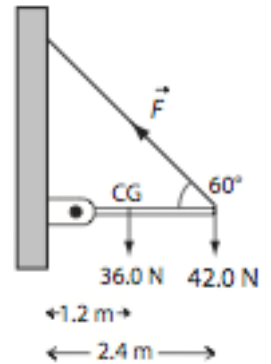
Answers:

- (5.3 N)
- (37 cm OR 0.37 m)
- $6.1 \times 10^2 \text{ N}$
- (130 N)
- (200 N)
- (93 kg)
- (10. N)
- (0.625 m)
- (left support =  $2.61 \times 10^3 \text{ N}$  down , right support =  $3.74 \times 10^3 \text{ N}$  up)
- $F_{\text{right}} = 320 \text{ N}$  ,  $F_{\text{left}} = 580 \text{ N}$

### Worksheet 3.3 – Torque not at 90° –

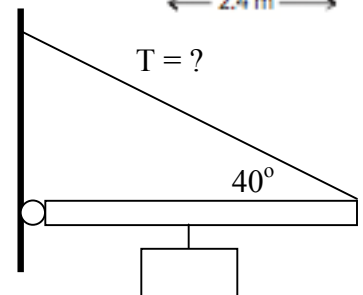
*A lot of this work can be done DIRECTLY on the diagram provided. Please remember to resolve ALL forces into their dimensions*

1. A small 42.0 N sign is suspended from the end of a hinged rod, which is 2.40 m long and uniform shape as shown in the diagram below and to the right. What tension force exists in the rope holding up both the rod and the sign? The rope makes an angle of 60° with the 36.0 N rod.

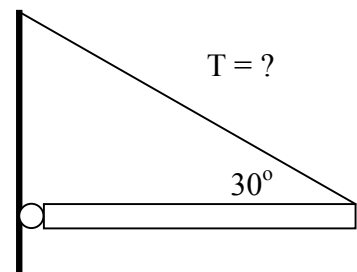


2. A beam of negligible mass is attached to a wall by means of a hinge. Attached to the centre of the beam is a 400. N weight. A rope also helps to support this beam as shown in the diagram.

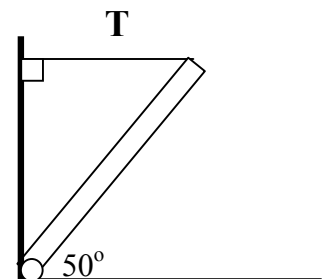
- a) What is the tension in the rope?
- b) What are the vertical and horizontal forces that the wall exerts on the beam?



3. Find the tension in the rope supporting the 200. N hinged uniform beam as shown in the diagram.

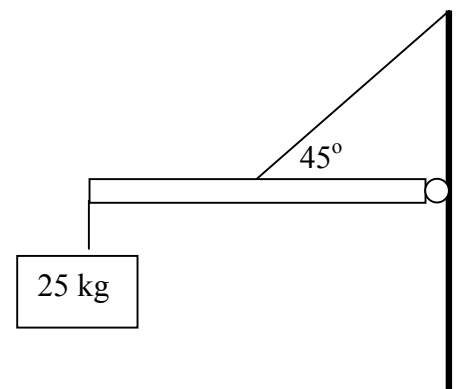


4. Find the tension in the rope supporting the 200. N hinged uniform beam as shown in the diagram.

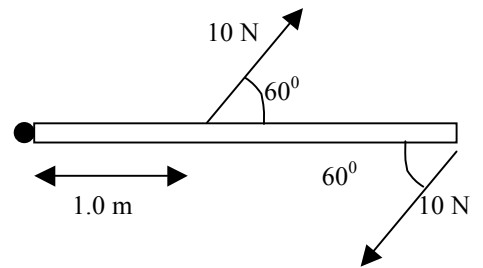


5. A uniform beam (mass = 22 kg) is supported by a cable that is attached to the centre of the beam as shown in the diagram.

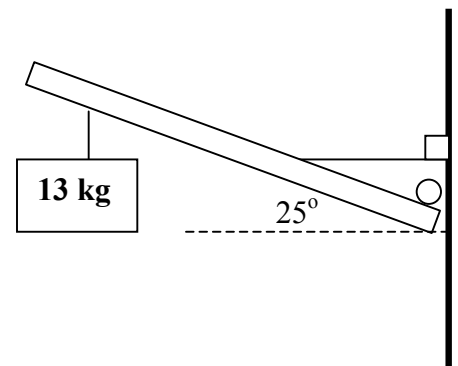
- a) find the tension in the cable.
- b) find the horizontal and vertical forces acting on the hinge.



6. The diagram below shows the top view of a door that is 2.0 m wide. Two forces are applied to the door as indicated in the diagram. What is the net torque on the door with respect to the hinge?



7. A 2.6 m uniform beam (mass of 9.0 kg) is attached to a wall by a hinge and supported by a rope. A 13 kg mass hangs from the beam 2.2 m from the hinge. Find the tension in the rope that is attached to the beam 1.1 m from the wall.



Answers:

1. (69 N)
2. a. (311 N) b. (V: 200 N, H: 238 N)
3. (200. N)
4. (83.9 N)
5. a. (1000 N) b. ( $F_x = 710$  N,  $F_y = 250$  N)
6. (8.66 Nm clockwise)
7. (770 N)