Unit 1: Kinematics in 2D

6 – Vector Addition and Subtraction

When we draw vectors we represent them as arrows with directions

Whenever we add vectors we use... Tip-to-tail method

To find the total or resultant vector, simply draw...

5 m/s [N]

5 m/s 40° [N of E]

from

5 m/s 50° [W of S]

$50°$ [W of N]

Vector Addition Methods:

1. Tip-to-Tail (for drawing general direction)
2. Adding Components (for magnitude) and Trigonometry (for accurate direction)

Add the vectors and find their resultant magnitudes and directions

1) 2.2 m South and 1.8 m North

Resultant vector

0.4 m [S]

2) 220 m North and 80 m West

$d_R^2 = 80^2 + 220^2$

$d_R^2 = 54800$

$d_R = 234\text{ m}$

$\tan\theta = \frac{80}{220}$

$\theta = \tan^{-1}\left(\frac{80}{220}\right)$

$\theta = 20°$ [W of N]

angle touching

When adding vectors does it matter which one you add first?

Ex1) : A student in a canoe is trying to cross a 45 m wide river that flows due East at 2.0 m/s. The student can paddle at 3.2 m/s.

without current t = 14 s

with current t = 14 s

a. If he points due North and paddles how long will it take him to cross the river?

Vert. Vel: $V_b = 3.2\text{ m/s}$

Vert. d: $d = 45\text{ m}$

$V = \frac{d}{t}$

$3.2 = \frac{45\text{ m}}{t}$

$\boxed{t = 14\text{ sec}}$

$V_w = 2\text{ m/s}$

$V_b = 3.2\text{ m/s}$

45 m

b. What is his total velocity relative to his starting point in part a?

$\vec{V}_{Total} = \vec{V}_{boat} + \vec{V}_{current}$

3.2 m/s + 2 m/s

$V_T^2 = 3.2^2 + 2^2$

$V_T = 3.77\text{ m/s}$

$\tan\theta = \frac{2}{3.2}$

$\theta = 32°$ [E of N]

c. If he needs to end up directly North across the river from his starting point, what heading should he take?

V_{water} [E]

V_b [?]

V_{Total} [N]

2 m/s opp

hyp 3.2 m/s

$\sin\theta = \frac{opp}{hyp}$

$\sin\theta = \frac{2}{3.2}$

direction of boat

$\boxed{\theta = 38.7°\ [W\ of\ N]}$

d. How long will it take him to cross the river at this heading?

$3.2^2 = 2^2 + V_T^2$

$V_T = 2.5\text{ m/s}$ [N]

width of river 45 m [N]

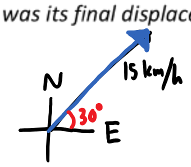
$V_y = \frac{d_y}{t}$

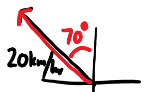
$t = \frac{d_y}{V_y} = \frac{45\text{ m}}{2.5\text{ m/s}} = \boxed{18\text{ sec}}$

Vector Addition – Trig Method

In the previous example we added perpendicular vectors which gave us a nice simple right triangle. In reality it's not always going to be that easy.

Ex2) A bird flies at 15 km/h 30° N of E for 2.5 hr and then changes heading and flies at 20 km/h 70° W of N for 1.5 hr. What was its final displacement?



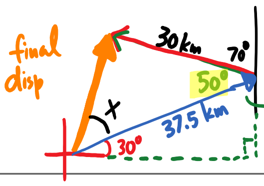


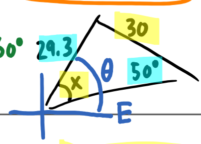
$d_1 = V \times t = 15\,km/h\,(2.5\,hr)$ $\qquad$ $d_2 = 20\,km/h\,(1.5\,hr)$

$d_1 = 37.5\,km$ $\qquad$ $d_2 = 30\,km$

$d_f^2 = 30^2 + 37.5^2 - 2(30)(37.5)\cos(50°)$

$d_f = 29.3\,km$



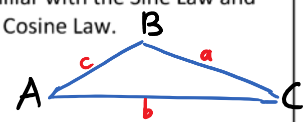


$\frac{\sin x}{30} = \frac{\sin 50°}{29.3}$

$\angle x = 51.7°$

$\angle \theta = 51.7 + 30 = 81.7°$ [N of E]

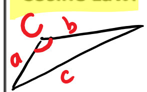
In order to solve non-right angle triangles, we will need to be familiar with the Sine Law and the Cosine Law.



Sine Law:

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Cosine Law:



$c^2 = a^2 + b^2 - 2ab\cos(\angle C)$

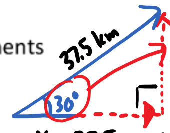
angle

length

Vector Addition – The Component Method

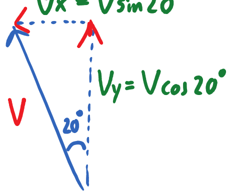
There is another method that we can use when adding vectors. This method is a very precise, stepwise approach; however, it is the only way we can add 3 or more vectors.

- **Draw** each vector
- **Resolve/break** each vector into x and y components
- Find the **total sum** of x and y vectors
- **Add** the x and y vectors
- **Solve** using trig



$y = 37.5\sin 30° = 18.75\,km$

$x = 37.5\cos 30° = 32.5\,km$

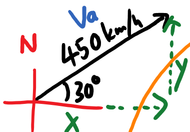


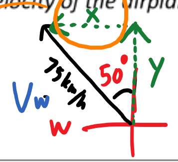
$V_x = V\sin 20°$

$V_y = V\cos 20°$

REMEMBER: When using x and y components…

Ex3. An airplane heading at 450 km/h, 30° north of east encounters a 75 km/h wind blowing towards a direction 50° west of north. What is the resultant velocity of the airplane relative to the ground?

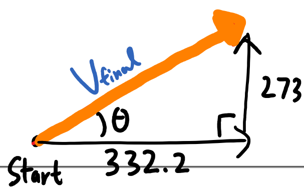




$V = \frac{d\ km}{t\ h}$ km/h

	X-Component	Y-Component
Air Velocity	$V_{ax} = 450\cos(30°) = 389.7$	$V_{ay} = 450\sin(30°) = 225$
Wind Velocity	$V_{wx} = -75\sin(50°) = -57.45$	$V_{wy} = 75\cos(50°) = 48.2$
Resultant	332.2 (E)	273.2 (N)

Total Resultant:



$V_f^2 = (273.2)^2 + (332.2)^2$ $\qquad$ $\tan\theta = \frac{273.2}{332.2}$

$V_f = 430\,km/hr$ $\qquad$ $\theta = 39.4°$ [N of E]

$\vec{A}$ $-\vec{A}$

Vector Subtraction

With vectors a negative sign indicates... it points in the opposite direction

When subtracting vectors we still draw them tip to tail, except... we reverse the negative vector!!

$V_1 - V_2 = V_1 + [-V_2]$

$\rightarrow - \uparrow = \rightarrow + \downarrow \Rightarrow$ $V_1 - V_2$

We generally subtract vectors when dealing with a change in a vector quantity.

ie, ΔV, Δd

Recall:

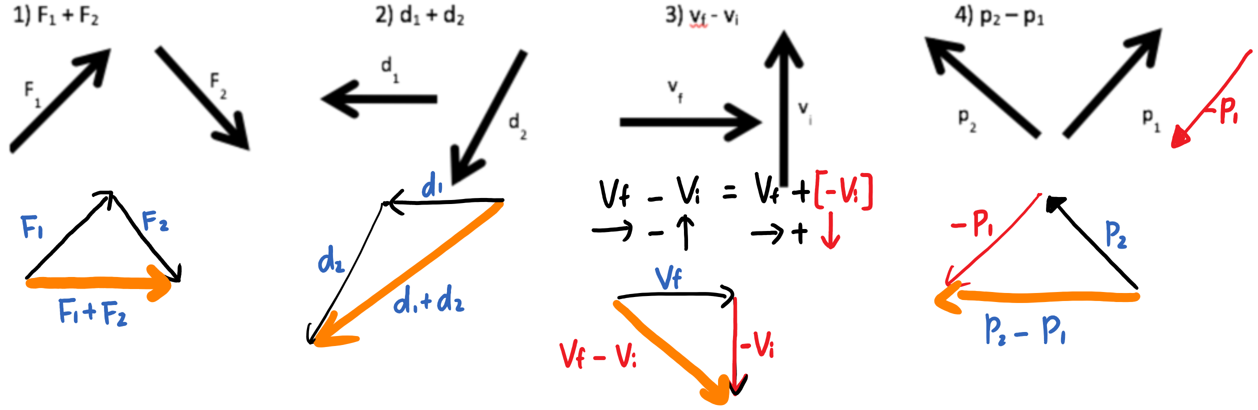
Change = final − initial

$\Delta V = V_f - V_i$

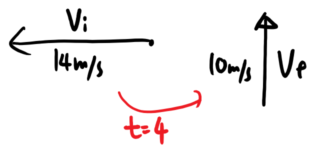
$\Delta d = d_f - d_i$

Draw the Following

1) $F_1 + F_2$ 2) $d_1 + d_2$ 3) $v_f - v_i$ 4) $p_2 - p_1$



Ex 4: A cyclist is traveling at 14 m/s west when he turns due north and continues at 10 m/s. If it takes him 4.0 s to complete the turn what is the magnitude and direction of his acceleration?

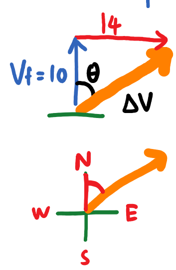


$a_{avg} = \frac{\Delta V}{t}$

$\Delta V = V_f - V_i = V_f + [-V_i]$

$\uparrow - \leftarrow = \uparrow + \rightarrow$

"delta"



$\Delta V^2 = 14^2 + 10^2$

$\Delta V = 17.2$ m/s

$\tan\theta = \left(\frac{14}{10}\right)$

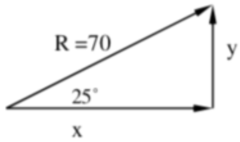
$\boxed{\theta = 54.5^\circ\ [E\ of\ N]}$

$\therefore\ a = \frac{\Delta V}{t} = \frac{17.2\ m/s}{4\ sec} = \boxed{4.3\ m/s^2}$

Worksheet 1.6 – Vectors and Nagvigation

For each question, find the value of x, y, R and/or theta as needed (R is the resultant vector)

1.



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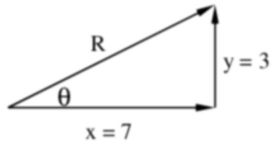
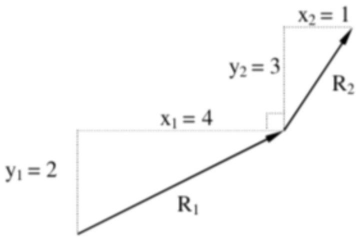
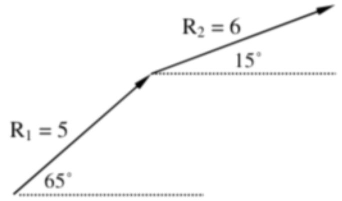


3.



4.



Find $R_{resultant}$

Draw and add the vectors

5) 8m [N] and 5m 30° [N of E]

6) 200m/s 20° [W of S] and 15m/s 20° [W of N]

The Change "Δ" Of A Quantity a.k.a. Vector Subtraction

This deals with the change of a quantity, which can be solved by vector subtraction. We will deal only with $\mathbf{\Delta v = v_f - v_i}$ in these questions but the concept will appear several more times in this course. Remember that each term is a vector (therefore, do not expect to simply subtract the values!!)

7) If a car that was originally going 40. m/s towards the east took 5.0 s to turn and go 30. m/s towards the south, what is the acceleration of the car?

8) What is the acceleration of a car that changes from 60. m/s to the north to 60. m/s to an angle of 45° East of North in a time of 3.0 s?

9) What is the acceleration of a ball that bounces off a wall in 0.30 s if its incoming velocity is 60. m/s and its recoil velocity is 50. m/s?

10) A car is traveling at 100 km/h, due northwest. The driver puts on the brakes and turns the corner. Four seconds later, he is heading east at 50 km/h. What is the average acceleration?

The Across the River Problem

11) A boat can travel 2.30 m/s in still water. If the boat heads directly across a river with a current of 1.50 m/s:

a) What is the velocity of the boat relative to the shore?

b) At what angle compared to straight across is it traveling?

c) How far from its point of origin is the boat after 8.0 s?

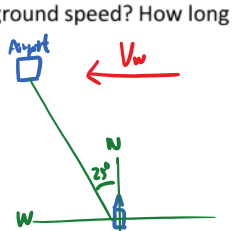
d) At what upstream angle (compared to straight across) must the boat travel in order to the other bank directly opposite its starting point? How fast across the stream is it traveling?

Vector problems (Component or Sine-Cosine Law Solutions)

12) A plane with an air speed of 400 km/hr wants to go north but a wind of 70 km/hr is blowing west. What must be the plane's heading (to go north)? What will be its resulting ground speed?

13) A seagull flying with an air speed of 10 km/h is flying north but suddenly encounters a wind of 5 km/h at 20° south of east. What will be the new direction and airspeed of the seagull?

14) A plane that can fly at 250 km/h wishes to reach an airport that has a bearing of 25° W of N from its present location. If there is a 50.0 km/h wind blowing directly to the west what should be the heading of the plane. What will be its ground speed? How long would it take to get to the airport if it were 560 km away?



250 km/h   50 km/h   ?

V_{air} (plane) + V_w (wind) = V_{final} (ground)

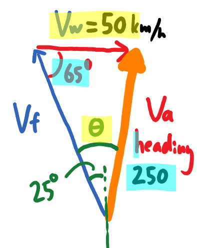
? (heading) + ⟵ = ↖

$V_a + V_w = V_f$

$V_a + \leftarrow = \nwarrow$

$V_a = \nwarrow - \leftarrow$

V_a (heading) $= \nwarrow + \rightarrow$



$$\frac{\sin\theta}{50} = \frac{\sin 65}{250}$$

$\theta = 10.4°$

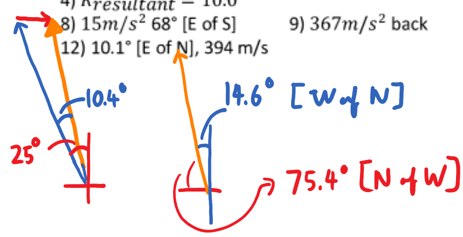
Answer key

1) x=63; y=30.   2) R=7.6 and θ=23°   3) $R_1 = 4.47; R_2 = 3.16$ 4) $R_{resultant} = 10.0$

5) 11.3 m 22° [E of N]   6) 188 m/s 23° [W of S]   7) $10 m/s^2$ 53° [W of S]   8) $15 m/s^2$ 68° [E of S]   9) $367 m/s^2$ back

10) $9.7 m/s^2$ 30° [S of E]   11) a. 2.7m/s 33° b. 33° c. 21.6m d. 41°   12) 10.1° [E of N], 394 m/s

13) 9.5km/h 30° [E of N]   14) 267km/h and 14.6° [W of N] ;2.1h

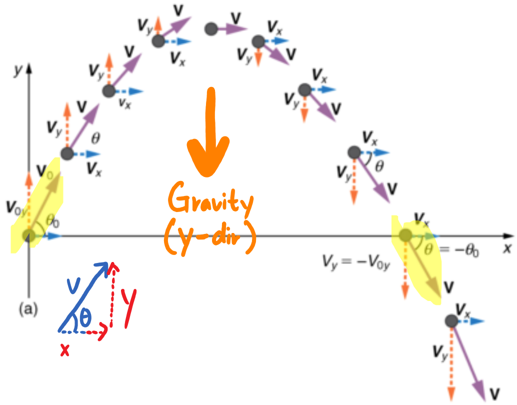


Vector and Kinematics Notes

7 – Projectile Motion 2D

An object launches into the air tends to follows a parabolic path. If you break down the velocity into x and y components you will discover that both sides are perpendicular ⊥ and therefore totally independent.

Fun Fact: if an object is caught at the same height as it was launched. Its landing (Vf) speed must equal to it launching speed (Vi) with opposite angle.



x-components	y-components
• No net Force in the x direction • acceleration is always zero ⇒ constant speed. • The only equation you can every use is $\boxed{V_x = \frac{d_x}{t}}$	• Always a constant acceleration of $-9.8\ m/s^2$ due to earth's gravitational pull. • Need to use the BIG 3 Equations $V_f = V_i + at$ $d = V_i t + \frac{1}{2}at^2$ $V_f^2 = V_i^2 + 2ad$

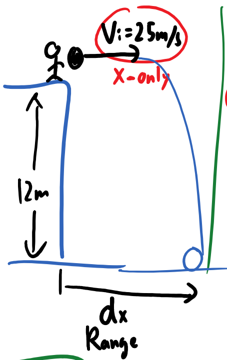
The only value that can ever be on both sides is time because it is scaler and has no direction

> Ex 1: A student sits on the roof of their house which is 12 m high. She can launch water-balloons from a slingshot at 25 m/s. If she fires a water-balloon directly horizontally:
> a. How long will it be airborne?
> b. How far will it travel?

* How long it is airborne only depends on: Vertical (y) direction

* How far it travels in the x-direction depends only on: V_x and t

a)



$\boxed{\text{Y-dir}}$

$V_{iy} = \emptyset$

$a_y = -9.8\ m/s^2$

$d_y = -12\ m$

$t = ?$

$d = \overset{\emptyset}{V_i t} + \frac{1}{2}at^2$

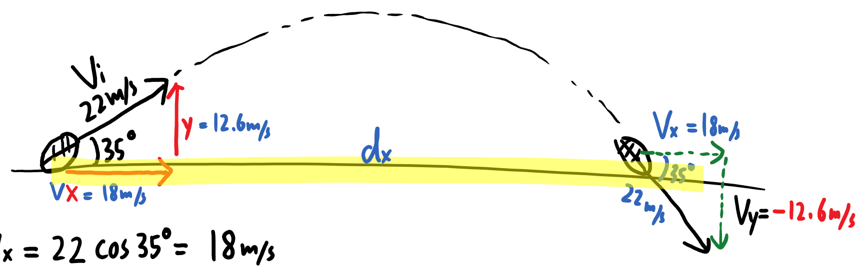
$-12 = \frac{1}{2}(-9.8)t^2$

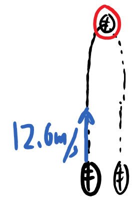
$\boxed{t = 1.56\ \text{sec}}$

b) $\boxed{\text{X-dir}}$ → constant vel. $V_x = \frac{d_x}{t}$ $25 = \frac{d_x}{1.56}$ $\boxed{d_x = 39\ m}$

Ex 2: A quarterback launches a ball to his wide receiver by throwing it at 22.0 m/s at 35° above horizontal.

a. How far downfield is the receiver?
b. How high does the ball go?
c. At what other angle could the quarterback have thrown the ball and reached the same displacement?





$V_x = 22 \cos 35° = 18 \text{ m/s}$

$V_y = 22 \sin 35° = 12.6 \text{ m/s}$

a) Range (d_x)

$\boxed{\text{y-dir}}$ time: $V_i = 12.6 \text{ m/s}$

$V_f = -12.6 \text{ m/s}$

$a_y = -9.8$

$t = ?$

$V_f = V_i + at$

$-12.6 = 12.6 + (-9.8)t$

$t = 2.57 \text{ sec.}$

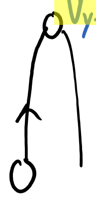
$\boxed{\text{x-dir}}$

$V_x = \frac{d_x}{t}$

$18 = \frac{d_x}{2.57}$

Range $= \boxed{46.4 \text{ m}}$

b) max height (d_y)



$V_{yf} = \emptyset$ (@ max)

$a = -9.8 \text{ m/s}^2$

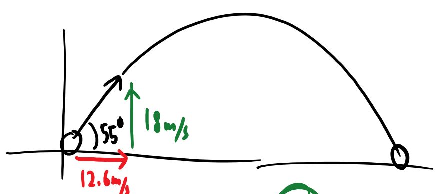
$d = ?$

$V_y = 12.6 \text{ m/s}$

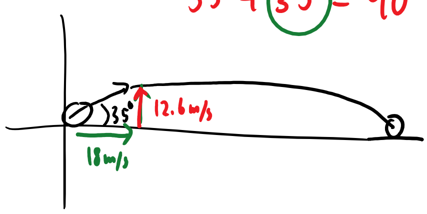
$V_f^2 = V_i^2 + 2ad$

$0 = 12.6^2 + 2(-9.8)\,d$

$\boxed{d_{ymax} = 8.1 \text{ m}}$



$55° + 35° = 90°$

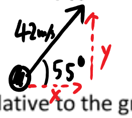


Ex 3: A cannon sits on a 65 m high cliff (typical Trask...so typical...). A cannonball is fired at 42 m/s 55° above the horizontal.

a. How long is it airborne?

b. What is its final velocity?

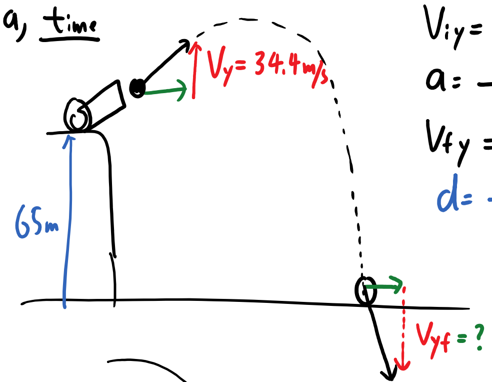
c. What is its maximum height relative to the ground below?



$V_x = 42\cos 55° = 24$ m/s

$V_y = 42\sin 55° = 34.4$ m/s

a) time



$V_{iy} = 34.4$

$a = -9.8$ m/s²

$V_{fy} = ?$

$d = -65$ m

$\boxed{V_f \rightarrow t}$

① $V_f^2 = V_i^2 + 2ad$

$V_f^2 = 34.4^2 + 2(-9.8)(-65)$

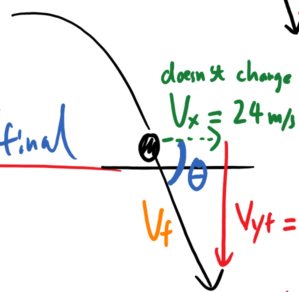
$V_f = -49.57$ m/s (down)

② $V_f = V_i + at$

$-49.57 = 34.4 + (-9.8)t$

$\boxed{t = 8.6 \text{ sec}}$

b) Vfinal



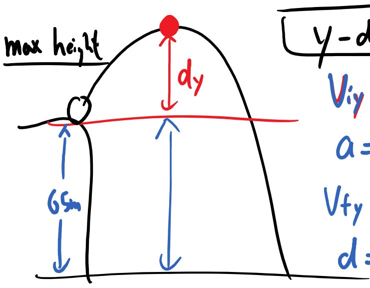
$V_f^2 = 24^2 + (49.57)^2$

$\tan\theta = \left(\frac{49.57}{24}\right)$

$\boxed{V_f = 55.1 \text{ m/s}}$

$\boxed{\theta = 64° \text{ below horizontal}}$

c) max height



$\boxed{y\text{-dir}}$

$V_{iy} = 34.4$ m/s

$a = -9.8$ m/s²

$V_{fy} = \emptyset$ @ max.

$d = ?$

$V_f^2 = V_i^2 + 2ad$

$\emptyset = 34.4^2 + 2(-9.8)d$

$d = 60.4$ m

Max $d_y = 60.4\text{ m} + \underset{\text{hill}}{65\text{ m}} = \boxed{125.4\text{ m}}$

Worksheet 1.7 - Projectiles

Solve all problems on your own paper showing all work!

1. A golf ball was struck from the first tee at Lunar Golf and Country Club. It was given a velocity of 48 m/s at an angle of 40° to the horizontal. On the moon, $g = -1.6\ m/s^2$.
   (a) What are the vertical and horizontal components of the ball's initial velocity? ***(Vx = 36.85 m/s; Vyo = 30.8 m/s)***
   (b) For what interval of time is the ball in flight? ***(38.6 sec)***
   (c) How far will the ball travel horizontally? ***(1418 m)***

2. A rock is thrown horizontally from the top of a cliff 98 m high, with a horizontal speed of 27 m/s.
   (a) For what interval of time is the rock in the air? ***(4.47 sec)***
   (b) How far from the base of the cliff does the rock land? ***(121 m)***
   (c) With what velocity does the rock hit? ***(51.5m/s, 61.3° below horizontal)***

3. A batter hits a ball giving it a velocity of 48 m/s at an angle of 50° above the horizontal.
   (a) What are the vertical and horizontal components of the ball's initial velocity? ***(30.8m/s, 36.8 m/s)***
   (b) How long is the ball in the air? ***(7.50 sec)***
   (c) What is the horizontal distance covered by the ball while in flight? ***(231 m)***
   (d) What velocity does the ball have at the top of its trajectory? ***(30.8 m/s horizontal only)***

4. A ball is thrown with a velocity of 24 m/s at an angle of 30° to the horizontal.
   (a) What are the vertical and horizontal components of the initial velocity? ***(12 m/s, 20.8 m/s)***
   (b) How long is the ball in the air? ***(2.45 sec)***
   (c) How far away will the ball land? ***(50.9 m)***
   (d) To what maximum height will the ball rise? ***(7.34 m)***
   (e) With what velocity will the ball land? ***(24 m/s 30° below hoiz)***

5. A diver takes off with a speed of 8.0 m/s from a 3.0 m high diving board at 30° above the horizontal. How much later does she strike the water? ***(1.28 sec)***

6. On level ground, a ball is thrown forward and upward. The ball is in the air 2.0 s and strikes the ground 30 m from the thrower. What was the ball's initial velocity? (***17.9 m/s 33^{o} above horiz***)