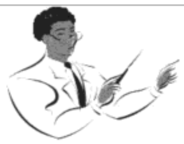
Lesson 13: Adding Probabilities



You learnt the meaning of complementary events and how to find the probability of the complement of an event In the last lesson.

In this lesson, you will:

- define mutually exclusive events
- identify the addition law and works out the probabilities of mutually exclusive events
- use the concept of adding probabilities to calculate probabilities of events.

We learnt earlier that an **event** is a set of outcomes. It is a subset of the sample space for an activity or experiment.

Life is full of random events. We need to get a feel for them to be smart and successful people.

The tossing of a coin, throwing of a dice, lottery drawings and picking a ball are all examples of events.

When we say events, we mean one or more outcomes.

Events are of different types.

When an event corresponds to a single outcome of the activity, it is often called a **simple event.**

Examples of simple events

1. Drawing the queen of spades from a deck of standard cards.
2. Getting a tail when tossing a coin
3. Rolling a 5 in throwing a dice

An event can include several outcomes.

Examples

1. Choosing a King from a deck of cards (any of the 4 Kings) is also an event.
2. Rolling an even number (2, 4, 6) is an event.

When two events cannot happen at the same time they are called mutually exclusive **events.** This means that you cannot get both events at the same time. It is either one or the other, but not both.

A common example of a mutually exclusive event is a coin flip. Either the coin will come up heads or tails. Since the coin coming up heads means that it will not come up tails. That makes the coin flip a mutually exclusive event. It will either one side or the other, it cannot be both.

Here are other examples of mutually exclusive events.
no both !!

1. Turning left or right
2. Heads and tails
3. Kings and Aces

Let's look at the probabilities of Mutually Exclusive events.

When two events let say Events A and B, are mutually exclusive it is **impossible** for them to happen together:

$$P(A \text{ and } B) = \varnothing$$

"The probability of A and B together equals $\varnothing$ (Impossible)"

But the probability of A **or** B is the sum of the individual probabilities:

$$P(A \text{ or } B) = P(A) + P(B)$$

"The probability of A **or** B equals the probability of A plus the probability of B"

Example 1: A Deck of Cards

In a Deck of 52 Cards:

- the probability of a King is 4 $P(\text{King}) = \frac{4}{52} = \frac{1}{13}$
- the probability of an Ace is also $P(\text{Ace}) = \frac{4}{52} = \frac{1}{13}$

When we combine those two Events:

- The probability of a card being a King **and** an Ace is **0** (Impossible).
- The probability of a card being a King **or** an Ace is (

They can be written like this:

P(King and Ace) = ~~0~~ (Impossible)

P(King or Ace) = ($\frac{1}{13} + \frac{1}{13}$) = $\boxed{\frac{2}{13}}$

A special notation is used to denote the word "and" and "or" in the rule.

Instead of "and" you will often see the symbol $\cap$ (which is the "Intersection" symbol used in Venn Diagrams)

So, P(A and B) is written as P($A \underset{\text{and}}{\cap} B$)

Instead of "or" you will often see the symbol $\cup$ (the "Union" symbol)

So, P(A or B) is written as P($A \underset{\text{or}}{\cup} B$)

Example 3: Scoring Goals

Q

If the probability of:

- scoring no goals (Event "A") is **20%**
- scoring exactly 1 goal (Event "B") is **15%**

Then:

a) • The probability of scoring no goals **and** 1 goal is _____

- The probability of scoring no goals **or** 1 goal is

They can be written like this:

a, $P(A \underset{\text{and}}{\cap} B) = \boxed{\emptyset}$

b, $P(A \underset{\text{or}}{\cup} B) = P(A) + P(B) = 20\% + 15\% = \boxed{35\%}$

So, to find the probability that one or the other of two mutually exclusive events will occur, add their individual probabilities.

This is called the Addition Rule.

Now let us have other examples.

Example 3

36 total

If two dice are tossed, the event A of rolling a total of 6 and the event B of rolling a total of 9 are mutually exclusive.

4,5 5,4
3,6 6,3

3,3
1,5 5,1
2,4 4,2

Find the probability of A or B occurring.

Solution: P(A or B) = P(A) + P(B)

$$= \frac{5}{36} + \frac{4}{36}$$

$$= \frac{9}{36} = \boxed{\frac{1}{4}} = \boxed{25\%}$$

Example 4

Yes / No

Which of the following represents a pair of mutually exclusive events when a die is rolled?

even #

No (a) obtaining an even number or obtaining a 4 $A = \{2, 4, 6\}$ $B = \{4\}$ No : There is a common element.

No (b) obtaining an odd number or obtaining a 3

Yes (c) obtaining a number less than 3 or obtaining a number more than 5

1, 2 6

No (d) obtaining a multiple of 2 or obtaining a multiple of 3

No (e) obtaining a factor of 6 or obtaining a multiple of 6

d) $A = \{2, 4, 6\}$
$B = \{3, 6\}$ Not mutually Exclusive.

$A = \{1, 2, 3, 6\}$
$B = \{6\}$

Solution:

Example 5

Total = 8 cars

In a certain parking lot, there are 4 Toyota, 3 Isuzu and 1 Hyundai.
What are the chances of either a Toyota or an Isuzu car will leave the parking lot?

Solution:

A Toyota cannot be included with Isuzu. A given vehicle cannot be a Toyota and an Isuzu at the same time. Thus the two vehicles are mutually exclusive.

Thus,

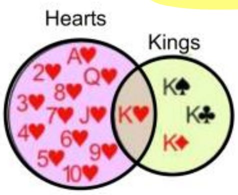
P(Toyota or Isuzu) = P(Toyota) + P(Isuzu)

$$= \frac{4}{8} + \frac{3}{8}$$

$$= \boxed{\frac{7}{8}}$$

Now let's see what happens when events are **not Mutually Exclusive**.

Example: Hearts and Kings



Hearts **and** Kings together is only the King of Hearts: 

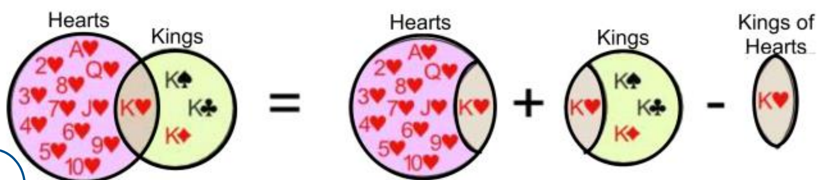
But Hearts **or** Kings is:

- all the Hearts (13 of them)
- all the Kings (4 of them)

] = 17

But that counts the King of Hearts twice **!**

So we correct our answer, by subtracting the extra "and" part:



16 Cards = 13 Hearts + 4 Kings - the 1 extra King of Hearts

You can count them to make sure this works.

As a formula this is:

$$\boxed{P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)}$$

"The probability of A **or** B equals the probability of A **plus** the probability of B **minus** the probability of A **and** B"

Here is the **same formula**, by using the symbol U and ∩.

$$\boxed{P(A \underset{or}{\cup} B) = P(A) + P(B) - P(A \underset{and}{\cap} B)}$$

Now look at Example 6.

16 people study French, 21 study Spanish and there are 30 altogether. Work out the probabilities!

Solution:

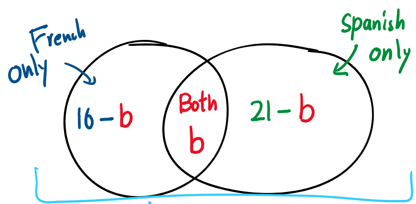
This is definitely a case of **not** Mutually Exclusive (you can study French and Spanish).

both

Let's say **b** is how many study both languages:

- people studying French Only must be $(16-b)$
- people studying Spanish Only must be $(21-b)$

And we get:



And we know there are **30** people, so:

$$(16-b)+b+(21-b)=30$$

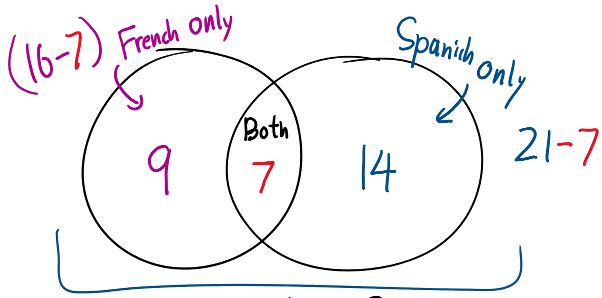
French 16 + Spanish $21 = \boxed{37}$

$$37 - b = 30 \quad -37$$

$$-b = -7$$

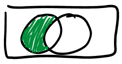
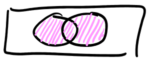
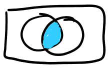
$$\boxed{\text{both} = 7 \text{ ppl}}$$

And we can put in the correct numbers:



$$9+7+14 = 30 \text{ ppl.}$$

So we know all this now:

- P(French) = 16/30 
- Try • P(Spanish) = 21/30
- P(French Only) = 9/30 
- Try • P(Spanish Only) = 14/30
- Try • P(French or Spanish) =  = 30/30 (Union)
- Try • P(French and Spanish) =  = 7/30 (Overlap)

Lastly, let's check with our formula:

P(A or B) = P(A) + P(B) – P(A and B)

Put the values in: $= \frac{16}{30} + \frac{21}{30} - \frac{7}{30}$

$$\boxed{P(A \text{ or } B) = \frac{30}{30} \checkmark}$$

Yes, it works!

Remember:

Mutually Exclusive

- A **and** B together is _Impossible_: **P(A and B) = 0**
- A **or** B is the _sum_ of A and B: **P(A or B) = P(A) + P(B)**

Non- Mutually Exclusive

- A **or** B is the _sum_ of A and B _minus_ A **and** B: **P(A or B) = P(A) + P(B) - P(A and B)**

NOW DO PRACTICE EXERCISE 13

Practice Exercise 13

1. For the rolling of a die, decide whether the following pairs of events are mutually exclusive or not:

no overlap.

   (a) A = {3, 5} ; B = {2, 4} Yes

   (b) A = factor of 3 ; B = factor of 4 A = {1, 3} B = {1, 2, 4} No

   (c) A = less than 3 ; B = more than 3

   (d) A = even number ; B = factors of 6

2. For a standard pack of cards, do the following pairs of events overlap?

   (a) A = king ; B = jack

   (b) A = ~~a~~ 6 ; B = ~~a~~ 3

   (c) A = heart ; B=: queen

   (d) A = black ; B = ~~a~~ 5

3. A die is rolled. Find the probability of the scores being:

   (a) either a 3 or a 5

   (b) an odd or even

   (c) a multiple of 3 or a multiple of 4

   (d) divisible by 4 or 6

   (e) multiple of 5 or factor of 6

   (f) multiples of 2 or multiples of 4

   (g) at least 3.

4. From a standard pack of cards, find the probability of drawing:

   (a) a club or a spade

   (b) a red or a black

   (c) a card which is not black

   (d) a 2 or a club

   (e) a 4 or a card less than 7 (excluding ace)

CORRECT YOUR WORK, ANSWER ARE AT THE END OF TOPIC 2