Lesson 14: Multiplying Probabilities

You learnt about mutually exclusive events and how to work out their probabilities using the addition rule.

In this lesson, you will:

- define independent events
- multiply probabilities of independent events
- find the probability of independent events using the multiplication rule.

Let us start this lesson by considering the following situation.

It is critical that the engine of a single-engine plane does not fail during a flight. These planes often have two **independent** electrical systems. In the event that one fails, for example, due to a faulty spark plug, the second system will still be able to keep the plane in flight.

Now let us consider another situation which involves an experiment where a jar contains one red, one black and one white marble. One marble is withdrawn and then **replaced**. A second withdrawal is made. What is the probability of drawing a white, then a red?

There are a couple of things to take note about this experiment. Withdrawal of one marble from the jar, replacing it, then, withdrawing again from the same jar is a **compound event**. Since the first marble was replaced, withdrawing a red marble on the first try has no effect on the probability of withdrawing a red pair on the second try. No matter what colour was taken in the first draw the second draw could be of any colours, because the first marble was **replaced.** The occurrence of one event does not affect the occurrence of the other. Thus the two draws are **independent** of each other.

Two events **A** and **B** are _Independent events_ if the fact that the occurrence of one event **A** _does not affect_ the probability of the other **B**.

Here are some other examples of independent events.

(a) The events that a coin landing heads on one toss **AND** tails on another toss. The result of one toss does not affect the result of the other, so the events are independent.

(b) Landing on _head_ after tossing a coin **AND** rolling a _5_ on a single 6-sided die.

(c) Choosing a marble from a jar **AND** landing on _tail_ after tossing a coin.

(d) Choosing a _3_ from a deck of cards, _replacing_ it, **AND** then choosing an _Ace_ as the second card.

(e) Rolling a _4_ on a single 6-sided die, **AND** then rolling a _5_ on a second roll of the die.

Finding the Probability of independent Events

To find the probability of two independent events that occur in sequence, find the probability of each event occurring separately, and then multiply the probabilities. This multiplication rule is defined symbolically below. Note that multiplication is represented by AND.

Multiplication Rule: When two events, A and B, are independent, the Probability of both events occurring is:

$$\boxed{P(A \text{ and } B) = P(A) \bullet P(B)}$$

Example 1

A coin is tossed and a single sided die is rolled. Find the probability of landing on the head side of the coin and rolling a 4 on the die.

Solution: First, find the Probability of each individual event.

H and 4

2

$P(\text{Head}) = \frac{1}{2}$

$P(4) = 1/6$

$P(\text{Head and } 4) = P(H) \cdot P(4)$

$= \frac{1}{2} \cdot \frac{1}{6}$

$= \boxed{\frac{1}{12}}$

Example 2

A card is chosen at random from a deck of 52 cards. It is then replaced and a second card is chosen. What is the probability of choosing a Queen and then a nine?

Solution: First, find the Probability of each individual event.

$P(\text{Queen}) = \frac{4}{52}$

$P(9) = 4/52$

$P(\text{Queen and } 9) = P(Q) \bullet P(9)$

$= \frac{4}{52} \bullet \frac{4}{52}$

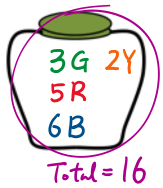
$= \frac{16}{2704}$

$= \boxed{\frac{1}{169}}$

Example 3

A jar contains 3 green, 5 red, 2 yellow and 6 blue marbles. A marble is chosen at random from the jar. After replacing it, a second marble is chosen. What is the probability of choosing a red and then a blue marble?

Solution: First, find the Probability of each individual event.



P(Red) = $\frac{5}{16}$

P(Blue) = $\frac{6}{16}$

P(Red and Blue) = $\frac{5}{16} \bullet \frac{6}{16}$

$= \frac{30}{256}$

$= \boxed{\frac{15}{128}}$

Example 4

An experiment consists of spinning the spinner below three times. For a spin, all outcomes are equally likely.

1, 3, 5

(a) What is the probability of spinning an odd number three times?
(b) What is the probability of spinning a 5 at least once?

Solution:

(a) For each spin, the probability is: P(odd) = $\frac{3}{5}$

So, the probability of spinning an odd number three times is:

P(odd, odd, odd) = $\frac{3}{5} \bullet \frac{3}{5} \bullet \frac{3}{5}$

$= \boxed{\frac{27}{125}}$ //

Spin 3 times: at least one 5

No 5 / One 5 / two 5s / three 5s

(b) Think of this: P(at least one 5) + P(no 5) = 1

For each spin, P(not 5) = $\frac{4}{5}$

P(not 5, not 5, not 5) = $\frac{4}{5} \cdot \frac{4}{5} \cdot \frac{4}{5} = 0.512$

Subtract from 1 to find the probability of spinning at least one 5.

So, P(at least one 5) = $1 - 0.512 = \boxed{0.488}$

Example 5

Charles drawer contains one pair of socks with each of the following colours: red, blue, green, black, yellow and blue. Each pair is folded together in a matching set. Charles goes into the sock drawer and picked a pair of socks without looking. He replaced this pair and then picked another pair of socks. What is the probability that Charles will pick the red pair of socks both times?

Solution: First, find the Probability of each individual event.

$$P(\text{Red}) = \frac{1}{6}$$

$$P(\text{Red and Red}) = P(\text{Red}) \cdot P(\text{Red})$$

$$= \frac{1}{6} \cdot \frac{1}{6}$$

$$= \boxed{\frac{1}{36}}$$

Example 6

A nationwide survey found that 72% of people in Port Moresby like pizza. If 3 people are selected at random, what is the probability that all three like pizza? Round off your answer to the nearest per cent.

Solution: Let x represent the event of randomly choosing a person who likes pizza from Port Moresby.

$$P(x) \cdot P(x) \cdot P(x) = 0.72 \times 0.72 \times 0.72 = 0.37 = \boxed{37\%}$$

Therefore, the probability that all three like pizza is

Sometimes some events can be "dependent".

Two events **A** and **B** are dependent **events** if the fact that the occurrence of one event **A** does affect the occurrence of the other event **B** so that the probability is change.

To calculate the probability of dependent events occurring, do the following:

1. Calculate the probability of the first event.
2. Calculate the probability that the second event would occur if the first event had happened already.
3. Multiply the probabilities.

If **A** and **B** are dependent **events**, then **P(A and B)=** $P(A) \cdot P(B \text{ after } A)$

Suppose you draw 2 marbles without replacement from a bag that contains 3 purple and 3 orange marbles. On the first draw,

−1 P

Before first draw

1st P(purple) = $\frac{3}{6} = \frac{1}{2}$

The sample space for the second draw depends on the first draw.

If the first draw was purple, then the probability of the second draw being purple is

2nd P(purple) = $\frac{2}{5}$

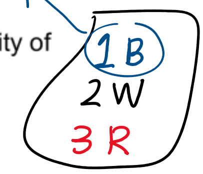
After first draw

So the probability of drawing two purple is

P(1st purple, 2nd purple) = $\frac{1}{2} \cdot \frac{2}{5} = \boxed{\frac{1}{5}}$

Now let us solve problems on the probability of dependent events.

Example 7 Try.



A bag contains one blue, two white and three red marbles. Find the probability of drawing a blue and then a red, if the first marble drawn is not replaced.

Solution:

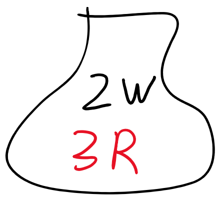
The first draw P(blue) = $\frac{1}{6}$

The marbles available for the second draw depend on what was selected in the first draw. With the blue not available in the bag, the number of possible outcomes reduces to 5. Hence, in draw 2, P(Red) = 3/5

Thus,

P(B and then R) = $\frac{1}{6} \cdot \frac{3}{5}$

$= \boxed{\frac{1}{10}}$



✱ P(1st Blue and 2nd Blue) = ∅

NOW DO PRACTICE EXERCISE 14

Practice Exercise 14

1. Four cards are chosen from a standard deck of 52 playing cards with replacement.

   What is the probability of choosing 4 hearts in a row?

2. A jar contains 6 red balls, 3 green balls, 5 white balls and 7 yellow balls. Two balls are chosen from the jar, with replacement.

   (a) What is the probability that both balls chosen are green?

   (b) What is the probability of choosing a red and a yellow ball?

3. Spin a spinner numbered 1 to 7, and toss a coin.

   What is the probability of getting an odd number on the spinner and a tail on the coin?

4. A survey showed that 65% of all the students dislikes eating vegetables. If 4 children are chosen at random, what is the probability that all 4 children dislike eating vegetables? Round off your answer to the nearest per cent.

5. Two cards are chosen from a deck of 52 cards without replacement.

   What is the probability of choosing two kings?

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3