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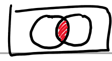
Lesson 15: Union and Intersection of Two Events

You learnt to identify independent and dependent events in lesson 14. You also learnt to find the probability of independent and dependent events using the multiplication rule.

In this lesson you will:

- review the concepts of union and intersection of two events
- find the probability of the intersection and union of two events.



Let us start by understanding the concepts of Union (or (+)) and Intersection (and (x)).

Suppose the experiment is throwing a die, then the sample space is {1, 2, 3, 4, 5, 6}.

From the sample space, we can have events such as:

Event A = {1, 3}

Event B = {4, 5, 6}

To find the **union** (or) of A and B, in symbol $A \cup B$ (or) (read as A union B), we need to find a set whose elements are elements that is in A or element that is in B.

Thus AUB = { 1, 3, 4, 5, 6 } (or)

In another case, if suppose Event C = {1, 2, 3, 6} and Event D = {2, 4, 6}

Then CUD (or) = { 1, 2, 3, 6, 4 }

Another concept that you need to learn is **intersection.**

To find the **intersection** (and) of A and B, in symbol $A \cap B$ (read as A intersects B), C∩D you need to find the element that is in A and that is in B too.

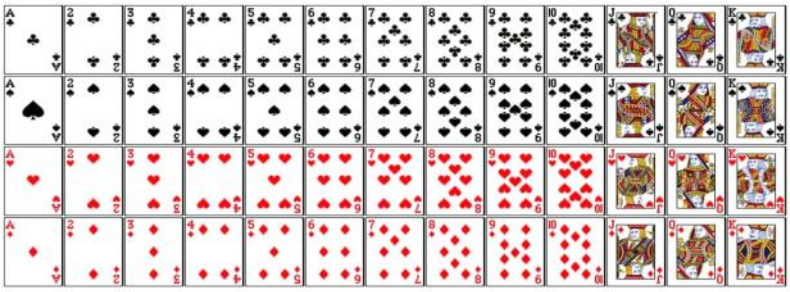
Thus A∩B (and) = ∅ (nothing)

Φ is symbol for null Set. It indicates that the two sets have no common Element.

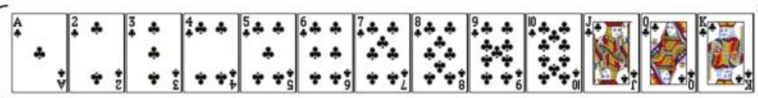
If you are asked to find C∩D (and), then C∩D = { 2, 6 }.

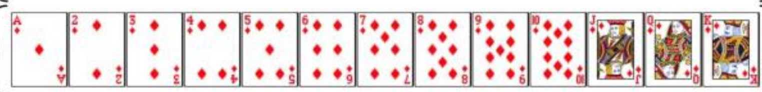
Since the elements 2 and 6 are both in C and D.

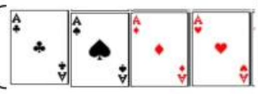
Let us have another example by considering the pack of cards below:



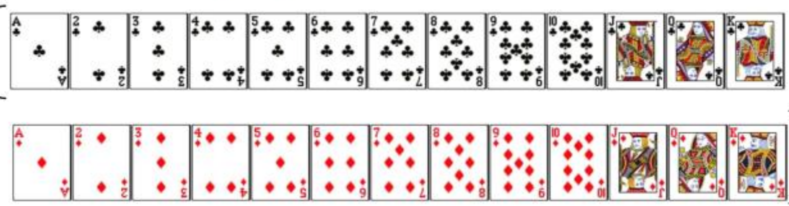
Let us have the following events:

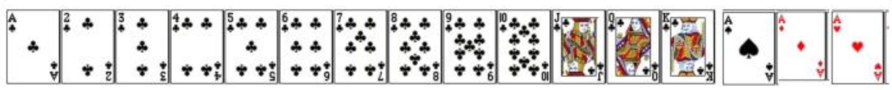
(club) A = 

(Diamond) B = 

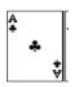
(Aces) C = 

Thus,

AUB = 

AUC = 

Club and diamond
A∩B = Ø

Club and Ace
A∩C = 

Let us use the idea of union and intersection in probability.

Union of Events

The union of events includes not only the probability of A and B, but also the probability of only A and the probability of only B.

$$\boxed{P(A\overset{\text{or}}{\cup}B) = P(A) + P(B) - P(A\underset{\text{overlap}}{\overset{\text{and}}{\cap}}B)}$$

The last term is important since it removed items that appear in both A and B, thereby avoiding double counting.

Consider the following example, meant to clarify the importance of the **- P(A∩B)** term in the formula for **P(AUB).**

Example 1

When rolling a die, what is the probability of rolling a number greater than 1 or a number that is even?

+

Solution: The presence of the word "OR" is a clue that the question is asking about the union of events. ⇒ +

Let Event A = rolling a number greater than 1
Let Event B = rolling a number that is even

5 items

Since there are five possibilities and six total numbers that could be rolled,

Greater than 1

The Probability of rolling a number greater than 1 is $\underbrace{1, (2, 3, 4, 5, 6)}_{6 \text{ total}}$

$$P(A) = \frac{5}{6}$$

Since there are three possibilities (i.e., 2, 4, 6) and six total numbers that could be rolled (i.e., 1, 2, 3, 4, 5, 6)

3

The Probability of rolling a number that is even is

$$P(B) = \frac{3}{6}$$

$$P(AUB) \neq \frac{5}{6} + \frac{3}{6} = \frac{8}{6}$$

Since:

(i) this number is greater than one, which is Impossible for a probability

(ii) this number double counts the even numbers greater than 1 (i.e., 2, 4, 6)

we need to remove the double counting!

(>1) and (even)

P(A∩B) = rolling a number greater than 1 and even

$$(2, 4, 6) \quad = \frac{3}{6}$$

Hence using the formula

$$\overset{\text{or}}{P(AUB)} = \overset{>1}{P(A)} + \overset{\text{even}}{P(B)} - \overset{>1 \text{ and even}}{P(A\cap B)}$$

Greater than 1

(2, 3, 4, 5, 6)

$$= \frac{5}{6} + \cancel{\frac{3}{6}} - \cancel{\frac{3}{6}}$$

even

(2, 4, 6)

$$= \boxed{\frac{5}{6}}$$

Therefore the probability of rolling a number greater than 1 or a number that is even is

Example 2

When drawing a card from a standard 52-card deck, what is the probability of drawing a red card or an ace?

Solution: The presence of the word "or" is a clue that the question is asking about the union of events.

Let Event A = drawing an ace

Let Event B = drawing a red card

Since there are four aces in each deck of 52 cards

$$P(A) = \frac{4}{52}$$

Since there are four suits and two of them are red (or 26 red cards in a deck of 52)

$$P(B) = \frac{26}{52}$$

Since there are 2 red aces in a deck of 52 cards)

P(A∩B) = the probability of drawing a red ace = $\frac{2}{52}$

It is important to subtract P(A∩B), otherwise you would double count the red aces.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{4}{52} + \frac{26}{52} - \frac{2}{52} = \frac{\cancel{28}^{\cancel{14}\;7}}{\cancel{52}_{\cancel{26}\;13}} = \boxed{\frac{7}{13}}$$ Red or Ace

Therefore, the probability of drawing a red card or an ace is

and (x)

Intersection of events

The intersection of events includes only the probability that both events A and B occur simultaneously. If event A occurs alone or if event B occurs alone, this does not fall within the intersection of events A and B.

Formula: P(A∩B) = probability of the intersection of events A and B

Prob. of B given that A already happened.

and

$$\boxed{P(A \cap B) = P(A) \cdot P(B|A)}$$

or (A, B are dependent)

$$\boxed{P(A \cap B) = P(B) \cdot P(A|B)}$$

If events A and B are mutually exclusive (i.e., if one event occurs then the other cannot occur), the formula can be simplified:

P(A∩B) = probability of the intersection of events A and B

$$\boxed{P(A\cap B) = P(A) \bullet P(B)}$$

(A, B Independent)

Example 3

What is the probability of drawing a red card from a standard 52 card deck and rolling an even number on a fair die?

Solution: Since drawing a red card and rolling an even number are entirely unrelated and have no bearing on each other, the two events are Independent and you can use the simplified formula for the intersection of events.

Let Event A = drawing a red card

Let Event B = rolling an even number

The probability of drawing a red card is: $P(A) = \frac{26}{52} = \frac{1}{2}$

1, 2, 3, 4, 5, 6

The probability of rolling an even number is: $P(B) = \frac{3}{6} = \frac{1}{2}$

The probability of drawing a red card and rolling an even number is:

$$P(A\underset{\text{and}}{\cap}B) = P(A) \bullet P(B) = \frac{1}{2} \bullet \frac{1}{2} = \boxed{\frac{1}{4}}$$

To conclude this discussion, remember the following:

at least 3 ⇒ 3 or 4 or 5 or 6

The union of two events combines the outcomes in both, written A or B (either can happen). The union happens if either A or B or both occur. The word *or*, *either*… or, *at least* is associated with UNION. It also implies Addition Rule. ⊕ at most

The intersection of two events is the outcomes that are in both at the same time, written A and B (both must happen). The intersection of two events is the event that occur in both. The words *and*, *both*… *and*, *and then* is associated with INTERSECTION. It also implies Multiplication Rule. ⊗

NOW DO PRACTICE EXERCISE 15

Practice Exercise 15

1. The table below shows the sample space of an experiment of rolling a pair of dice.

BLACK DIE \ RED DIE	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1,4)	(1,5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2,4)	(2,5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3,4)	(3,5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4,4)	(4,5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5,4)	(5,5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6,4)	(6,5)	(6, 6)

List down the elements of each of the following events: Complete the given set, one is already given.

(i) A = the sum of the two upper faces is 4
= {(3, 1), ______, ______,……}

(ii) B = the sum is 11
B = {(6, 5), (5,6), ~~……~~}

(iii) C = double (the same number on both die)
C = {(1, 1), ______, ______,……}

(iv) D = sum is greater than or equal to 10
D = {(6, 4), ______, ______,……}

Total 10: (5,5) (6,4) (4,6)
Total 11: (5,6) (6,5)
Total 12: (6,6)

(v) E = sum is at least 10
E = {(6, 4),______, ______,……}

Same

more than 10
Total 11
Total 12

(vi) F = ~~sum is~~ at most 5
F = {(1, 1), ______, ______,……}

Total 2 : (1,1)
Total 3 : (1,2) (2,1)
Total 4 : (1,3) (3,1) (2,2)
Total 5 : (3,2) (2,3) (1,4) (4,1)

2. Using the events given in number 1, find each of the following.

(i) $A \cup B$

(ii) $C \cup D$

(iii) $A \cup F$

(iv) $A \cap B$

(v) $A \cap C$

(vi) $D \cap E$

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3. Find each of the following from question 1.

(i) $P(A \cup B)$

(ii) $P(C \cup D)$

(iii) $P(A \cup F)$

(iv) $P(A \cap B)$

(v) $P(A \cap C)$

(vi) $P(D \cap E)$

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4. Ten cards are numbered 1 to 10. One is drawn at random. Find the probability that the number is…

(i) greater than 7 or less than 3

(ii) greater than 6 or odd

5. A bag contains 2 black and 1 blue balls. A ball is drawn, its color noted and then replaced. A second ball is then drawn. Find the probability of drawing:

(i) a black ball and another black ball.

(ii) a black and then a blue

6. Rework number 5, if the first ball drawn is **not replaced**.

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3