P10-14

September 10, 2018 12:59 PM

P.10

1.2 Exercise Set

1. For each number, find its square root. If there is no square root, write ∅ (empty set).

a) $\sqrt{49}$ ____ b) $\sqrt{81}$ ____

c) $\sqrt{225}$ ____ d) $\sqrt{2500}$ ____

e) $\sqrt{-9}$ → no sol. f) $\sqrt{\frac{4}{25}}$ ____

g) $\sqrt{\frac{121}{9}}$ ____ h) $\sqrt{\frac{225}{49}}$ ____

i) $\sqrt{0.09}$ ____ j) $\sqrt{0.0625}$ ____

2. Simplify, if possible.

a) $\sqrt{25}$ ____ b) $\sqrt{144}$ ____

c) $-\sqrt{\frac{16}{9}}$ ____ d) $\sqrt{0.36}$ ____

e) $-\sqrt{\frac{36}{81}}$ ____ f) $\sqrt{-16}$ ____

g) $\sqrt{1}$ ____ h) $\sqrt{0.0064}$ ____ $\sqrt{625} = 25$

i) $\sqrt{0}$ ____ 2 j) $\sqrt{6.25} = \sqrt{\frac{625}{100}} = \frac{\cancel{25}\,5}{\cancel{10}\,2}$ 5/2

3. Simplify.

a) $\sqrt{9}+\sqrt{16}$ ____ b) $\sqrt{9+16}$ ____

c) $\sqrt{25}-\sqrt{36}$ ____ d) $\sqrt{1}+\sqrt{9}$ ____

e) $-\sqrt{16}-\sqrt{4}$ ____ f) $-\sqrt{25-9}$ ____

g) $-\sqrt{8^2}+\sqrt{15^2}$ ____ h) $-\sqrt{8^2+15^2}$ ____

i) $\sqrt{5^2+12^2}$ ____ j) $\sqrt{5^2}+\sqrt{12^2}$ ____

HW P.10 Q 1-5 (Right)

P 11

4. Determine the length of each side of the square.

(x)
Length = $\sqrt{\text{Area}}$

a) 64 cm²

b) 625 m²

HW P.10 Q1-5 (Right)

P 11

(x)
Length = $\sqrt{Area}$

4. Determine the length of each side of the square.

a) 64 cm²

b) 625 m²

c) 0.0049 mm²

d) 0.01 ft²

5. Determine the rational square root of each number, if it exists. (Example: $\sqrt{9} = 3$)

a) 10 ______ **b)** $\sqrt{100}$ ______

c) 1000 ______ **d)** $\sqrt{36}$ ______

e) 360 ______ **f)** $\sqrt{3600}$ ______

g) 0.04 ______ **h)** $\sqrt{0.004}$ — ______

i) 0.0004 ______ **j)** $\sqrt{0.81}$ ______

k) 0.081 ______ **l)** $\sqrt{0.0081}$ ______

6. A square piece of land has a total area of 2.89 miles². Determine:

a) The length of each side of the piece of land.

b) The perimeter of the piece of land.

7. The amount a child consumes in calories each day is based on the formula $C = 600\sqrt{A}$, where C = number of calories and A = age.

a) Determine the calories needed for a $C = ?$

i) one year old

ii) nine year old

b) Determine the age of a person who consumes

i) 1200 calories per day

ii) 2400 calories per day

P.12

8. A rectangle has a length twice as long as it is wide, and an area of 242 m². Determine the length and width of the rectangle. (Area = length × width)

$l = 2w$

$\boxed{A = 242\,m^2}$ w

$\boxed{\begin{array}{l} W = 11\,m \\ l = 22\,m \end{array}}$

Area = $l \times w$

Area = $(2w) \times w$

$\frac{242}{2} = \frac{2w^2}{2}$

$121 = w^2$

$11 = w$

$3 \times 3 = 3^2$

$W \times W = W^2$

$\sqrt{121} = 11$

9. A rectangle has a length 25% longer than the width, and an area of 80 cm². Determine the length and width of the rectangle.

10. A right triangle made by cutting a diagonal on a square has an area of 242 cm². What is the perimeter of the original square?

11. A triangle is made by cutting a diagonal on a rectangle $33\frac{1}{3}$ % longer than it is wide. If the area of the triangle is 54 cm², determine the length and width of the original triangle.

10. A right triangle made by cutting a diagonal on a square has an area of 242 cm². What is the perimeter of the original square?

11. A triangle is made by cutting a diagonal on a rectangle $33\frac{1}{3}$% longer than it is wide. If the area of the triangle is 54 cm², determine the length and width of the original triangle.

12. The area of a circle is determined by the equation $A = \pi r^2$. What is the radius of a circle that has an area of 49π cm²?

13. A semi-circle has an area of 32π cm². What is the diameter of the semi-circle?

14. The volume of a rectangular solid with a square base is 245 cm³. If the height of the rectangular solid is 5 cm, what is the length of the square base?

15. A rectangular solid has a base twice as long as it is wide, and a height of 8 cm. If the volume of the rectangular solid is 1936 cm³, what is its length and width?

HW P.11 Q 6, 10, 12

P. 13

1.3 Square Roots of Non-Perfect Squares

P.13

As we saw from the previous section, the square roots of **perfect squares** are rational numbers. For example $\sqrt{16} = \sqrt{4 \times 4} = 4$ because $4^2 = 16$. In this section we will look at **non-perfect squares**, which are irrational numbers. For example $\sqrt{5}$ is irrational, because no two equal rational numbers multiply out to 5. We can, however, use a calculator to find a decimal approximation of these values.

Ex1) $\sqrt{27} \approx \boxed{5.2}$

small $\sqrt{25}$ 2 $\sqrt{27}$ 9 $\sqrt{36}$ big

5 5.2 6

$\sqrt{1} = 1$

$\sqrt{4} = 2$

$\sqrt{9} = 3$

$\sqrt{16} = 4$

$\sqrt{25} = 5$

$\sqrt{36} = 6$

$\sqrt{49} = 7$

Approximating Square Roots with a Calculator

Ex2) $-\sqrt{59} \approx \boxed{-7.7}$

$\sqrt{49}$ 10 $\sqrt{59}$ 5 $\sqrt{64}$

7 7. 8

Example 1 Find the decimal approximation of $\sqrt{12}$.

▶ *Solution:* No two rational numbers multiply out to $\sqrt{12}$. So, by calculator, $\sqrt{12} = 3.464...$ which is an irrational number. To two decimal places, $\sqrt{12} \simeq 3.46$.

Try) $\boxed{\sqrt{110} \approx 10.5}$

$\sqrt{100}$ 10 $\sqrt{110}$ 11 $\sqrt{121}$

10 10.5 11

Approximating Square Roots without a Calculator

Not every square root is a whole number. In fact most square roots are non-perfect squares. However we can approximate square roots that are not whole numbers.

Example 2 Find the decimal approximation of $\sqrt{11}$ without a calculator.

Example 2 Find the decimal approximation of $\sqrt{11}$ without a calculator.

► *Solution*: $\sqrt{11}$ is not a whole number. Its value must lie between $\sqrt{9}$ and $\sqrt{16}$, that is $\sqrt{11}$ is between 3 and 4. Since $3^2 = 9$ and $4^2 = 16$, 11 is 2 units from 9 and 5 units from 16. Therefore $\sqrt{11}$ must be closer to 3 than to 4, say 3.3. Check $3.3^2 = 10.89$

Example 3 Find the decimal approximation of $\sqrt{110}$ without a calculator.

► *Solution*: $\sqrt{110}$ is not a whole number. Its value must lie between $\sqrt{100}$ and $\sqrt{121}$, that is $\sqrt{110}$ is between 10 and 11. Since $10^2 = 100$ and $11^2 = 121$, 110 is 10 units from 100 and 11 units from 121. Therefore $\sqrt{110}$ must be very close to 10.5. Check $10.5^2 = 110.25$

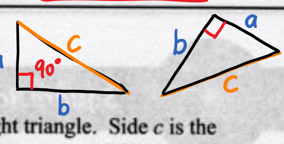
$\boxed{a^2+b^2=c^2}$

P.14

The Pythagorean Theorem: $a^2 + b^2 = c^2$

right-angle Triangle

longest side – "C"



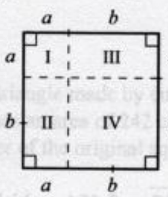
The Pythagorean Theorem describes the relationship between the sides of any right triangle. Side c is the **hypotenuse** (longest side) and sides a and b are the **legs** of the right triangle. There are over 300 ways of proving the Pythagorean Theorem. Below are two proofs.

Proof 1: Consider a square with sides $a + b$.

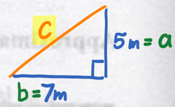
$a^2+b^2=c^2$ 🍎 = \$7

$c^2=a^2+b^2$ \$7 = 🍎

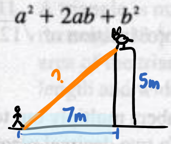
Drawing two inner squares and two inner rectangles:



$$\begin{aligned}\text{Area of big square} &= \text{Area I} + \text{Area II} + \text{Area III} + \text{Area IV}\\ &= a^2 + ab + ab + b^2\\ &= a^2 + 2ab + b^2\end{aligned}$$



Ex1)



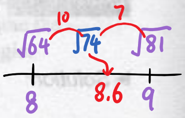
$c^2 = a^2+b^2$

$c^2 = 5^2+7^2$

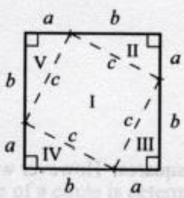
$c^2 = 25+49$

$\sqrt{c^2} = \sqrt{74}$

$c = \sqrt{74} = \boxed{8.6\text{ m}}$



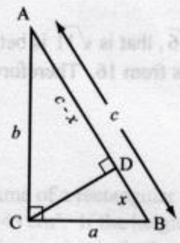
In the same square, draw four right triangles and an inner square:



$$\begin{aligned}\text{Area of big square} &= \text{Area I} + \text{Area II} + \text{Area III} + \text{Area IV} + \text{Area V}\\ &= c^2 + \tfrac{1}{2}ab + \tfrac{1}{2}ab + \tfrac{1}{2}ab + \tfrac{1}{2}ab\\ &= c^2 + 4(\tfrac{1}{2}ab)\\ &= c^2 + 2ab\end{aligned}$$

Since the area of both squares are equal: $a^2 + 2ab + b^2 = c^2 + 2ab \rightarrow a^2 + b^2 = c^2$

Proof 2: Objects of the same shape have sides that are proportional.



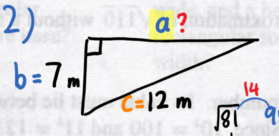
ΔABC, ΔACD and ΔCBD are similar to each other.

So $\dfrac{c}{a} = \dfrac{a}{x} \rightarrow a^2 = cx$

$\dfrac{c}{b} = \dfrac{b}{c - x} \rightarrow b^2 = c(c - x) = c^2 - cx$

Therefore $a^2 + b^2 = cx + c^2 - cx = c^2$

Ex2)



$a^2+b^2=c^2$

$a^2+7^2 = 12^2$

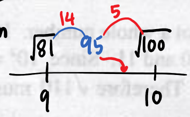
$a^2+49 = 144$

$-49 \quad -49$

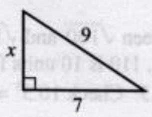
$\sqrt{a^2} = \sqrt{95}$

$a = 9.7\text{ m}$ ✓

9.8 m ✓



Example 4 Find x.



► Solution:

$x^2 + 7^2 = 9^2$

$x^2 = 9^2 - 7^2$

$x^2 = 32$

$x = \sqrt{32} \approx 5.6$