P15-19

September 10, 2018 12:55 PM

P.15

$\sqrt{25}$ $\sqrt{36}$ $\sqrt{49}$ $\sqrt{64}$ $\sqrt{81}$ $\sqrt{100}$

* estimate $\sqrt{x}$ to 2 decimals.

ex) $\sqrt{50}$ $a = \sqrt{49} = 7$

ex) $\sqrt{70}$ $a = 8$

Non-Calculator Methods of Approximating Square Roots

Heron's Method

$$\sqrt{x} \approx \frac{1}{2}\left[a + \frac{x}{a}\right]$$

closest perfect square root.

This method is named after the first-century Greek mathematician Heron of Alexandria who gave the first explicit description of iteratively computing the square root.

Ex1) estimate $\sqrt{34}$ to 2 decimals.

1. Find the closest perfect square a to non-perfect square x.
2. Use formula $\frac{1}{2}\left(a + \frac{x}{a}\right)$.
3. Repeat this formula to improve calculation.

$x = 34$ $a = \sqrt{36} = 6$

$$\sqrt{34} \approx \frac{1}{2}\left[6 + \frac{34}{6}\right]$$

$$= \frac{1}{2}\left[\frac{36}{6} + \frac{34}{6}\right]$$

$$= \frac{1}{2} \cdot \frac{70}{6} = \frac{35}{6}$$

$$\therefore \boxed{\sqrt{34} \approx 5.83}$$

$$\begin{array}{r} 5.83 \\ 6\overline{)35.00} \\ -30 \\ \hline 50 \\ -48 \\ \hline 20 \\ 18 \\ \hline 2 \end{array}$$

Example 5 Find $\sqrt{8}$.

► *Solution*: The closest perfect square to $\sqrt{8}$ is $\sqrt{9} = 3^2$ therefore $a = 3$

$$\frac{1}{2}\left(a + \frac{x}{a}\right) = \frac{1}{2}\left(3 + \frac{8}{3}\right) = 2.83$$

$$\text{Improve: } \frac{1}{2}\left(2.83 + \frac{8}{2.83}\right) = 2.828...$$

Finding Square Roots Using an Algorithm

This method of computing square roots was taught in schools before the invention of calculators. While learning this algorithm may not be necessary in today's world with calculators, working out some examples could be a challenging exercise still.

1. Starting at the decimal point, separate the number into pairs of digits, to the left and right of the decimal point. No pair should straddle a decimal point.
2. Find the largest number whose square is equal to or less than the leading digit pair. Put that number to the left of the square root, and above the first digit pair.
3. Square that number, and subtract this square from the leading digit pair.
4. Bring down the next digit pair, and put it to the right of the difference you just calculated.
5. Multiply the number on the top by two, and put it to the left of the difference you calculated in step 3. Leave an empty place next to it, eg. 4 __.
6. Find the largest single-digit number to put in this blank decimal place such that this combined number times the single-digit number itself will be less than the current difference, eg. 4$\underline{N} \times \underline{N}$. Put this number in the blank space, and in the next decimal place on the top row.
7. Now subtract the product you just found.
8. Repeat for the next digit, starting at step 4.

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Example 6 Find $\sqrt{536}$ using an algorithm.

► *Solution:*

```
  )5 36.00
```

Step 1: Separate the number into pairs of digits, counting from the decimal point.

```
     2
2 )5 36.00
   4
   1
```

Steps 2,3: 2 is the largest number whose square is less than 5. Subtract 2×2 and the difference is 1.

```
     2 .
2 )5 36.00
   4 ↓
4_   1 36
```

Steps 4,5: Bring down the next pair of digits, 36. Double the number from the top row is 4. Place the number to the left of the 136 and leave a space.

```
     2  3.
2 )5 36.00
   4 ↓
43   1 36
     1 29
        7
```

Steps 6,7: 3 is the largest number to fill in the space since $43 \times 3 = 129 < 136$ and $44 \times 4 = 176$. Subtract 129; the difference is 7.

```
     2  3.
2 )5 36.00
   4 ↓  ↓
43   1 36
     1 29
46_     7 00
```

Repeat Steps 4,5: Bring down next pair of digits, 00. Double the number on the top row is 46. Place the number to the left of the difference and leave a space.

```
     2  3. 1
2 )5 36.00
   4 ↓  ↓
43   1 36
     1 29
461     7 00
        4 61
        2 39
```

Repeat Steps 6,7: 1 is the largest number to fill in the space since $461 \times 1 = 461 < 700$ and $462 \times 2 = 924 > 700$. Subtract, and the difference is 239. Repeat.

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1.3 Exercise Set

P.17 HW: Q $\boxed{1-2, 4}$

1. State if the square root of the number is rational or irrational.

$\sqrt{4}, \sqrt{9}$ $\sqrt{2}, \sqrt{0.7}$

a) $\sqrt{121} = 11$ R

b) $\sqrt{60}$ ______

c) $\sqrt{729}$ ______

d) $\sqrt{750}$ ______

e) $\sqrt{\frac{81}{5}}$ $\frac{\sqrt{81}}{\sqrt{5}} = \frac{9}{}$ Irrational

f) $\sqrt{\frac{8}{18}}$ $\frac{4}{9}$ $= \frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3}$ R

g) $\sqrt{1.6} \rightarrow \sqrt{\frac{16}{10}} = \frac{4}{}$ I

h) $\sqrt{\frac{24}{54}}$ ______

i) $\sqrt{0.9}$ ______

j) $\sqrt{0.09}$ ______

2. Between what two integers does the irrational number fall?

a) $\sqrt{75}$ ______

b) $\sqrt{110}$ ______

c) $\sqrt{90}$ $\sqrt{81}$ $\sqrt{100}$ 9 10 — 9 and 10

d) $-\sqrt{300}$ ______

e) $-\sqrt{160}$ ______

f) $-\sqrt{125}$ ______

g) $\sqrt{204}$ $\sqrt{196}$ $\sqrt{225}$ 14 15 — 14 and 15

h) $-\sqrt{470}$ ______

i) $\sqrt{300}$ ______

j) $-\sqrt{500}$ ______

* $\boxed{\text{use table on P.8}}$

3. Determine the closest integer to the irrational number.

a) $\sqrt{5}$ ______

b) $\sqrt{19}$ ______

c) $\sqrt{180}$ ______

d) $\sqrt{450}$ ______

e) $\sqrt{590}$ ______

f) $-\sqrt{5}$ ______

g) $-\sqrt{19}$ ______

h) $-\sqrt{180}$ ______

i) $-\sqrt{450}$ ______

j) $-\sqrt{590}$ ______

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P.18 Q4

4. Without using a calculator, determine the square roots of the irrational numbers to one decimal place.

a) $\sqrt{75}$ ______ b) $\sqrt{110}$ ______

c) $\sqrt{90}$ ______ d) $-\sqrt{300}$ ______

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e) $-\sqrt{160}$ -12.6 ✓ -12.7 ✓ f) $-\sqrt{125}$ ______

$\sqrt{144}$ $\sqrt{160}$ $\sqrt{169}$ 16 9 12 13

g) $\sqrt{204}$ ______ h) $-\sqrt{470}$ ______

i) $\sqrt{300}$ ______ j) $-\sqrt{500}$ ______

P.18

$$\sqrt{x} \approx \frac{1}{2}\cdot\left[a + \frac{x}{a}\right]$$

5. Without using a calculator determine the square roots of the irrational numbers to two decimal places. ~~(Hint: Start with your answers to question 4)~~, then apply Heron's Method)

P.18

a) $\sqrt{75}$ b) $\sqrt{110}$ 10.50

d, $-\sqrt{300}$ $a = 17$ $x = 300$

$$\sqrt{300} \approx \frac{1}{2}\left[\frac{17}{1}\cdot\frac{\times 17}{\times 17} + \frac{300}{17}\right]$$

$$= \frac{1}{2}\left[\frac{289 + 300}{17}\right]$$

$$= \frac{1}{2} \times \frac{589}{17} = \frac{589}{34}$$

$$34\overline{)589.00} = 17.32$$

34
249
- 238
110
102
80
68

c) $\sqrt{90}$ ______ d) $-\sqrt{300}$ $\boxed{-17.32}$

$\sqrt{289} = 17$

e) $-\sqrt{160}$ ______ f) $-\sqrt{125}$ ______

HW Heron's Method
P.18) 5bf, 6a–d
Triangle

g) $\sqrt{204}$ ______ h) $-\sqrt{470}$ ______

i) $\sqrt{300}$ ______ j) $-\sqrt{500}$ ______

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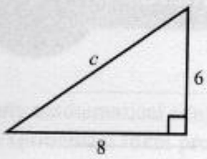
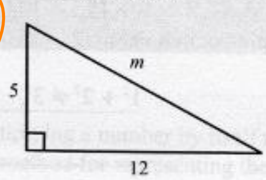
6. Using the Pythagorean Theorem, solve to one decimal place.

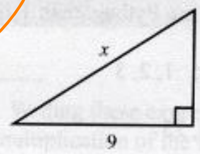
a)



b)

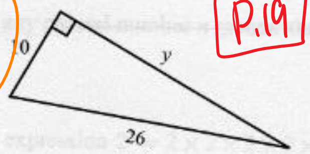


c)



$c =$ ______ $m =$ ______ $x =$ ______

d)



P.19 Try

e)

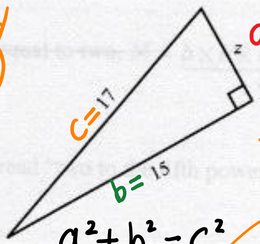


Table P.8

$a^2+b^2=c^2$

$a^2+15^2=17^2$

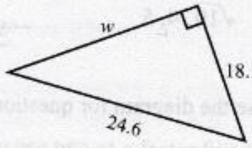
$a^2+225=289$

$-225 \quad -225$

$a^2=64$

$\boxed{a=8}$

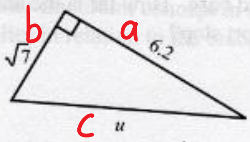
f)



$y =$ ______ $a/z =$ ______ $w =$ ______

P.19

g)



$c^2=a^2+b^2$

$u^2=6.2^2+\sqrt{7}^2$

$u^2=38.44+7$

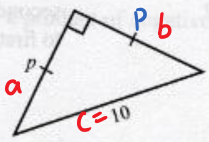
$u^2=45.44$ $\quad u=\sqrt{45.44}$

$u = \boxed{6.7}$

$\sqrt{36} \quad 9 \quad \sqrt{45} \quad 4 \quad \sqrt{49}$

$6 \quad 6.7 \quad 7$

h)



$a^2+b^2=c^2$

$p^2+p^2=10^2$

$\frac{2p^2}{2}=\frac{100}{2}$

$p^2=50$

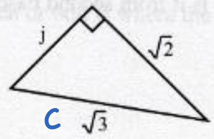
$\sqrt{p^2}=\sqrt{50}$ $\quad p=\sqrt{50}$

$p =$

$\sqrt{49} \quad 1 \quad \sqrt{50} \quad 14 \quad \sqrt{64}$

$7 \quad \boxed{7.1} \approx p \quad 8$

i)



$j =$

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