

Probability Review Key

April 5, 2019 8:39 AM

FOM 12 Chapter 5 Review Package

Formulas for this chapter

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \text{ if there is overlap}$$

$$\text{Probability of A given B} \quad P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

- A packet of 20 candies has 7 that are caramel. If a candy is chosen at random, determine
 - the probability that a caramel candy is chosen
 - the probability that a caramel candy is not chosen,

a) $\frac{7}{20}$ b) $\frac{13}{20}$
- Two dice are rolled. Determine the probability of the following.
 - The sum of the two dice is eight or ten. (make a table to help)

Eight: $\begin{matrix} (2-6) \\ (3-5) \\ (4-4) \end{matrix} \times 2$ } 5 ways } $\frac{8}{36}$

Ten: $\begin{matrix} (4-6) \\ (5-5) \end{matrix} \times 2$ } 3 ways }
 - The sum of the two dice is even or greater than nine

c) $P(\text{not } 7) = \text{Total} - P(7) = \frac{30}{36}$

count in sample space: $\frac{20}{36}$
- The months of the year are placed on a card and then the cards are well shuffled. What is the probability that a card drawn has
 - A month that starts with "J" $\frac{3}{12} = \frac{1}{4}$ Jan July June.
 - A month that end in "y" $\frac{4}{12} = \frac{1}{3}$ Jan Feb May July

Probability and Odds

- A card is randomly drawn from a deck of 52 cards.
 - What is the probability of drawing a face card? $P(\text{Face}) = \frac{12 \text{ Face}}{52} = \frac{3}{13}$
 - What are the Odds in favour of drawing a face card?
 - What are the Odds against drawing an ace?

b) Face : Non Face 12 : 40 \Rightarrow $\frac{3}{10}$

c) Not Ace : Ace 48 : 4 \Rightarrow $\frac{12}{1}$
- Youjin bought a box of a dozen doughnuts that contained five chocolate doughnuts, three cinnamon doughnuts, three jelly doughnuts, and one sugar doughnut. She randomly selects a doughnut from the box.
 - What are the odds in favour of her selecting a chocolate doughnut? $\frac{5}{7}$ (cho : not cho)
 - What are the odds against her selecting a jelly doughnut? $\frac{9}{3}$ (Not Jelly : Jelly)
 - What is the probability of her selecting a cinnamon doughnut or a sugar doughnut?

c) $\frac{4}{12} = \frac{1}{3}$
- A weather forecaster says that the odds against rain tomorrow are 7 : 13. What is the probability of rain tomorrow?

 $P(\text{Rain}) = \frac{13 \text{ (Rain)}}{7+13 \text{ (Total)}} = \frac{13}{20}$

No Rain | Rain
- A high school football coach is trying to decide which quarterback he should start in next week's game. He examines the win: loss ratio for the two quarterbacks. Based on this information, which quarterback should he start? Explain your answer.

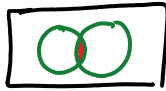
Player	Wins	Losses
Matthew	8	5
Azam	7	4

Winning %

$$\frac{8}{13} = 0.62 \approx 62\%$$

$$\frac{7}{11} = 0.64 \approx 64\% \rightarrow \text{starts Azam}$$

Ch. 5 Prob Test on April 15th Monday.



Mutually Exclusive Events P(A or B)

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

8. If $P(A) = 1/2$, $P(B) = 1/5$, and $P(A \text{ and } B) = 0$, find $P(A \text{ or } B)$.

$$P(A \text{ or } B) = P(A) + P(B) - \emptyset = \frac{1}{2} + \frac{1}{5} = \frac{5}{10} + \frac{2}{10} = \frac{7}{10}$$

$$P(A \text{ and } B) = 0.7$$

9. If $P(A) = 0.3$, $P(B) = 0.8$, and $P(A \text{ or } B) = 0.4$, find $P(A \text{ and } B)$.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \quad 0.4 = 0.3 + 0.8 - P(A \text{ and } B)$$

10. A card is drawn from a standard deck Determine the probability that

(a) a face card or a club is drawn

(b) a five of diamond or a Queen is drawn

(c) a black card or diamond is drawn

(d) a Jack or a red card is drawn

a) $\frac{12}{52} + \frac{13}{52} - \frac{3}{52} = \frac{22}{52}$

b) $\frac{1}{52} + \frac{4}{52} = \frac{5}{52}$

c) $\frac{26}{52} + \frac{13}{52} = \frac{39}{52} = \frac{3}{4}$

d) $\frac{4}{52} + \frac{26}{52} - \frac{2}{52} = \frac{28}{52} = \frac{7}{13}$ *over count!!*

Conditional Probability

11. Given: $P(A \text{ and } B) = 0.4$ and $P(B) = 0.65$, find $P(A|B) = \frac{0.4}{0.65} = 0.62$

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

12. Let A and B be events such that $P(A) = 0.6$, $P(B) = 0.4$, and $P(A \text{ and } B) = 0.3$

a. Find $P(A \text{ or } B)$

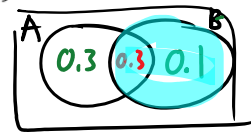
$$P(A \text{ or } B) = 0.3 + 0.3 + 0.1 = 0.7$$

b. Find $P(A|B)$

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{0.3}{0.4} = \frac{3}{4}$$

c. $P(B|A)$

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{0.3}{0.6} = 0.5$$



13. The student population of a high school this year is given in the table below.

	Girls	Boys	Total
Senior 2	250	350	600
Senior 3	250	200	450
Senior 4	215	210	425
Total	715	760	1475

If a student is selected at random from the school, find

(a) $P(\text{boy}) = \frac{760}{1475} = 52\%$

(b) $P(\text{Senior 4 student}) = \frac{425}{1475}$

(c) $P(\text{girl} | \text{Senior 3}) = \frac{250}{450}$

(d) $P(\text{senior 3} | \text{girl}) = \frac{250}{715}$

Tree Diagrams to Solve Probability

14. Use the given tree diagram to determine:

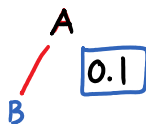
(a) $P(B)$

$$0.4(0.1) + 0.6(0.1) = 0.1$$

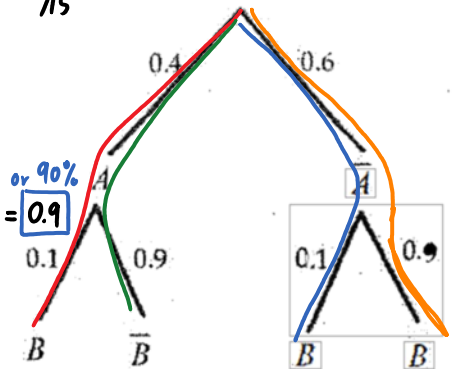
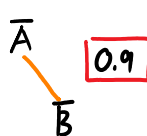
(b) $P(\bar{B})$

$$0.4(0.9) + 0.6(0.9) = 0.9$$

(c) $P(B|A)$



(d) $P(\bar{B}|\bar{A})$



(below)

15. There are two bus routes between Luke's home and downtown where he works. Luke uses route A two-thirds of the time, and route B one-third of the time. On route A, 10% of the buses are late, and on route B, 20% of the buses are late.
- Draw a tree diagram to represent the information.
 - What is the probability the Luke is late for work on a particular day?
 - Given that Luke arrives late, what is the probability he travelled by route A?

(below)

16. Bottle A has 4 red, 5 white and 6 black balls. Bottle B has 5 red, 6 white and 7 black balls. A coin is flipped to determine which bottle a ball is to be selected from.
- Represent the information on a tree diagram.
 - What is the probability of selecting a black ball?
 - If a black ball is selected, what is the probability that it came from bottle A?

Classifying Events

17. The experiment is to study the athletic activities and the level of fitness of a person. The first event is that the person plays several different sports on a regular basis. The second event is that the person enjoys a high level of fitness. Independent or dependent?
18. The experiment is to flip a coin and to roll a six-sided die. The first event is that a tail is flipped. The second even is that a one is rolled. Independent or dependent?
19. The experiment is rolling a die. The first event is that the number is less than four. The second event is that the number is odd. Mutually exclusive or not mutually exclusive?
- You can have odd and less than 4
20. The experiment is cutting a deck of cards. The first event is that the card is red. The second event is that the card is a black face card. Mutually exclusive or not mutually exclusive?

Independent/Dependent Events P(A and B)

21. When rolling a six-sided die and cutting a deck of cards, what is the probability of obtaining a six and a heart?
- $$P(\text{Six and Heart}) = P(\text{Six}) \cdot P(\text{Heart}) = \frac{1}{6} \cdot \frac{13}{52} = \frac{1}{24}$$
22. When tossing a coin and cutting a deck of cards, what is the probability of obtaining a head and a queen?
- $$P(\text{Head and Queen}) = \frac{1}{2} \cdot \frac{4}{52} = \frac{1}{26}$$
23. The probabilities that Emma will pass grade 12 math and Grade 12 physics this semester are 0.85 and 0.75. If these events are independent, find the probability that she will pass
- pass math 0.85 Not pass math 0.15 pass phys 0.75 not pass phys 0.25
- both math and physics
 $0.85 \times 0.75 = 0.64$
 - math but not physics
 $0.85 (0.25) = 0.21$
 - physics but not math
 $0.75 (0.15) = 0.11$
 - neither math nor physics
 $0.15 (0.25) = 0.038$

24. The letters of the alphabet are placed on tiles and put in a bag. A tile is chosen, replaced, and then another tile is chosen, determine the probability of

(a) A vowel then another vowel $\frac{5}{26} \cdot \frac{5}{26} = \frac{25}{676}$

(b) A consonant and then another consonant $\frac{21}{26} \cdot \frac{21}{26} = \frac{441}{676}$

25. Two cards are drawn without replacement from a standard deck of 52 cards. Determine the probability of the following events

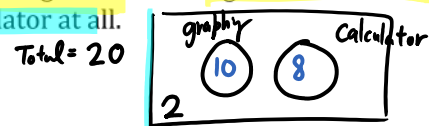
(a) club and then a diamond $\frac{13}{52} \cdot \frac{13}{51} = 6.4\%$ (b) red card and then a black card $\frac{26}{52} \cdot \frac{26}{51} = \frac{13}{51} = 25\%$

(c) red ace and then another red ace $\frac{2}{52} \cdot \frac{1}{51} = \frac{1}{1326}$ (d) 2 of clubs and then 2 of clubs = \emptyset
 $\frac{1}{52} \cdot \frac{0}{51}$

Venn Diagrams to Solve Probability

26. In a math class of 20 students, ten students have graphing calculators, eight students have scientific calculators, and two students have no calculator at all.

(a) Represent the information using a Venn diagram.



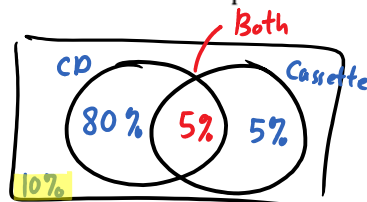
If a student is selected at random from the class, what is the estimated probability that:

(b) the student has a calculator? $P(GC \text{ or } SC) = \frac{10}{20} + \frac{8}{20} = \frac{18}{20} = \frac{9}{10} = 90\%$

(c) the student has a graphing calculator, given that the student has a calculator. $P(GC | C) = \frac{10}{18} = \frac{5}{9} = 56\%$

27. At a second-hand music store, researchers found that the probabilities that a teenage buys a particular item are as follows:

CD	85%
Cassette	10%
Both	5%

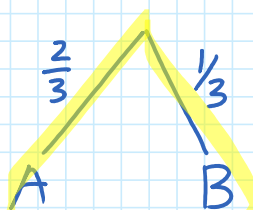


$100\% - 80\% - 5\% - 5\%$

What is the probability that a teenager, chosen at random, does not buy a CD or a cassette?

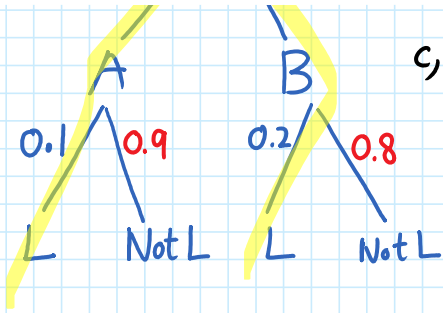
$P(\text{neither}) = 10\%$

15



b) $P(\text{Late}) = \frac{2}{3}(0.1) + \frac{1}{3}(0.2) = \frac{2}{15}$

c) $P(A | \text{Late}) = \frac{P(A \text{ and Late})}{P(\text{Late})} = \frac{\frac{2}{3}(0.1)}{\frac{2}{15}} = \frac{1}{2}$



$$\text{c, } P(A | \text{Late}) = \frac{P(A \text{ and Late})}{\underbrace{P(\text{Late})}_{\text{part b}}} = \frac{\frac{2}{3}(0.1)}{\frac{2}{15}} = \boxed{\frac{1}{2}}$$