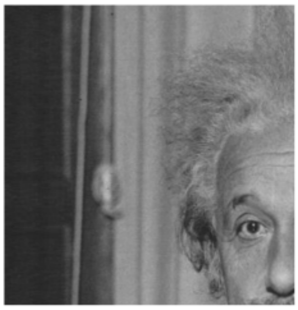
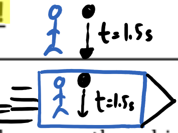
Unit 5 Special Relativity Day 1
Frame of Reference and the Postulates

Albert Einstein (1879-1955)

- Born in Germany
- After graduating from Swiss Polytechnic School in 1901, he went to work in Swiss Patent Office. In his *spare* time, continued to work in independent studies in theoretical physics
- 1905 (at 26) earned his Ph. D in Physics and published 4 papers that revolutionized Physics including "Special Theory of relativity"



All motions are relative!!



What is my present speed?

a) 0 m/s ✓ relative to the room.

b) 465 m/s ✓ ← speed of Earth rotation.

b) 30,000 m/s ✓ Earth's speed around the sun!!

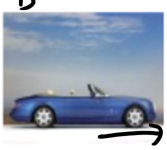
You are flying on a plane…

- If you jump up will you fly across the cabin?
- If you throw a tennis ball to the front of the cabin and then to the back will there be a difference in how fast it flies?

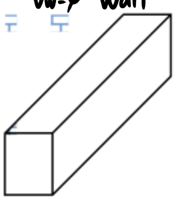
A

$V_A = 24 m/s$

B



$V_B = 10 m/s$



How fast is Car A going relative to brick wall?

$+24 m/s$

How fast is Car A going relative to Car B?

$V_A - V_B = 24 - 10 = +14 m/s$

How fast is Car B going relative to brick wall?

$+10 m/s$

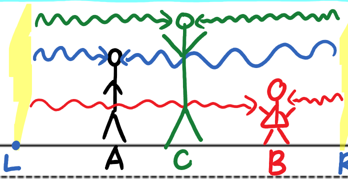
How fast is Car B going relative to Car A?

$V_B - V_A = 10 - 24 = -14 m/s.$

General Rule for Relative Velocity: $V_{A \text{ relative to } B} = \vec{V}_A - \vec{V}_B$

Let's put together the **first POSTULATE of Special Relativity….**

- The measurement of position and velocity depends on the motion of the source and the observer! (Frame of Reference!!)
- There is no preferred frame of reference. All Frames of reference are equally valid
- What occurs in one reference frame may occur in a different order in another reference frame!!!
- **Simultaneous** in one reference frame is not always simultaneous in another reference frame.



obs A : L before R

obs B : R before L

obs C : R and L simultaneous

Inertial Frame of Reference is a reference frame in which Newton 1st Law is valid.

↳ a=∅ at rest or constant speed.

2 postulates (assumptions) of Special Relativity:

1. All the laws of ~~nature~~ Physics are the same in all Inertial Frame of Reference.
2. Speed of light is a constant, independent of the motion of the light source or the observer.

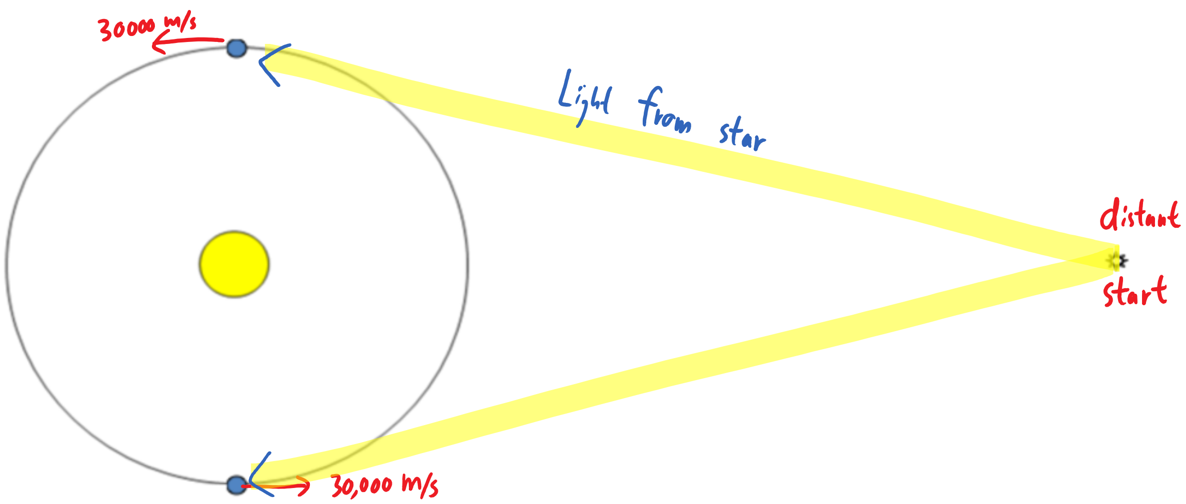
Speed of light "c" = 299,792,458 m/s = $\boxed{3\times10^{8}\ m/s}$ //

1m = dist that light can travel in $\frac{1}{299\,792\,458}$ sec

Michelson-Morley Experiment 1887 (the search for Ether)

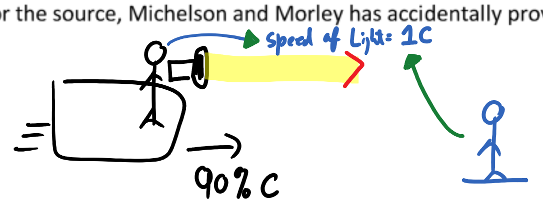
What? They attempted to measure the difference in the speed of light coming from a distant star!

Why? At the end of the nineteenth century, scientists found that light behaves as a wave. Waves need a medium to travel in, what about light? It was theorized that outer space must be filled with a special transparent medium then called Ether. Scientists then set out to determine the speed of the Earth relative to this absolute frame of reference (speed of the Earth relative to the ether it was moving through).



Result? They couldn't detect any difference in the speed of light coming from the distant star!!
They thought they had completely failed!!............

Wait Maybe there is no Ether and speed of light is Constant. regardless of the speed of the observer or the source, Michelson and Morley has accidentally proved Einstein's 2nd Postulate

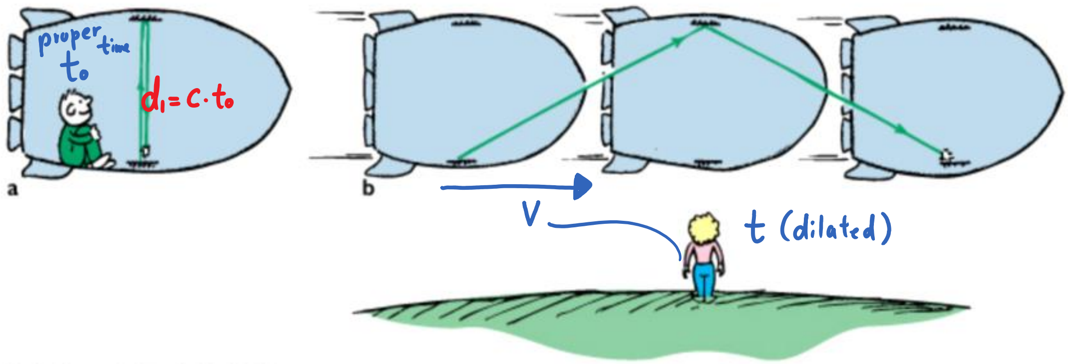


Next Class: Circular Motion Test !!!

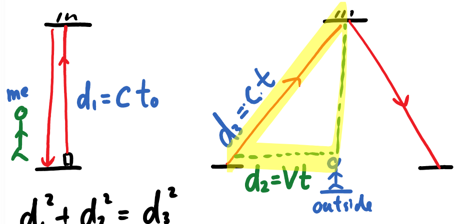
Unit 5 Special Relativity Day 2

Consequence of Special Relative: (1) Time Dilation - moving clock run slowly!!!

Time Dilation - the phenomenon whereby an observer finds that another's clock which is physically identical to their own is ticking at a __slower__ rate as measured by their own clock (assuming motion is uniform, and **a** = 0).



You have a glass rocket traveling at a constant speed v relative to a stationary observer on Earth. Inside the rocket are a laser and a light detector. There is an observer inside the rocket who times how long it takes for the light to hit the detector. Likewise there is another observer on Earth who times the same event.



$$d_1^2 + d_2^2 = d_3^2$$

$$c^2 t_0^2 + V^2 t^2 = c^2 t^2$$

$$c^2 t_0^2 = c^2 t^2 - V^2 t^2$$

$$c^2 t_0^2 = t^2 (c^2 - V^2)$$

$$t^2 = \frac{c^2 t_0^2 / c^2}{[c^2 - V^2]/c^2}$$

$$t^2 = \frac{t_0^2}{1 - \frac{V^2}{c^2}}$$

Time dilation Eq.

$$\boxed{t = \frac{t_0}{\sqrt{1 - \frac{V^2}{c^2}}}}$$

gamma

$$\gamma = \frac{1}{\sqrt{1 - \frac{V^2}{c^2}}}$$

$$\boxed{t = \gamma t_0}$$

Time Dilation Formula

$$t = \frac{t_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

t : **Dilated time** interval measured by an observer who is in motion with respect to the events

t_o : **proper time** interval between two events, as measured by an observer who is at rest with respect to the events.

v : relative speed between the two observers

c : speed of light in a vacuum (m/s)

Remember these effects only become noticeable near the speed of light!

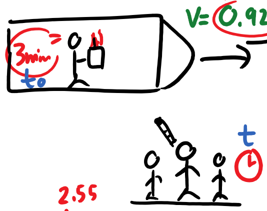
Imagine that in the distant future being able to ride on a spaceship that can travel close to the speed of light.

You embark on a 2.0 year journey to the distant stars travelling at 0.99999 C. Upon your return everyone that has been left behind has aged by 50 – 60 years!. You are now "younger" that all your friends and your parents

Ex) The spacecraft is moving past the earth at a constant speed that is 0.92 times the speed of light. The astronaut, Mr. Cheung, is preparing his lunch by adding water to his cup noodle and waiting for 3mins. How long does it take the noodle to cook according to an observer on earth who is looking through a telescope?

proper time t_0

$V = 0.92c$ $\quad C = 3\times10^8$ m/s.



$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\gamma = \frac{1}{\sqrt{1-\frac{0.92^2c^2}{c^2}}}$$

$$\gamma = \frac{1}{\sqrt{1-0.92^2}} = 2.55$$

$t = \gamma t_0$ (2.55, 3mins)

$\boxed{t = 7.65 \text{ mins}}$

Ex) Alpha Centauri, a nearby star in our galaxy is 4.5 light-years away. If a rocket leaves for Alpha Centauri at a speed of 0.95 c relative to the earth, by how much will the passengers have aged, according to their own clock, when they reach their destination? (1.4 years)

$d = 4.5\text{ ly} \times (3\times10^8 \text{ m/s} \cdot 60\cdot60\cdot24\cdot365)$

$d = 4.5\text{ ly} = 4.5 \times c\cdot \text{yr}$

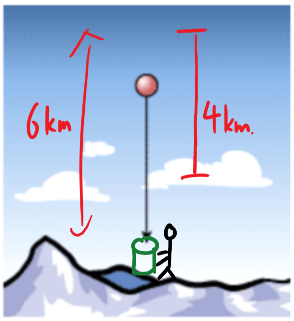
$$t_{earth} = \frac{d}{v} = \frac{4.5\cdot \cancel{c}\cdot \text{yr}}{0.95\cancel{c}} = \boxed{4.74 \text{ years}}$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{0.95^2c^2}{c^2}}} = \frac{1}{\sqrt{1-0.95^2}} = 3.$$

$t = \gamma t_0 \qquad 4.74\text{ yr} = 3.2\,(t_0)$

$\boxed{t_0 = 1.5\text{ yr}}$

Evidence of Time Dilation – Decay of Muons



Elementary particles called muons (heavy electrons) are known to be produced in the upper atmosphere (6-8km above the Earth's surface) during collisions of cosmic rays with nuclei of atoms in air molecules. They have a short life-time (2×10^{-6}s), but travel with a very high speed of 0.988c.

$2\,\mu C \qquad \frac{2}{1000000}$ sec

a) How far can a muon travel on average? Can they reach ground level?

$d = Vt = 0.988 \times (3\times10^8 \text{ m/s}) \cdot (2\times10^{-6}\text{ sec}) = 593\text{ m}.$

b) Yet they are detected here on Earth in relative abundance!!!!! Lets fix our calculation!!

$\boxed{t_0 = 2\times10^{-6}\text{ sec}}$ (measured in a Lab when the muon is at rest) $V=0$

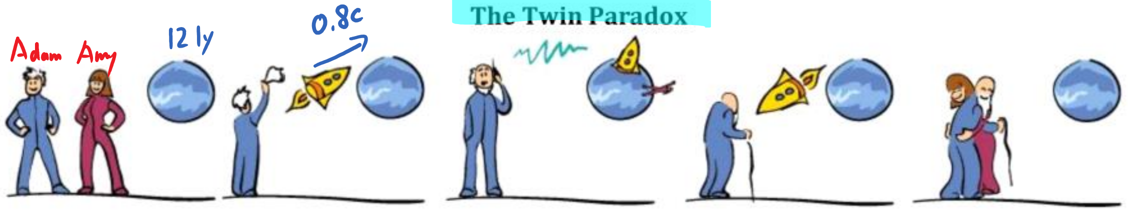
when the muon is going at 98.8 % c , its clock runs slower !!!

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{0.988^2\cancel{c^2}}{\cancel{c^2}}}} = \frac{1}{\sqrt{1-0.988^2}} = 6.47$$

$t = \gamma t_0 = 6.47\,(2\times10^{-6}) = 1.3\times10^{-5}\text{ sec}$

$d = V\cdot t = 0.988\,(3\times10^8 \text{ m/s}) \times 1.3\times10^{-5}\text{ sec} \approx 4\text{ km}.$

The Twin Paradox



At age 20, one of a set of twins leaves on a journey to a star system 12 light years away, travelling at v = 0.80c. The round-trip requires 30 yeas of earth time, how long did the journey take for the travelling twin?

(Adam ⇒ t (dilated))

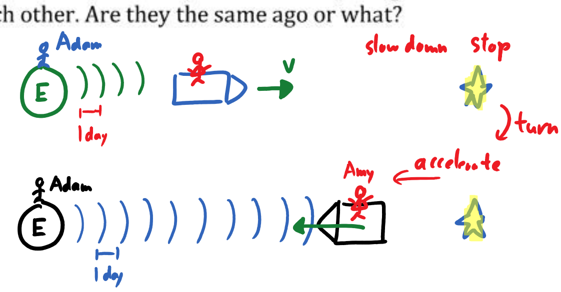
$$\gamma = \frac{1}{\sqrt{1-0.8^2}} = 1.67 \qquad t = \gamma t_0$$

$$30\text{ yr} = 1.67\, t_0$$

$$t_0 = 18\text{ yr.}$$

	left.	Return.
Adam	20 +30	50
Amy	20+18	38.

However, in the traveler's reference frame, it is Earth that is moving, so Earth clocks run slow. Each twin measures the clock of the other as running slow. Hence, they both think the other one is aging slower. When the space ship returns, what are the age of each twin? They can't both be younger that each other. Are they the same ago or what?



The resolution of this paradox is that the Einstein's theory of Time Dilation only works between constant speed / at rest reference frame. As soon as the space ship accelerates, and it must do that to reverse its velocity and return to Earth, the frame of reference of the space traveler is no longer inertial. So, even though the Earth twin may be aging slowly according to the traveler when the ship is coasting, he will age very rapidly at other times.

The bottom line here is that the space-traveler really does return to Earth at biological age 38 to greet the now 50-year-old twin he left behind.

$V < C$ (time dilation)

$V = C$: $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{c^2}{c^2}}} = \frac{1}{\sqrt{1-1}} = \frac{1}{\sqrt{0}} = \infty$

so your time is slower by ∞ ??!! ⇒ your time has stopped !!!!!

$V > C$: $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ greater than C $= \frac{1}{\sqrt{1-\#}} = \frac{1}{\sqrt{-\#}}$ = undefine !!!

bigger than 1

$$\boxed{\text{WS \#2}}$$

Special Relativity: **Worksheet 2** **Time Dilation**

1. An astronaut is circling the Earth in the ISS at a high speed. He measures the time it takes for a baseball to be thrown up, and fall back down in his Earth gravity simulated environment. On Earth, a NASA engineer observes this event through a telescope, and measures the time for this event as well. Who measures the proper time of this event correctly? Explain your answer.

2. A bird flies 15m/s E relative to the Earth against a 5m/s W wind. A river flows at a velocity of 8m/s E relative to a stationary observer watching this event on the shore. What is the speed of the bird relative to the water? (2m/s E)

3. An observer in a fixed frame of reference is watching an event in a spaceship moving with a velocity of 0.866c. If the observer in the moving frame measures the event to take a time of 5.0s to occur, what amount of time will the "fixed" observer measure? (10.s)

4. An observer watching a spaceship moving at 0.33c observes an event to take 2.7s. What is the proper time of the event as viewed by someone on the spaceship? (2.5s)

5. A law enforcement officer in an intergalactic police car turns on a flashing light and sees it generate a flash every 1.5s. A person on the Earth measures the time between flashes as 2.5s. How fast is the police car moving relative to the Earth? (0.80c)

6. Ronin the astronaut's wife gives birth to a child the day he leaves for a 7.00 year long space mission. Assuming negligible acceleration, how fast will Ronin have to travel so that he is the same age as his child when he returns from space, given that his current age is 38.0? (hints what is Ronin and his son's ago when he come back?) (0.988c)

Ronin 38 $\xrightarrow{7yr}$ 45 $t_0 = 7yr$

kid 0 $\longrightarrow$ 45 $t = 45yr.$

$t = \gamma t_0$ $\gamma = \frac{t}{t_0} = \frac{45}{7} = 6.42857....$

7. A beam of muons have a proper lifetime of 2.2×10^{-6}s. If they are measured to move with a speed of 0.99c, find how far they would travel before decaying assuming they undergo time dilation. (4.7×10^3m)

8. Your friend, a test pilot for the NASA, says that he is younger now by a whole second because of the flight he took on a top-secret aircraft. If he tells you that the flight lasted 5 hours how fast must the plane be? (0.0105c or $3.162 \times 10^6 m/s$)

9. Look at the equation for time dilation.
   a) Do you believe based on the equation that it is possible to travel **faster than** the speed of light? Why or why not
   b) Do you believe based on the equation that it is possible to travel **at the** speed of light? Why or why not?
   c) Is there such a speed that you can plug into the equation to make time run backwards? What does this tell you about some properties of time?

8 pilot : 5 hr $= t_0 =$ 18000 sec

You : 5 hr 1 sec. $= t =$ 18001 sec

$t = \gamma t_0$ $\gamma = \frac{18001}{18000} = 1.000056....$

$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

$0.999... = 1 - \frac{v^2}{c^2}$

$\sqrt{\frac{v^2}{c^2}} = \sqrt{1.1 \times 10^{-4}}$

$\frac{v}{c} = 0.0105$ $V = 0.0105\,c$

$= 1\%\ c$

$= \boxed{3.16 \times 10^6 m/s}$

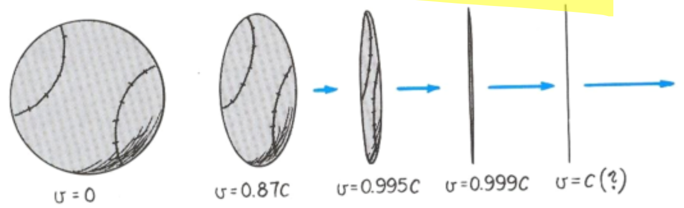
Unit 5 Special Relativity Day 3

Consequence of Special Relative: (2) Length Contraction - moving objects are shorter!!!

Length Contraction – Moving object contract in the direction of motion.

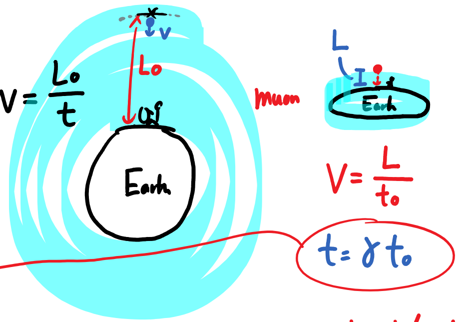
Length Contraction – Moving object contract in the direction of motion.

This is how a high speed baseball will look to a stationary observer as it reaches different speeds.



The reason that it takes so little time for high-speed space ships to travel across space is because space has been ________ contracted. **(shorten)** in the direction of motion.

Moving at high speeds actually shortens the distance that has to be travelled!



Earth $V = \frac{L_o}{t}$

$V = \frac{L}{t_o}$

$\frac{L_o}{t} = \frac{L}{t_o}$

$t = \gamma t_o$

$\frac{L_o}{\gamma \cancel{t_o}} = \frac{L}{\cancel{t_o}}$

$L = \frac{L_o}{\gamma} = \boxed{L_o \sqrt{1 - \frac{v^2}{c^2}}}$

Contracted

proper length / rest length

Length Contraction Formula

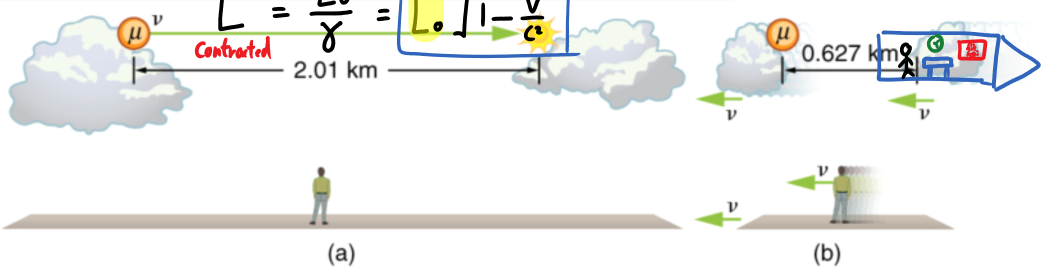
$$L = L_o\sqrt{1 - \frac{v^2}{c^2}}$$

L : **Length of object** as measured when moving

L_o : **Proper Length** of object measured by observer who is at rest with the object

v : relative speed between the two observers

c : speed of light in a vacuum (m/s)



(a) The Earth-bound observer sees the muon travel 2.01 km between clouds. (b) The muon sees itself travel the same path, but only a distance of 0.627 km. The Earth, air, and clouds are moving relative to the muon in its frame, and all appear to have smaller lengths along the direction of travel

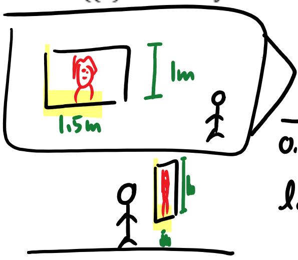
Ex) A particle is traveling through the Earth's atmosphere at a speed of $0.750c$. To an Earth-bound observer, the distance it travels is 2.50 km. How far does the particle travel in the particle's frame of reference?

rest length. L_o

$$\gamma = \frac{1}{\sqrt{1-0.75^2}} = 1.51$$

$$L = L_o/\gamma = \frac{2.5\text{ km}}{1.51} = \boxed{1.65\text{ km}}$$

Ex) A rectangular painting measures 1.00 m tall and 1.50 m wide. It is hung on the side wall of a spaceship which is moving past the Earth at a speed of 0.90 c. (a) What are the dimensions of the picture according to the captain of the spaceship? (b) What are the dimensions as seen by an observer on the Earth? ((a) 1.00 m by 1.50 m, (b) 1.00 m by 0.65 m)



b) $\gamma = \frac{1}{\sqrt{1-0.9^2}}$

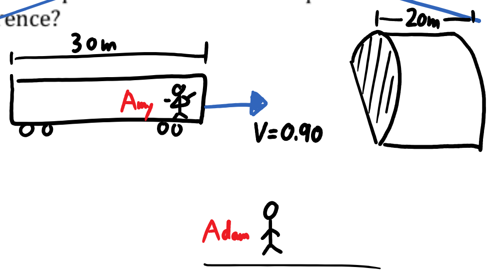
$\gamma = 2.3$

length $L = \frac{L_o}{\gamma} = \frac{1.5\text{ m}}{2.3}$

$L = 0.654$ m

Ex) How fast would a 6.0 m-long sports car have to be going past you in order for it to appear only 5.5 m long?

~~Ex) A particle is traveling through the Earth's atmosphere at a speed of 0.750c. To an Earth-bound observer, the distance it travels is 2.50 km. How far does the particle travel in the particle's frame of reference?~~



Summary

- All observers agree upon relative speed.
- Distance depends on an observer's motion. Proper length L_o is the distance between two points measured by an observer who is at rest relative to both of the points. Earth-bound observers measure proper length when measuring the distance between two points that are stationary relative to the Earth.
- Length contraction L is the shortening of the measured length of an object moving relative to the observer's frame:

Unit 5 Special Relativity Day 4

Consequence of Special Relative: (4) Mass Increase

Energy and mass are related to each other. "Two sides of the same coin".

ENERGY can be transformed into MASS and MASS can be transformed into ENERGY!

E_k

For 200 years scientists were wondering how our sun could keep "burning" without running out of fuel!

The answer lies in the conversion of hydrogen into helium and the loss of a little mass each time.

+H, +H, fusion, He

$m_H \times 2$ = initial mass

less than

m_{He} = final mass + Energy

Missing mass: $E=mc^2$

The sun is actually losing 4 million tons of mass every second!

Don't worry ... the sun has a total mass of 2 x 10^{30} kg ... which means it will last for about another 5 Billion year !!

Bomb, Fission

$t = \gamma t_0$

$L = \frac{L_0}{\gamma}$

Mass Increase Formula

$$m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}$$

$m = \gamma m_0$

E_k 0.99c, 0.9900001c

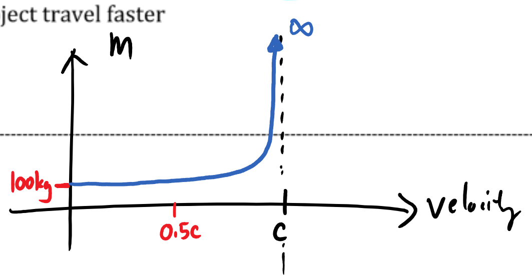
m :	**mass of object** as measured when moving
m_o :	**Proper mass (rest)** of object measured by observer who is at rest with the object
v :	relative speed between the two observers
c :	speed of light in a vacuum (m/s)

Back to relativity... How do make something move?

In order to make an object move faster you need to apply more Kinetic Energy.

At **low speeds** most of the energy will go into increasing the object's velocity. However some of the energy that the object receives will be converted to mass. The object will become little more massive.

At **high speeds** most of the energy will go into increasing the object's mass and very little will go into making the object travel faster



As the object becomes more massive, it becomes harder to move even faster. Giving it even more energy will only move it a little faster, but will make it even more massive.

In order to make an object travel at the speed of light you would need infinite amount of energy!!

Therefore no object that has mass can ever travel at the speed of light!

Ex) Calculate the mass of an electron when it has a speed of (a) $\boxed{4.00 \times 10^7 \text{ m/s}}$ in the CRT of a television set, and (b) $\boxed{0.98\ c}$ in an accelerator used for cancer therapy. The rest mass of an electron is 9.11×10^{-31} kg.

a) $\gamma = \dfrac{1}{\sqrt{1 - \dfrac{(4\times10^7)^2}{(3\times10^8)^2}}} = 1.009$ $m = \gamma m_0$

$m = 9.18 \times 10^{-31}$ kg

b) $\gamma = \dfrac{1}{\sqrt{1 - 0.98^2}} = 5.025$

$m = \gamma m_0 = 4.58 \times 10^{-30}$ kg

9.11×10^{-31}

Ex) A particle of mass $1.6x10^{-24}$kg travels with velocity of 0.65 c. Calculate its rest mass.

Ex) What speed would an ~~object~~ 68 kg Ricky. have to travel to increase its mass ~~by 100%~~? 225000 kg (Statue of Liberty)

$m = \gamma m_0$ $\gamma = 3462$

$\gamma = \dfrac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}}$

$(3462)^2 = \dfrac{1}{1 - \dfrac{v^2}{c^2}}$

$1 - \dfrac{v^2}{c^2} = \dfrac{1}{(3462)^2}$

$1 - \dfrac{1}{3462^2} = \dfrac{v^2}{c^2}$

$0.99999992 = \dfrac{v^2}{c^2}$

$V = 0.99999996\ c$

$m = \gamma m_0$

Ex) What is the momentum of a 70 kg person travelling at 0.90c? *(Be careful here!)*

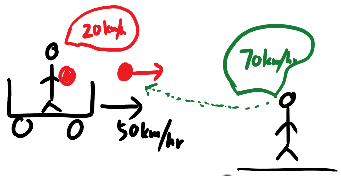
$\vec{p} = mV$ slow

$\vec{p} = \gamma m_0 V$ $v \sim c$

$\gamma = \dfrac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}} = \dfrac{1}{\sqrt{1 - 0.9^2}} = 2.3$

$\vec{p} = 2.3(70\text{ kg}) \cdot (0.9 \cdot 3\times10^8 \text{ m/s})$

$\boxed{\vec{p} = 4.3 \times 10^{10} \text{ kg m/s}}$



Unit 5 Special Relativity Day 5

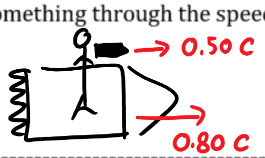
Consequence of Special Relative: (4) Relativistic Addition of Velocity

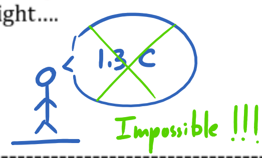
One of light's most important roles as a limiting velocity follows from this: no matter how hard we try, it is impossible to accelerate something through the speed of light. More generally, the speeds of things are divided into two groups:

things that travel	things that travel
below the speed of Light	at "C"

We Can't **accelerate** **anything so that it crosses the barrier of the speed of light!!**

Yet it looks like it would be pretty easy to violate the limiting character of the speed light by accelerating something through the speed of light....





Classical Velocity Addition

Ex, A girl is riding in a sled at a speed 1.0 m/s relative to an observer. She throws a snowball first forward, then backward at a speed of 1.5 m/s relative to the sled.

v: velocity of sled relative to earth
u′: velocity of the snowball relative to the sled.
u: velocity of the snowball relative to the earth-bound observer

Classical Velocity Addition

$$u = v + u'$$

Thus:

Pic 1: $U = 1\,m/s + 1.5\,m/s = 2.5\,m/s$

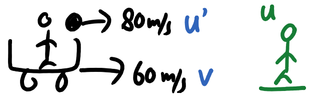
Pic 2: $U = 1\,m/s\ (R) - 1.5\,m/s\ [L] = -0.5\,m/s$

Relativistic Velocity Addition

According to experiment and the second postulate of relativity, light from the car's headlights moves away from the car at speed c and towards the observer on the sidewalk at speed c. Classical velocity addition is not valid.

Otherwise the speed of the headlight coming toward the observer on sidewalk would be $u = 2c$

Which is IMPOSSIBLE!!



$$\frac{60\cdot 80}{(3\times10^8)^2} = 5.3\times10^{-14}$$

$$1+\square \approx 1$$

Relative Velocity

$$u = \frac{v+u'}{1+\frac{vu'}{c^2}}$$

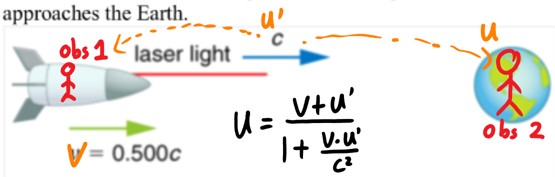
v : **relative velocity between two observer**

u': **velocity of an object relative to one observer**

u : **velocity of an object relative to the other observer**

Note that the term $\frac{vu'}{c^2}$ becomes very small at low velocities, and $u = \frac{v+u'}{1+\frac{vu'}{c^2}}$ gives a result very close to classical velocity addition. No wonder the classical velocity addition seems correct in our experience.

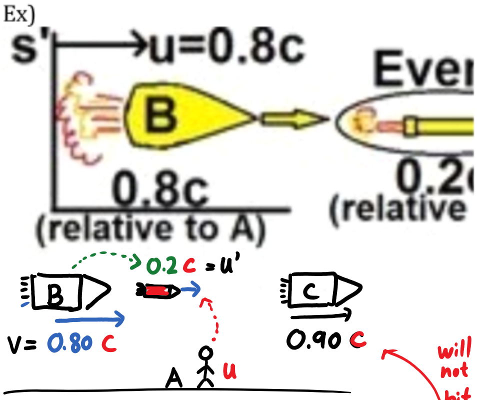
Ex) Suppose a spaceship heading directly towards the Earth at half the speed of light sends a signal to us on a laser-produced beam of light. Given that the light leaves the ship at speed "c" as observed from the ship, calculate the speed at which it approaches the Earth.



$$u = \frac{v+u'}{1+\frac{v\cdot u'}{c^2}}$$

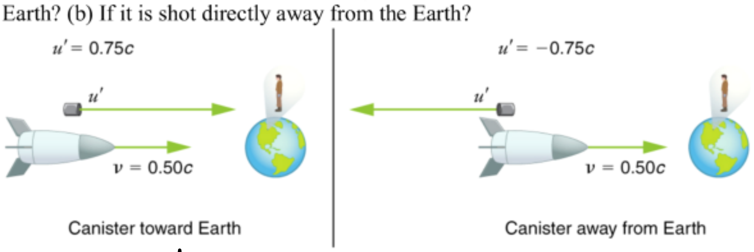
$$u = \frac{0.5c+1c}{1+\frac{0.5\cancel{c}\cdot 1\cancel{c}}{\cancel{c^2}}} = \frac{1.5c}{1+0.5} = \boxed{c}$$

Ex)



$$u = \frac{0.8c+0.2c}{1+\frac{0.8\cancel{c}\cdot 0.2\cancel{c}}{\cancel{c^2}}} = \frac{1c}{1+0.16} = \boxed{0.86c}$$

Ex) Suppose the spaceship in the previous example is approaching the Earth at half the speed of light and shoots a canister at a speed of 0.750c. (a) At what velocity will an Earth-bound observer see the canister if it is shot directly towards the Earth? (b) If it is shot directly away from the Earth?



$$u = \frac{v+u'}{\left(1+\frac{vu'}{c^2}\right)}$$

$$u\left[1+\frac{vu'}{c^2}\right] = v+u'$$

$$u + \frac{uvu'}{c^2} = v+u'$$

$$u - v = u' - \frac{uvu'}{c^2}$$

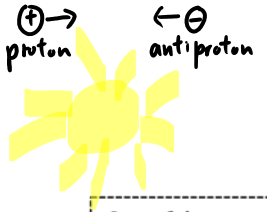
$$u = \frac{0.5c - 0.75c}{1+\frac{0.5\cancel{c}\cdot(-0.75\cancel{c})}{\cancel{c^2}}} = \boxed{-0.40\,c}$$

3×10^8 m/s

$$u - v = u'\left[1-\frac{vu}{c^2}\right]$$

$$\boxed{u' = \frac{u-v}{1-\frac{vu}{c^2}}}$$

$\boxed{\text{WS\# 4, 5}}$



$$E = mc^2$$

$$m = \gamma m_0 \quad \text{(}m_0\text{: rest mass)}, \quad \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Unit 5 Special Relativity Day 6

Consequence of Special Relative: (5) Relativistic Energy

One of the most astonishing results of special relativity is that mass and energy are equivalent, in the sense that a gain or loss of mass can be regarded equally well as a gain or loss of energy.

The Total **Energy** of an object $E_T = \gamma m_0 c^2 = \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}}$

The rest **Energy**, (if the object is not moving) $E = m_0 c^2$

Ex1) How much energy is in Mr.Cheung's 1.33 kg stuffed Yoda?

$$E = m_0 c^2 = 1.33\,kg\,(3\times10^8)^2 = \boxed{1.2\times10^{17}\ J}$$

Ex2) How long will that run a 60-W light bulb?

$$P = \frac{E}{t} \qquad 60\ W = \frac{1.2\times10^{17}\ J}{t} \qquad t = 2\times10^{15}\ sec = 63\ \text{million years.}$$

Ex3) the sun radiates electromagnetic energy at $3.92 \times 10^{26} W$. How much mass does the sun lose in 1 year?

$$P = \frac{E}{t} \qquad 3.92\times10^{26} = \frac{E}{\underbrace{3.15\times10^7\ sec}_{1\ year}} \qquad E = 1.2\times10^{34}\ J = m(c^2)$$

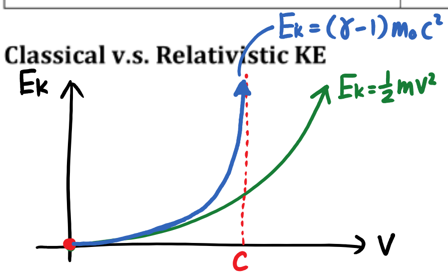
$$\text{mass} = \frac{1.2\times10^{34}}{c^2} = 1.37\times10^{17}\ kg$$

If the object is moving, then the total Energy is $E_T = E_0 + KE$

Relativistic Kinetic Energy

$E_T = E_0 + E_k$
 $E_T = \gamma m_0 c^2$, $E_0 = m_0 c^2$
 $E_k = \gamma m_0 c^2 - m_0 c^2$
 $\boxed{E_k = m_0 c^2(\gamma - 1)}$
	Mass and Energy are the same
	 • A change in one, means a change in the other
	 • For example, you pick up your backpack and increase its Ep
	   • Since the Energy increases, the mass must increase.
	   • So when you pick up your backpack, it is actually heavier than when it is on the ground

Classical v.s. Relativistic KE



Classical : $E_k = \frac{1}{2}mv^2$

Relativistic : $E_k = (\gamma - 1)m_0c^2$, $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

As a consequence: Objects with mass **cannot reach the speed of light. Because it would take an** infinite (∞) **amount of energy !!!!**

$E_0 = m_0c^2$ (rest) $\qquad \boxed{E_T = \gamma m_0 c^2}$ (when moving; includes Ek already !!) $\qquad \boxed{E_k = (\gamma-1)m_0c^2}$

Special Relativity: **Worksheet 1** **Reference Frames**

1. If I am standing perfectly still on the sidewalk what is my speed relative to the sidewalk?

2. There are three cars. Car A travels North at 25km/h. Car B travels North at 75km/h. Car C travels South at 50km/h.

   a) What is the speed of car A relative to car B?
   b) What is the speed of car A relative to car C?
   c) What is the speed of car B relative to car C?

3. A bird and an airplane are flying in opposite directions. The speed of the bird relative to the airplane is 1000m/s. If the speed of the airplane relative to the ground is 950m/s what is the speed of the bird relative to the ground?

4. I am traveling at 5km/h relative to you. How fast are you traveling relative to me?

5. I am running inside an airplane at 15km/h towards the tail. My speed relative to the ground is 500km/h. What is the speed of the airplane relative to the ground?

6. Please sketch (roughly) the path of a projectile fired on Earth at 45º. Now please sketch the path of that same projectile if fired inside an airplane flying at a constant altitude with a constant velocity of 1000km/h.

7. You are inside a truck trailer heading down the highway at a constant 100km/h. You decide to test which direction you are moving in by placing a tennis ball on a frictionless surface inside the closed trailer. In which direction will the ball roll?

8. You and a friend are inside a rocket ship moving at a constant speed of 30,000m/s. He is seated in the front of the 100m long ship and you are seated in the back. What is your speed relative to your friend?

9. You and another car are headed for a head on collision. Your speedometer reads 50km/h. You calculate that your speed relative to the other care is 200km/h. How fast is the other car traveling? If you were to measure the speed of light from his headlights what would that speed be?

10. Please state in your own words Einstein's two postulates.

Ans) 1) 0 km/h 2) -50 km/h, 75 km/h, 125 km/h 3) -50 m/s 4) -5 km/h 5) 515 km/h 7) Won't move 8) 0 m/s 9) 150 km/h, 3x10^8 m/s

Special Relativity: **Worksheet 2** **Time Dilation**

1. An astronaut is circling the Earth in the ISS at a high speed. He measures the time it takes for a baseball to be thrown up, and fall back down in his Earth gravity simulated environment. On Earth, a NASA engineer observes this event through a telescope, and measures the time for this event as well. Who measures the proper time of this event correctly? Explain your answer.

2. A bird flies 15m/s [E] relative to the Earth against a 5m/s [W] wind. A river flows at a velocity of 8m/s E relative to a stationary observer watching this event on the shore. What is the speed of the bird relative to the water? (2m/s E)

3. An observer in a fixed frame of reference is watching an event in a spaceship moving with a velocity of 0.866c. If the observer in the moving frame measures the event to take a time of 5.0s to occur, what amount of time will the "fixed" observer measure? (10.s)

4. An observer watching a spaceship moving at 0.33c observes an event to take 2.7s. What is the proper time of the event as viewed by someone on the spaceship? (2.5s)

5. A law enforcement officer in an intergalactic police car turns on a flashing light and sees it generate a flash every 1.5s. A person on the Earth measures the time between flashes as 2.5s. How fast is the police car moving relative to the Earth? (0.80c)

6. Ronin the astronaut's wife gives birth to a child the day he leaves for a 7.00 year long space mission. Assuming negligible acceleration, how fast will Ronin have to travel so that he is the same age as his child when he returns from space, given that his current age is 38.0? (0.988c)

7. A beam of muons have a proper lifetime of 2.2×10^{-6} s. If they are measured to move with a speed of 0.99c, find how far they would travel before decaying assuming they undergo time dilation. (4.7×10^{3}m)

8. A beam of muons is injected into a storage ring that uses electromagnetic fields to maintain the muons in a circular motion. If the ring has a radius of 50m, and the muons are injected with a velocity of 0.95c, how many revolutions of the ring will an "average" muon make before it decays, assuming muon proper lifetime = 2.2×10^{-6}s. (6 full revolutions)

Special Relativity: **Worksheet 3** **Length Contraction**

1. A rocket ship at rest has a length of 75 m. What is its length relative to the earth when it has a speed of 0.92 c relative to the earth?

2. An electron travels in an accelerator tube at a speed of 0.998 c relative to the earth. The length of the tube, as measured in the earth frame of reference, is 15.7 m. In the frame of reference of the electron, what is the length of the tube?

3. At what speed does an object need to be travelling in order that its length will be measured to be half its rest length?

$$\gamma = \frac{1}{\sqrt{1-0.776^2}} = 1.58 \qquad L = \frac{L_0}{\gamma} = \frac{135\,m}{1.58} = 85\ m$$

4. Two rocket ships, Champion and Charlatan, are each measured to be 135 m long when at rest. If these two ships pass each other in space with a relative speed of 0.776 c:
   a. What is the length of Champion according to measurements made by astronauts in Charlatan?
   b. What is the length of Charlatan according to measurements made by astronauts in Champion?

5. A spaceship, travelling past earth at a speed of 0.856 c, is measured to be 37 m long by observers on earth. What is the rest length of the ship?

6. A rocket ship at rest is 85.3 m long and 4.52 m wide, and is travelling with a velocity that is parallel to its length.
   a. What is its speed if its length is measured to be 52.4 m?
   b. What is its width at this speed?

Ans: 1) 29.4m, 2) 0.992m, 3) 0.866c, 4a) 85.15m, 4b) 85.15m, 5) 71.6m, 6a) 0.789c, 6b) 4.52m

Special Relativity: **Worksheet 4** **Mass Increase**

1. What is the mass of a 0.142 kg baseball that is pitched by Optimus Prime at 2.89×10^8 m/s? By how much does the mass increase? Where does this mass come from? (**0.529 kg, 0.387 kg, from the KE**)

2. What speed would an object have to travel to increase its mass by 50%? ***(v = 0.745 c)***

3. What is the momentum of a 5.0 kg rock travelling at 0.99c? *(Be careful here!)* ***(*1.05 x 10^{10} kgm/s)**

4. A 12500 kg (rest mass) spaceship is travelling at 0.99c. What is the spaceship kinetic energy? *(Be careful here!)* *(***3.90 x 10^{21} J**)

5. What is the momentum of an electron traveling at 0.980c? ($\mathbf{1.346} \times \mathbf{10^{-21}}$ kg m/s)

6. Find the velocity of a proton that has a momentum of 4.48 × 10−19 kg·m/s. ($\mathbf{2.00} \times \mathbf{10^{8}}$ m/s)

Special Relativity: **Worksheet 5** **Relativistic addition of velocity**

1. Suppose a spaceship heading straight towards the Earth at 0.750c can shoot a canister at 0.500c relative to the ship. (a) What is the velocity of the canister relative to the Earth, if it is shot directly at the Earth? (b) If it is shot directly away from the Earth? **0.909c, 0.400c,**

2. Repeat the previous problem with the ship heading directly away from the Earth. **-0.400c, -0.909c**

3. If a spaceship is approaching the Earth at 0.100c and a message capsule is sent toward it at 0.100c relative to the Earth, what is the speed of the capsule relative to the ship? **0.198c**

4. If two spaceships are heading directly towards each other at 0.800c, at what speed must a canister be shot from the first ship to approach the other at 0.999c as seen by the second ship? **0.991c**

5. A missile is shot from one spaceship towards another, it leaves the first at 0.950c and approaches the other at 0.750c . What is the relative velocity of the two ships? **-0.696c away**

Special Relativity: **Worksheet 6** **Relativistic Energy**

1. A 25 kg pikachu is accelerated to a speed of 0.98c.
   a. What would the mass of Pikachu be at this speed? (126 kg)
   b. How much energy would be associated with the PikaPika at rest? At this speed? ***(2.25 x 10^{18} J, 1.13 x 10^{19} J)***

2. Mr.Cheung (85 kg rest mass) is running so fast he has a (dilated) mass of 89 kg. What is his kinetic energy, and what is his velocity? ($\mathbf{3.6 \times 10^{17} J, 0.296c}$)

3. An electron (m_i = 9.109 × 10^{-31} kg) is accelerated from rest to a speed of 0.9995 c in a particle accelerator. Determine the electron's (a) rest energy, (b) total energy, and (c) kinetic energy. **(8.19 × 10^{-14} J, 2.59 × 10^{-12} J, 2.51 × 10^{-12} J)**