ENERGY & MOMENTUM
PROVINCIAL EXAMINATION ASSIGNMENT

ANSWER KEY / SCORING GUIDE

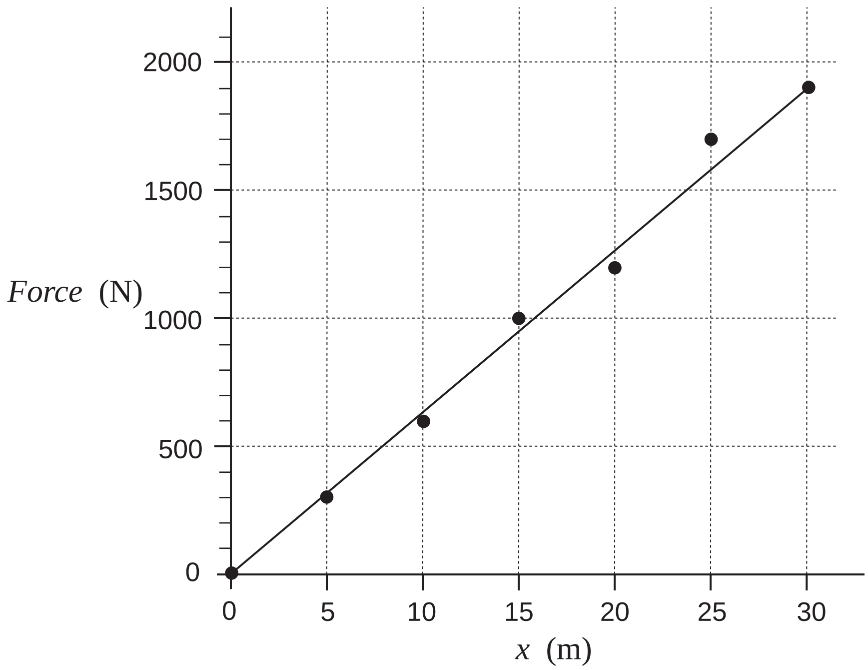
PART A: Multiple Choice (each question worth ONE mark)

Q	K	Q	K
1.	B	16.	B
2.	D	17.	C
3.	B	18.	B
4.	B	19.	C
5.	A	20.	D
6.	D	21.	D
7.	C	22.	B
8.	C	23.	C
9.	C	24.	D
10.	C	25.	C
11.	A	26.	C
12.	C	27.	C
13.	A	28.	A
14.	B	29.	C
15.	A	30.	C

1. A daredevil is attached by his ankles to a bungee cord and drops from the top of a bridge. The force exerted on the daredevil by the bungee cord is measured against the change in length, x, of the cord as the cord is stretched, slowing the daredevil's fall.

Force (N)	0	300	600	1 000	1 200	1 700	1 900
x (m)	0	5	10	15	20	25	30

a) Plot a graph of force vs. change in length on the graph below. **(2 marks)**

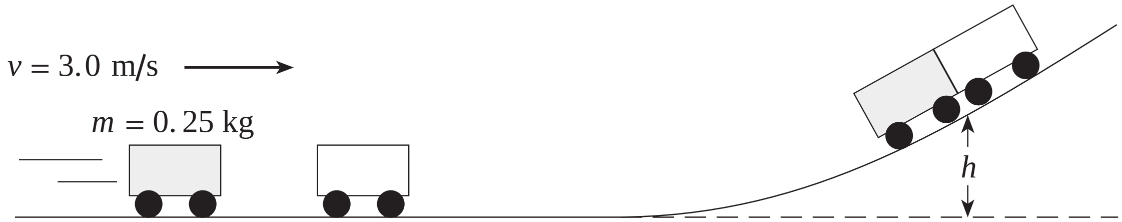


b) Use the graph to determine the work done by the bungee cord during its stretch. **(3 marks)**

$$Area = \frac{1\,900 \cdot 30}{2} = 28\,500 \text{ J}$$

$$= 2.9 \times 10^4 \text{ J}$$ ← **3 marks**

2. A $0.25\ \text{kg}$ cart travelling at $3.0\ \text{m/s}$ collides with and sticks to an identical stationary cart on a level track. (Ignore friction.)

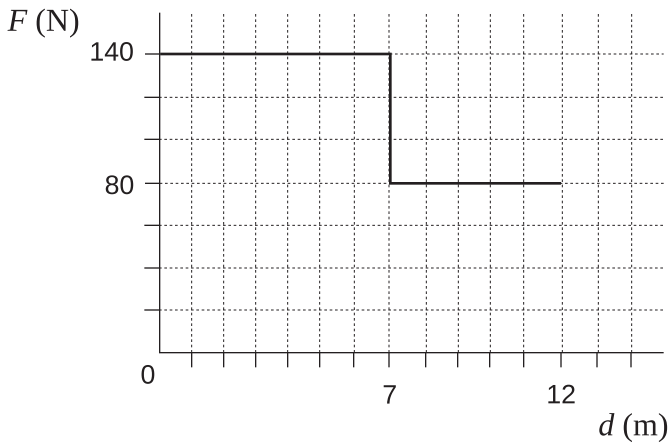


To what height h do the combined carts travel up the hill? **(7 marks)**

$$\left.\begin{aligned} p_i &= p_f \\ mv_i &= (2m)v_f \\ v_f &= \frac{v_i}{2} \\ &= 1.5\ \text{m/s} \end{aligned}\right\} \leftarrow \mathbf{3\tfrac{1}{2}\ marks}$$

$$\left.\begin{aligned} \Delta E_P &= -\Delta E_k \\ (2m)gh_{\text{max}} &= \frac{1}{2}(2m)\left(v_f\right)^2 \\ h_{\text{max}} &= \frac{\left(v_f\right)^2}{2g} \\ &= 0.11\ \text{m} \end{aligned}\right\} \leftarrow \mathbf{3\tfrac{1}{2}\ marks}$$

3. Starting from rest, a farmer pushed a cart 12 m. The graph shows the force F which he applied, plotted against the distance d.



a) How much work did the farmer do moving the cart 12 m? **(3 marks)**

W = *area bounded by graph*

$= (140 \text{ N} \times 7.0 \text{ m}) + (80 \text{ N} \times 5.0 \text{ m})$ ← **2 marks**

$= 980 \text{ J} + 400 \text{ J}$

$= 1\,380 \text{ J}$ ← **1 mark**

b) After the farmer had pushed the 240 kg cart 12 m, it was moving with a velocity of 2.2 m/s. What was the cart's kinetic energy? **(2 marks)**

$E_k = \frac{1}{2}mv^2$ ← **1 mark**

$= \frac{1}{2}(240 \text{ kg})(2.2 \text{ m/s})^2$

$= 580 \text{ J}$ ← **1 mark**

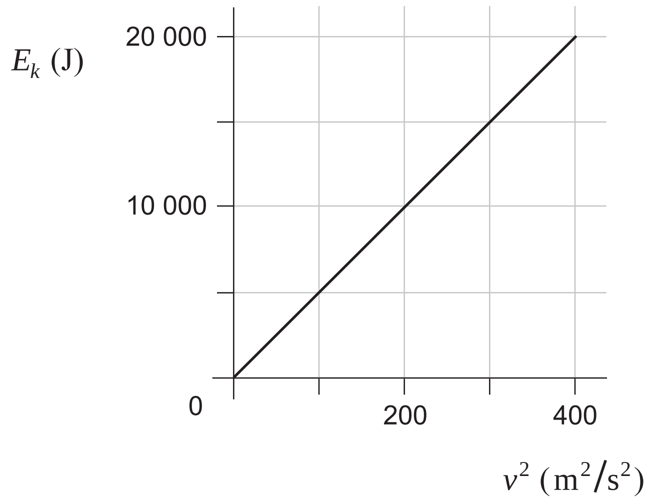
c) What was the efficiency of this process? **(2 marks)**

$Efficiency = \dfrac{E_{out}}{E_{in}}$ ← **1 mark**

$= \dfrac{580 \text{ J}}{1\,380 \text{ J}}$

$= 0.42$ or 42% ← **1 mark**

4. A student plots the graph below, showing the kinetic energy E_k of a motorbike versus the square of its velocity v^2.



a) What is the slope of this graph? **(2 marks)**

$$slope = \frac{\Delta E_k}{\Delta v^2}$$
$$= \frac{20\ 000\ \text{J}}{400\ \text{m}^2/\text{s}^2}$$
$$= 50\ \text{J}/\text{m}^2/\text{s}^2 \qquad \leftarrow \textbf{2 marks}$$

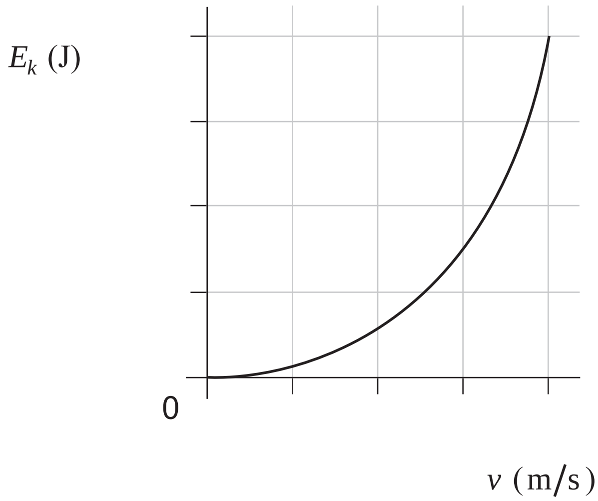
or 50 kg

b) What does the slope represent? **(2 marks)**

From the graph: $E_k = kv^2, \quad \therefore \left(E_k = 50\ v^2\right)$ $\leftarrow$ **1 mark**

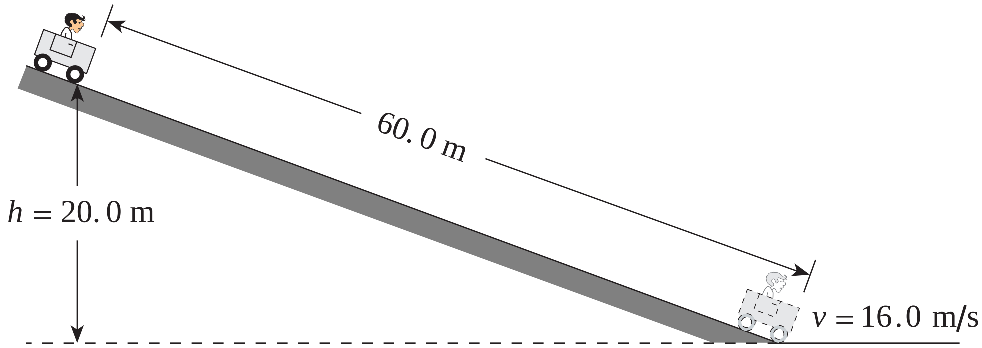
But $E_k = \frac{1}{2}mv^2$**, therefore the slope represents one half the mass of the motorbike.** $\leftarrow$ **1 mark**

c) Using the axes below, sketch the graph of kinetic energy E_k versus velocity v for this motorbike. There is no need to plot any data points. **(1 mark)**



← **1 mark**

5. A 170 kg cart and rider start from rest on a $20.0\ \text{m}$ high incline.



a) How much energy is transformed to heat? **(5 marks)**

$$\Delta E = 0$$

$$Ep = E_k + Heat \qquad \leftarrow \textbf{2 marks}$$

$$mgh = \tfrac{1}{2} mv^2 + Heat \qquad \leftarrow \textbf{1 mark}$$

$$170(9.8)20.0 = \tfrac{1}{2}(170)16.0^2 + E_h \qquad \leftarrow \textbf{1 mark}$$

$$33\,320 = 21\,760 + E_h$$

$$1.16 \times 10^4\ \text{J} = E_h \qquad \leftarrow \textbf{1 mark}$$

b) What is the average force of friction acting on the cart? **(2 marks)**

$$E_h = \text{work done by friction}$$

$$11560 = F_f \cdot d$$

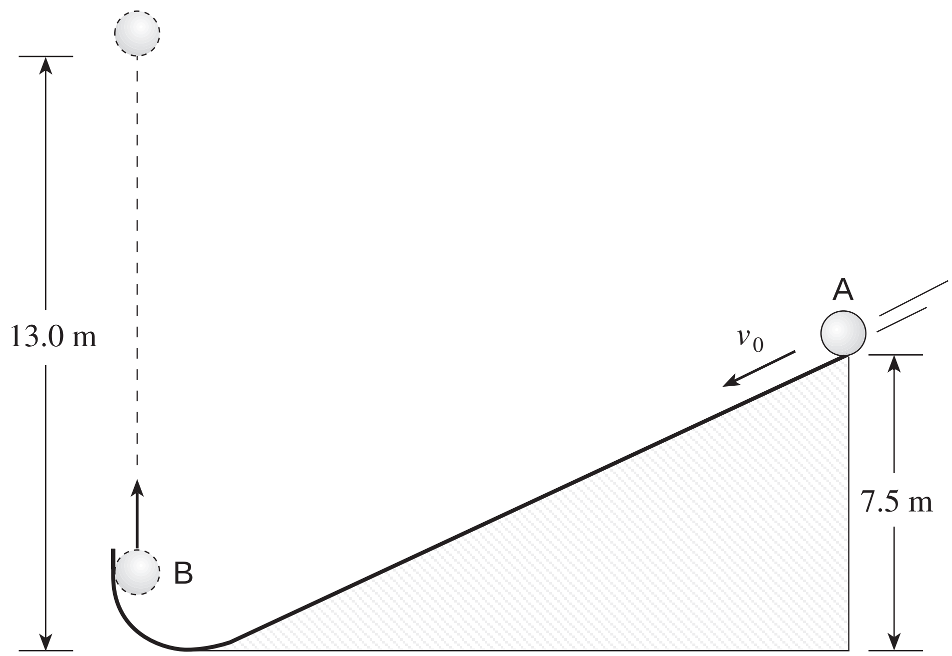
$$\therefore F_f = \frac{11\,560}{60.0}$$

$$F_f = 193\ \text{N}$$

$$F_f = 190\ \text{N} \quad \leftarrow \textbf{2 marks}$$

6. A 0.50 kg ball starting from position A which is 7.5 m above the ground, is projected down an incline as shown. Friction produces 10.7 J of heat energy.

The ball leaves the incline at position B travelling straight upward and reaches a height of 13.0 m above the floor before falling back down.



What was the initial speed, v_0, at position A? Ignore air resistance. **(7 marks)**

$$E_{TA} = E_{Total} \quad \leftarrow \textbf{2 marks}$$

$$E_{K_A} + E_{P_A} = E_{P_{top}} + E_h$$

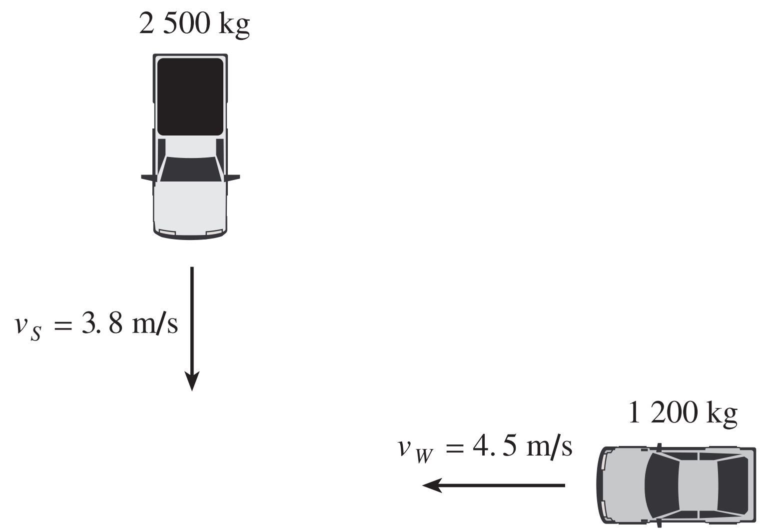
$$\frac{1}{2}mv^2 + mgh_A = mgh + E_h \quad \leftarrow \textbf{2 marks}$$

$$\frac{1}{2}\times 0.50\left(v^2\right) + 0.50\times 9.8\times 7.5 = 0.50\times 9.8\times 13\times +10.7 \quad \leftarrow \textbf{1 mark}$$

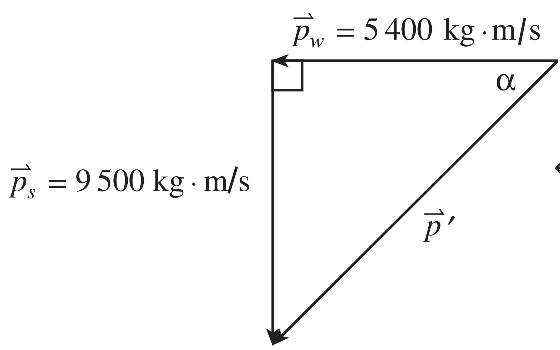
$$v^2 = \frac{74.4 - 36.75}{0.25} \quad \leftarrow \textbf{1 mark}$$

$$v = 12 \text{ m/s} \quad \leftarrow \textbf{1 mark}$$

7. Sally is driving south in her 2 500 kg pickup truck at 3.8 m/s when she collides with Willy driving west in his 1 200 kg car at 4.5 m/s.



The two vehicles lock together and slide over the wet parking lot. Find the speed and direction of the damaged vehicles immediately after the collision. **(7 marks)**



← **1 mark for momentums**

$(p')^2 = 5\,400^2 + 9\,500^2$ ← **1 mark for addition**

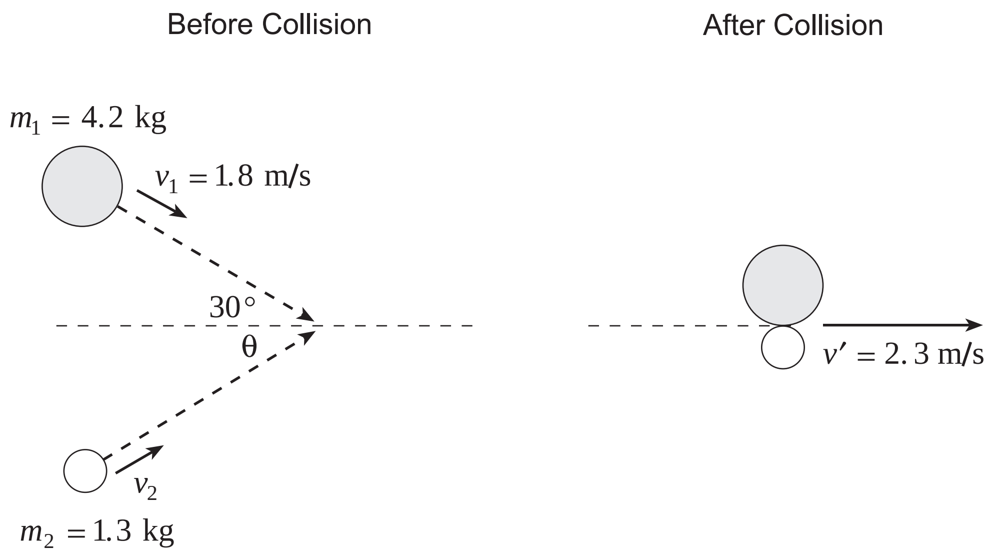
$p' = 10\,900 \text{ kg}\cdot\text{m/s}$ ← **2 marks for pythagorus**

$v' = \dfrac{10\,900}{(2\,500 + 1\,200)} = 3.0 \text{ m/s}$ ← **1 mark for dividing by 3700**

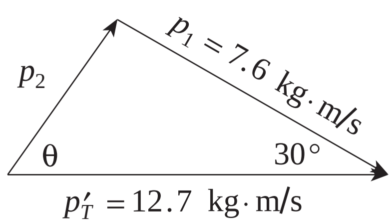
$$\left.\begin{array}{l} \tan\alpha = \dfrac{9\,500}{5\,400} \\ \alpha = 60^\circ \end{array}\right\}$$ ← **1 mark**

$v' = 3.0 \text{ m/s}, 60^\circ \text{ S of W}$ ← **1 mark**

8. Two steel pucks are moving as shown in the diagram. They collide inelastically.



Determine the speed and direction (angle θ) of the 1.3 kg puck before the collision. **(7 marks)**



$$\left.\begin{aligned} p_1 &= m_1 v_1 \\ &= 4.2 \times 1.8 \\ &= 7.6 \text{ kg} \cdot \text{m/s} \\ p_T' &= m_T v_1' \\ &= 5.5 \times 2.3 \\ &= 12.7 \text{ kg} \cdot \text{m/s} \end{aligned}\right\} \leftarrow \textbf{1 mark}$$

<u>Method 1:</u>

Cosine Law:

$$p_2{}^2 = \left(p_T'\right)^2 + p_1{}^2 - 2\,p_T' p_1 \cos 30^\circ$$

$$= 12.7^2 + 7.6^2 - 2 \times 12.7 \times 7.6 \times \cos 30^\circ$$

$$p_2{}^2 = 51.9$$

$$p_2 = \sqrt{51.9} = 7.20 \text{ kg m/s} \qquad \leftarrow \textbf{3 marks}$$

$$v_2 = \frac{p_2}{m_2} = \frac{7.20 \text{ kg m/s}}{1.3 \text{ kg}} = 5.5 \text{ m/s} \qquad \leftarrow \textbf{1 mark}$$

Sine Law:

$$\left.\begin{aligned}\frac{\sin\theta}{7.6} &= \frac{\sin 30^\circ}{7.2}\\ \sin\theta &= \frac{7.6\times\sin 30^\circ}{7.2}\\ \sin\theta &= 0.528\\ \theta &= 32^\circ\end{aligned}\right\}\leftarrow \textbf{2 marks}$$

$$v_2 = 5.5 \text{ m/s at } 32^\circ$$

<u>Method 2:</u> (one variation)

$$m_1v_1\cos 30^\circ + m_2v_2\cos\theta = m_Tv' \qquad \leftarrow \textbf{1 mark}$$

$$4.2(1.8)\cos 30^\circ + 1.3(v_2)\cos\theta = (4.2+1.3)(2.3) \qquad \leftarrow \textbf{1 mark}$$

$$v_2 = \frac{4.69}{\cos\theta} \qquad \leftarrow \textbf{1 mark}$$

$$m_1v_1\sin 30^\circ + m_2v_2\sin\theta = 0 \qquad \leftarrow \textbf{1 mark}$$

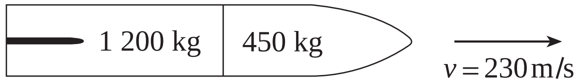
$$4.2(1.8)\sin 30^\circ + 1.3(v_2)\sin\theta = 0 \qquad \leftarrow \textbf{1 mark}$$

$$v_2 = \frac{2.91}{\sin\theta}$$

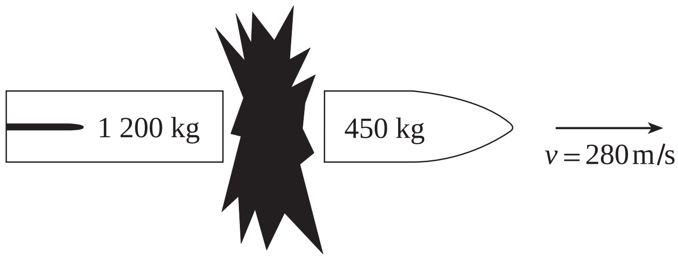
$$\left.\begin{aligned}\frac{4.69}{\cos\theta} &= \frac{2.91}{\sin\theta}\\ \frac{\sin\theta}{\cos\theta} &= \frac{2.91}{4.69}\\ \tan\theta &= 0.618\\ \theta &= 32^\circ\end{aligned}\right\}\leftarrow \textbf{1 mark}$$

$$\left.\begin{aligned}v_2 &= \frac{4.69}{\cos 31.8}\\ v_2 &= 5.5 \text{ m/s}\end{aligned}\right\}\leftarrow \textbf{1 mark}$$

9. A space vehicle made up of two parts is travelling at 230 m/s as shown.



An explosion causes the 450 kg part to separate and travel with a final velocity of 280 m/s as shown.



a) What was the momentum of the space vehicle before the explosion? **(2 marks)**

$\rho = mv$

$= (1\,200 + 450)230$

$= 3.8 \times 10^5$ kg m/s ← **2 marks**

b) What was the magnitude of the impulse on the 1 200 kg part during the separation? **(3 marks)**

$Impulse = \Delta p$

$= P_b - P_a$ ← **1 mark**

$= (450 \times 280) - (450 \times 230)$ ← **1 mark**

$= 2.3 \times 10^4$ N·s ← **1 mark**

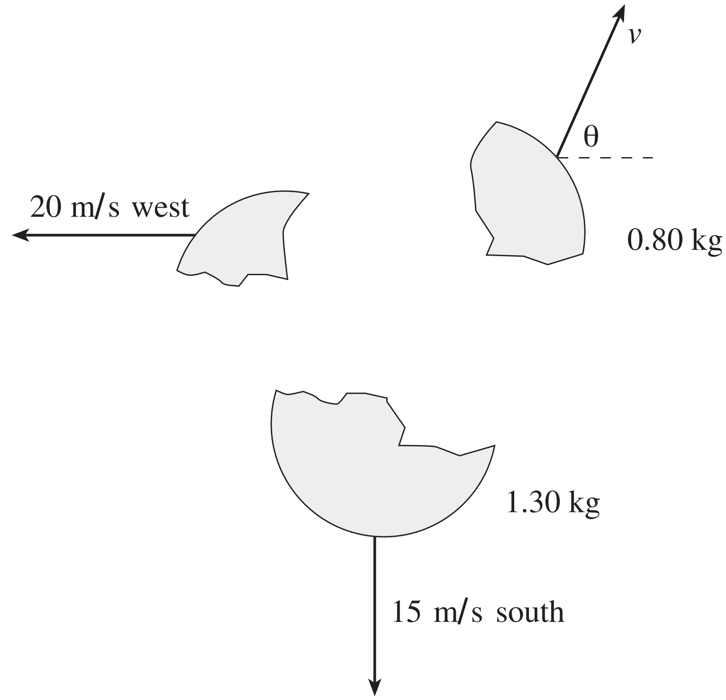
c) Using principles of physics, explain what changes occur, if any, to the
i) momentum of the system as a result of the explosion. **(2 marks)**

In an explosion, momentum must be conserved.

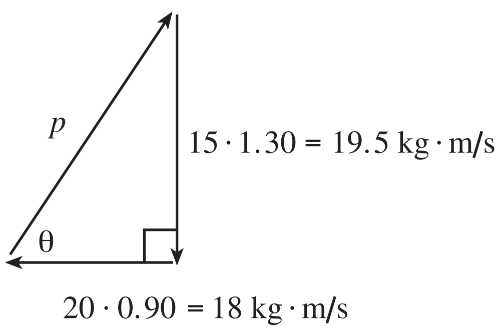
ii) kinetic energy of the system as a result of the explosion. **(2 marks)**

Since the explosion adds energy to the system, the system will gain kinetic energy.

10. A 3.00 kg object initially at rest explodes into three fragments as shown in the diagram below.



What are the speed and direction of the 0.80 kg fragment? **(7 marks)**



← **3 marks**

$$p^2 = 18^2 + 19.5^2$$

$p = 26.5 \text{ kg} \cdot \text{m/s}$ ← **1 mark**

$$v = \frac{p}{m}$$

$$= \frac{26.5}{0.80}$$

$= 33 \text{ m}/s$ ← **1 mark**

$$\theta = \tan^{-1}\left(\frac{19.5}{18}\right)$$

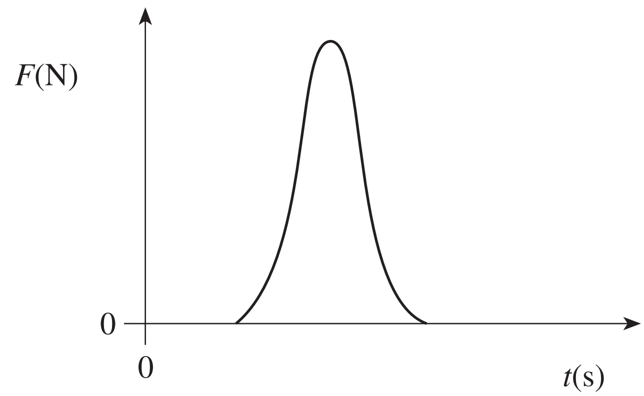
$= 47°$ ← **2 marks**

11. A 5.20 kg block sliding at 9.40 m/s across a horizontal frictionless surface collides head on with a stationary 8.60 kg block. The 5.20 kg block rebounds at 1.80 m/s. How much kinetic energy is lost during this collision? **(7 marks)**

$$\left.\begin{aligned} m_1v_1 + m_2v_2 &= m_1v_1' + m_2v_2' \\ (5.20)(9.40) &= (5.20)(-1.80) + (8.60)v_2' \\ v_2' &= 6.77 \text{ m/s} \end{aligned}\right\}$$ **4 marks**

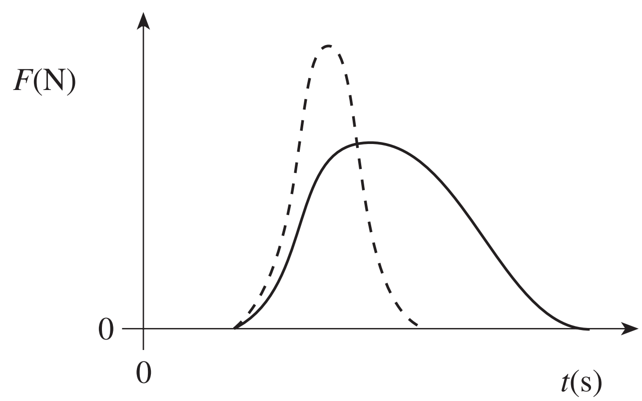
$$\left.\begin{aligned} E_k(\text{before}) &= \frac{1}{2}m_1v_1^2 = 229.7 \text{ J} \\ E_k(\text{after}) &= \frac{1}{2}m_1v_1^{2\prime} + \frac{1}{2}m_2v_2^{2\prime} \\ &= 8.424 + 197.2 \\ &= 205.6 \text{ J} \\ \text{LOST} &= 24.12 \text{ J} \\ \Delta E &= 24.1 \text{ J} \end{aligned}\right\}$$ **3 marks**

12. In sports such as golf, tennis and baseball, a player exerts a force over a time interval on a ball in order to give it a high speed, as shown on the graph.



Players are instructed to "follow through" on their swing. A weaker player may not exert as large a force but may give the ball a higher speed than a stronger player.

a) Sketch on the graph below how a weaker player can overcome the force handicap. **(1 mark)**



b) Explain how the player can impart a greater impulse on a ball. **(3 marks)**

By exerting a smaller force for a longer time, the weaker player may be able to deliver a greater impulse to the ball.