GRAVITATION
PROVINCIAL EXAMINATION ASSIGNMENT
ANSWER KEY / SCORING GUIDE

PART A: Multiple Choice (each question worth ONE mark)

Q	K	Q	K
1.	A	21.	D
2.	D	22.	A
3.	D	23.	B
4.	C	24.	C
5.	D	25.	A
6.	D	26.	A
7.	A	27.	B
8.	B	28.	C
9.	C	29.	C
10.	D	30.	A
11.	A	31.	C
12.	B	32.	B
13.	D	33.	D
14.	B	34.	D
15.	C	35.	B
16.	B	36.	D
17.	C	37.	B
18.	B	38.	D
19.	C	39.	B
20.	B	40.	A

1. A 4.2×10^3 kg spacecraft orbits a 5.6×10^{26} kg planet. If it takes the spacecraft 8.9×10^4 s to complete one orbit, how far is it from the planet's centre? **(7 marks)**

$$F_{net} = F_G$$

$$F_C = F_G \qquad \leftarrow \textbf{2 marks}$$

$$\frac{m_1 4\pi^2 r}{T^2} = \frac{Gm_1m_2}{r^2} \qquad \leftarrow \textbf{2 marks}$$

$$r^3 = \frac{Gm_2T^2}{4\pi^2} \qquad \leftarrow \textbf{1 mark}$$

$$= \frac{(6.67\times10^{-11})(5.6\times10^{26})(8.9\times10^4)^2}{4\pi^2} \qquad \leftarrow \textbf{1 mark}$$

$$r = 2.0\times10^8 \text{ m} \qquad \leftarrow \textbf{1 mark}$$

2.	a)	The space shuttle orbits the Earth in a circular path where the gravitational field strength is 8.68 N/kg. What is the shuttle's orbital radius? **(5 marks)**

$$F_g = mg = \frac{GMm}{r^2}$$ ← **2 marks**

$$g = \frac{GM}{r^2}$$ ← **1 mark**

$$r = \sqrt{\frac{GM}{g}}$$

$$= \sqrt{\frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{8.68}}$$ ← **1 mark**

$$= 6.78 \times 10^{6}\ \text{m}$$ ← **1 mark**

b) A space station that has 10 times the mass of the shuttle in a) orbits Earth at the same altitude. How does the orbital speed of the space station compare to that of the shuttle? (Check one response.) **(1 mark)**

- ☐ The space station's speed is less than the shuttle's speed.
- ☑ The space station's speed is the same as the shuttle's speed.
- ☐ The space station's speed is greater than the shuttle's speed.

c) Using principles of physics, explain your answer to b). **(3 marks)**

The force of gravity is the only force that provides the centripetal acceleration. Since both the gravitational force and the centripetal force are proportional to mass, the acceleration remains the same, therefore the speeds must be the same. $\left(\frac{v^2}{r}\right)$ ← **3 marks**

OR

Since $F_g = F_c$, $\frac{m_1 v^2}{r} = \frac{G m_1 m_{earth}}{r^2}$ **when you solve for *v*, the mass of the orbiting body cancels out. Speed is independent of the size of the orbiting mass.**

3. An 884 kg satellite in orbit around a planet has a gravitational potential energy of -5.44×10^{10} J.

The orbital radius of the satellite is 8.52×10^{6} m and its speed is 7.84×10^{3} m/s.

a) What is the mass of the planet? **(3 marks)**

$$E_p = -\frac{GMm}{r} \qquad \leftarrow \textbf{1 mark}$$

$$-5.44\times10^{10} = -\frac{6.67\times10^{-11}\times M\times884}{8.52\times10^{6}} \qquad \leftarrow \textbf{1}\tfrac{1}{2}\textbf{ mark}$$

$$M = 7.86\times10^{24}\ \text{kg} \qquad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

b) What is the kinetic energy of the satellite? **(2 marks)**

$$E_k = \frac{1}{2}mv^2 \qquad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$= \frac{1}{2}(884)\left(7.84\times10^{3}\right)^2 \qquad \leftarrow \textbf{1 mark}$$

$$= 2.72\times10^{10}\ \text{J} \qquad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

c) What is the total energy of the satellite? **(2 marks)**

$$E_T = E_k + E_p \qquad \leftarrow \textbf{1 mark}$$

$$= 2.72\times10^{10} + \left(-5.44\times10^{10}\right) \qquad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$= -2.72\times10^{10}\ \text{J} \qquad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

4. A spacecraft of mass 470 kg rests on the surface of an asteroid of radius 1 400 m and mass 2.0×10^{12} kg. How much energy must be expended so that the spacecraft may rise to a height of 2 800 m above the surface of the asteroid? **(7 marks)**

$$\Delta E = E_p' - E_p$$ ← **2 marks**

$$= \left(-G\frac{Mm}{r'}\right) - \left(-\frac{GMm}{r}\right)$$ ← **2 marks**

$$= \left(-\frac{6.67\times10^{-11}\times2.0\times10^{12}\times470}{(1\,400+2\,800)}\right) - \left(-\frac{6.67\times10^{-11}\times2.0\times10^{12}\times470}{1\,400}\right)$$ ← **2 marks**

$$= (-14.9) - (-44.8)$$

$$= 29.9 \text{ J}$$

$$= 30 \text{ J}$$ ← **1 mark**

5. a) Mars has a mass of 6.37×10^{23} kg and a radius of 3.43×10^{6} m. What is the gravitational field strength on its surface? **(4 marks)**

$$g = \frac{GM}{r^2} \quad \leftarrow \textbf{2 marks}$$

$$= \frac{6.67 \times 10^{-11}\left(6.37 \times 10^{23}\right)}{\left(3.43 \times 10^{6}\right)^2} \quad \leftarrow \textbf{1 mark}$$

$$= 3.61 \text{ N/kg} \quad \leftarrow \textbf{1 mark}$$

b) What thrust force must the rocket engine of a Martian lander exert if the 87.5 kg spacecraft is to accelerate upwards at 1.20 m/s^2 as it leaves the surface of Mars? **(3 marks)**

$$F_{net} = ma \quad \leftarrow \textbf{1 mark}$$

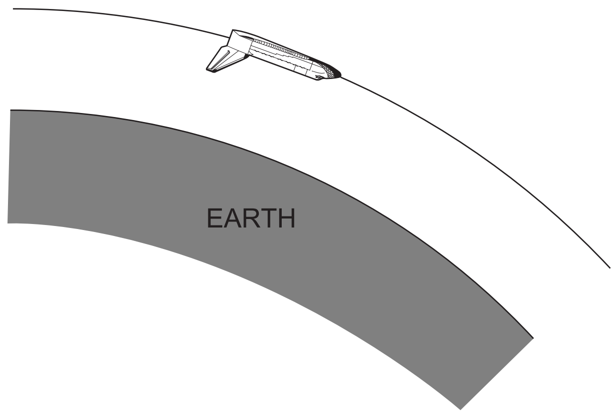
$$F_T - F_g = ma$$

$$F_T - mg = ma \quad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$F_T - 87.5(3.61) = 87.5(1.20) \quad \leftarrow \textbf{1 mark}$$

$$F_T = 421 \text{ N} \quad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

6. A space shuttle is placed in a circular orbit at an altitude of 3.00×10^{5} m **above** Earth's surface.



a) What is the shuttle's orbital speed? **(5 marks)**

$F_c = F_g$ ← **1 mark**

$$m\frac{v^2}{R} = \frac{GMm}{R^2}$$

$$v^2 = \frac{GM}{R}$$

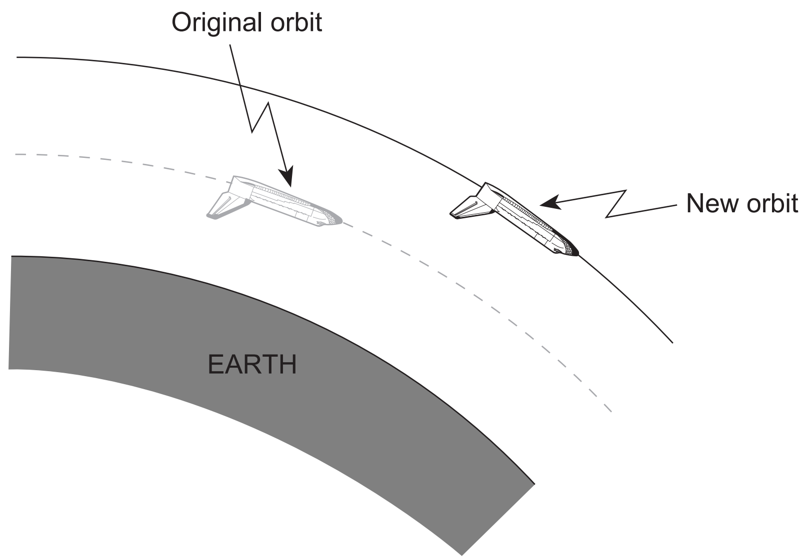
$$v = \sqrt{\frac{GM}{R}}$$

← **2 marks**

$$= \sqrt{\frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{6.68 \times 10^{6}}}$$ ← **1 mark**

$v = 7.73 \times 10^{3}$ m s ← **1 mark**

b) The space shuttle is then moved to a higher orbit in order to capture a satellite.



The shuttle's speed in this new higher orbit will have to be

- ☐ greater than in the lower orbit.
- ☑ less than in the lower orbit.
- ☐ the same as in the lower orbit.

(Check one response.) **(1 mark)**

c) Using principles of physics, explain your answer to b). **(3 marks)**

As the space shuttle moves further away from the earth's centre the force of gravity acting on the shuttle decreases. Since the centripetal force is provided by the force of gravity, it must decrease as well. ← 2 marks

The smaller centripetal force generates a smaller centripetal acceleration ← 1 mark which in turn requires a smaller orbital velocity.

7. A $650\ \text{kg}$ satellite in circular orbit around Earth has an orbital period of $1.5\times10^{4}\ \text{s}$.

a) What is the satellite's orbital radius? **(5 marks)**

$$F_G = F_C$$ ← **1 mark**

$$\frac{Gm M}{R^2} = m\frac{4\pi^2}{T^2}R$$ ← **2 marks**

$$R^3 = \frac{GMT^2}{4\pi^2} = \frac{6.67\times10^{-11}\left(5.98\times10^{24}\right)\left(1.5\times10^{4}\right)^2}{4\pi^2}$$

$$R = 1.3\times10^{7}\ \text{m}$$ ← **2 marks**

b) What is the gravitational potential energy of this satellite? **(2 marks)**

$$E_p = -\frac{GmM}{R}$$ ← **1 mark**

$$= \frac{-\left(6.67\times10^{-11}\right)(650)\left(5.98\times10^{24}\right)}{\left(1.3\times10^{7}\right)}$$

$$E_p = -2.0\times10^{10}\ \text{J}$$ ← **1 mark**

8. A 1 200 kg space probe is in a circular orbit around the Sun. The orbital radius is 7.0×10^9 m.

a) What is the orbital speed of this satellite? **(5 marks)**

$$F_{net} = ma \qquad \leftarrow \textbf{1 mark}$$

$$\frac{Gm_1m_2}{r^2} = \frac{m_1v^2}{r} \qquad \leftarrow \textbf{2 marks}$$

$$v^2 = \frac{Gm}{r}$$

$$v^2 = \frac{\left(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.98 \times 10^{30}\ \text{kg}\right)}{7.0 \times 10^9\ \text{m}} \qquad \leftarrow \textbf{1 mark}$$

$$v = 1.37 \times 10^5\ \text{m/s} \qquad \leftarrow \textbf{1 mark}$$

b) If the Sun collapsed to one-tenth its present radius without a change to its mass, the space probe's orbital radius will

- ☐ increase.
- ☐ decrease.
- ☑ stay the same.

(Check one response.) **(1 mark)**

c) Using principles of physics, explain your answer to b). **(3 marks)**

Since the mass of the sun has not changed and the distance between the two objects has not changed, then the force of gravity is still the same. The force of gravity is the net force; therefore the centripetal force must be the same.

9. What minimum energy is required to take a stationary 3.5×10^3 kg satellite from the surface of the Earth and put it into a circular orbit with a radius of 6.88×10^6 m and an orbital speed of 7.61×10^3 m/s? (Ignore Earth's rotation.) **(7 marks)**

$$E_{orbit} = \frac{1}{2}E_p$$

$$= \frac{1}{2}\left(-\frac{GmM}{R}\right)$$

$$= \frac{1}{2}\left(-\frac{6.67\times10^{-11}\left(3.5\times10^3\right)\left(5.98\times10^{24}\right)}{6.88\times10^6}\right)$$

$$= -1.01\times10^{11}\text{ J}$$ ← **4 marks**

$$E_{surface} = -\frac{GmM}{R}$$

$$= -\frac{6.67\times10^{-11}\left(3.5\times10^3\right)\left(5.98\times10^{24}\right)}{6.38\times10^6}$$

$$= -2.19\times10^{11}\text{ J}$$ ← **1 mark**

$$\Delta E = E_{orbit} - E_{surface}$$ ← **1 mark**

$$= \left(-1.01\times10^{11}\right) - \left(-2.19\times10^{11}\right)$$

$$= 1.17\times10^{11}\text{ J}$$

$$= 1.2\times10^{11}\text{ J}$$ ← **1 mark**

10. A 720 kg communication satellite is in synchronous orbit around the planet Mars. This synchronous orbit matches the period of rotation so that the satellite appears to be stationary over a position on the equator of Mars. What is the orbital radius of this satellite? **(7 marks)**

Planetary Data for Mars
Mass: 6.42×10^{23} kg
Period of rotation: 8.86×10^{4} s

$$F_{net} = ma_c$$

$$F = \frac{GmM}{R^2}$$ ← **1 mark**

$$F_g = \frac{m4\pi^2 R}{T^2}$$ ← **1 mark**

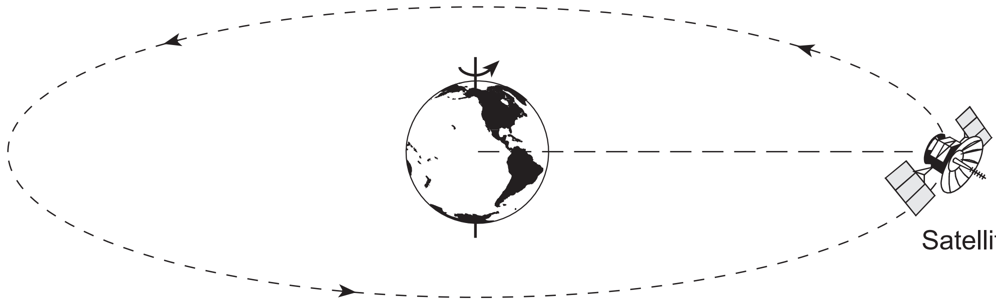
$$\frac{GmM}{R^2} = \frac{m4\pi^2 R}{T^2}$$ ← **2 marks**

$$R^3 = \frac{GMT^2}{4\pi^2}$$

$$R^3 = \frac{\left(6.67 \times 10^{-11}\right)\left(6.42 \times 10^{23}\right)\left(8.86 \times 10^{4}\right)^2}{4\pi^2}$$ ← **2 marks**

$$R = 2.0 \times 10^{7}\,\text{m}$$ ← **1 mark**

11. Geostationary satellites appear to remain stationary to an observer on Earth. Such satellites are placed in orbit far above the equator.



Using principles of physics, explain why such satellites all have the same orbital radius.

(4 marks)

The period of such satellites must be 24 hours to remain stationary over one point. ← 1 mark
The centripetal force is a gravitational force. ← 2 marks
For a period of 24 hours there is one orbital radius. ← 1 mark