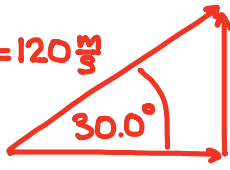


PROJECTILE MOTION (LAUNCHED AT ANGLES) - SOLUTIONS

1. a)


$$v_i = 120 \frac{\text{m}}{\text{s}}$$
$$v_{yi} = 120 \sin 30.0^\circ$$
$$= 60 \frac{\text{m}}{\text{s}}$$
$$v_x = 120 \cos 30.0^\circ$$
$$= 103.9230 \frac{\text{m}}{\text{s}}$$
$$\rightarrow 1.0 \times 10^2 \frac{\text{m}}{\text{s}}$$

b) VERTICAL

GIVEN:

$$v_i = 60. \frac{\text{m}}{\text{s}}$$

$$a = -9.8 \frac{\text{m}}{\text{s}^2}$$

$$d = 0$$

$$t = ?$$



$$d = v_i t + \frac{1}{2} a t^2$$

$$0 = v_i t + \frac{1}{2} a t^2$$

$$= t(v_i + \frac{1}{2} a t)$$

$$t = 0$$

$$v_i + \frac{1}{2} a t = 0$$

$$t = -\frac{2v_i}{a}$$

$$= -\frac{2(60.)}{-9.8}$$

$$= 12.2449 \text{ s}$$

$$12 \text{ s}$$

c) HORIZONTAL

GIVEN:

$$v = 103.9230 \frac{\text{m}}{\text{s}}$$

$$t = 12.2449 \text{ s}$$

$$d = ?$$

$$d = vt$$

$$= (103.9230)(12.2449)$$

$$= 1272.53 \text{ m}$$

$$\rightarrow 1300 \text{ m}$$

2. a) HORIZONTAL

GIVEN:

$$t = 4.3 \text{ s}$$

$$d = 55 \text{ m}$$

$$v_x = ?$$

$$v = \frac{d}{t}$$
$$= \frac{55}{4.3}$$
$$= 12.7907 \frac{\text{m}}{\text{s}}$$

VERTICAL

GIVEN:

$$t = 4.3 \text{ s}$$

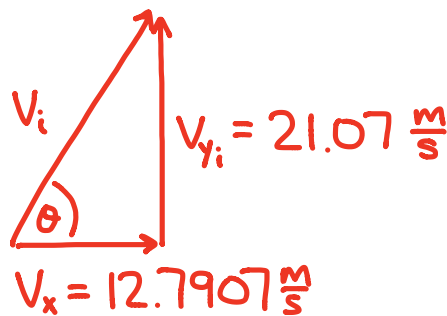
$$a = -9.8 \frac{\text{m}}{\text{s}^2}$$

$$d = 0$$

$$v_i = ?$$



$$d = v_i t + \frac{1}{2} a t^2$$
$$0 = v_i t + \frac{1}{2} a t^2$$
$$v_i = -\frac{1}{2} a t$$
$$= -\frac{1}{2} (-9.8) (4.3)$$
$$= 21.07 \frac{\text{m}}{\text{s}}$$



$$v_i^2 = (12.7907)^2 + (21.07)^2$$
$$v_i = \sqrt{(12.7907)^2 + (21.07)^2}$$
$$= 24.65 \frac{\text{m}}{\text{s}} \rightarrow 25 \frac{\text{m}}{\text{s}}$$

$$\tan \theta = \frac{21.07}{12.7907}$$
$$\theta = \tan^{-1} \left(\frac{21.07}{12.7907} \right)$$
$$= 59^\circ$$

$25 \frac{\text{m}}{\text{s}}$ 59° ABOVE THE HORIZONTAL

b) VERTICAL

GIVEN:

$$t = 4.3 \text{ s}$$

$$a = -9.8 \frac{\text{m}}{\text{s}^2}$$

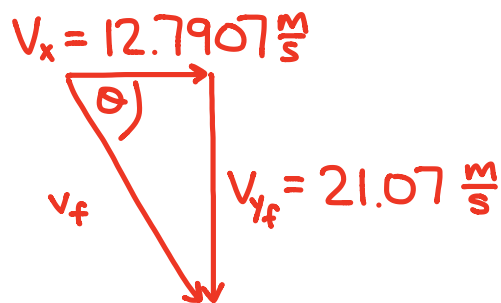
$$d = 0$$

$$v_i = 21.07 \frac{\text{m}}{\text{s}}$$

$$v_f = ?$$



$$\begin{aligned} v_f &= v_i + at \\ &= 21.07 + (-9.8)(4.3) \\ &= -21.07 \end{aligned}$$



$$\begin{aligned} v_i^2 &= (12.7907)^2 + (21.07)^2 \\ v_i &= \sqrt{(12.7907)^2 + (21.07)^2} \\ &= 24.65 \frac{\text{m}}{\text{s}} \rightarrow 25 \frac{\text{m}}{\text{s}} \end{aligned}$$

$$\begin{aligned} \tan \theta &= \frac{21.07}{12.7907} \\ \theta &= \tan^{-1}\left(\frac{21.07}{12.7907}\right) \\ &= 59^\circ \end{aligned}$$

$25 \frac{\text{m}}{\text{s}}$ 59° BELOW THE HORIZONTAL

CAN ALSO BE SOLVED BY SYMMETRY.

c) VERTICAL

$$a = -9.8 \frac{\text{m}}{\text{s}^2}$$

$$v_f = 0$$

$$v_i = 21.07 \frac{\text{m}}{\text{s}}$$

$$t = 2.15 \text{ s}$$

$$d = ?$$



$$v_f^2 = v_i^2 + 2ad$$

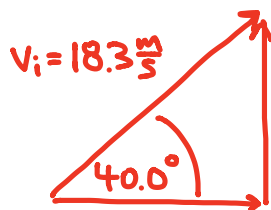
$$0 = v_i^2 + 2ad$$

$$d = -\frac{v_i^2}{2a}$$

$$= -\frac{21.07^2}{2(-9.8)}$$

$$= 23 \text{ m}$$

3. a)



$$v_i = 18.3 \frac{\text{m}}{\text{s}}$$

$$v_{yi} = 18.3 \sin 40.0^\circ$$
$$= 11.7630 \frac{\text{m}}{\text{s}}$$

$$v_x = 18.3 \cos 40.0^\circ$$

$$= 14.0186 \frac{\text{m}}{\text{s}}$$

VERTICAL

GIVEN:

$$v_i = 11.7630 \frac{\text{m}}{\text{s}}$$

$$a = -9.8 \frac{\text{m}}{\text{s}^2}$$

$$d = 0$$

$$t = ?$$



$$d = v_i t + \frac{1}{2} a t^2$$

$$0 = v_i t + \frac{1}{2} a t^2$$

$$= t(v_i + \frac{1}{2} a t)$$

$$t = 0$$

$$v_i + \frac{1}{2} a t = 0$$

$$t = -\frac{2v_i}{a}$$

$$= -\frac{2(11.7630)}{-9.8}$$

$$= 2.4006 \text{ s}$$

$$\rightarrow 2.4 \text{ s}$$

b) HORIZONTAL

GIVEN:

$$v = 14.0186 \frac{\text{m}}{\text{s}}$$

$$t = 2.4006 \text{ s}$$

$$d = ?$$

$$d = vt$$

$$= (14.0186)(2.4006)$$

$$= 33.6532 \text{ m}$$

$$\rightarrow 34 \text{ m}$$

c) VERTICAL

GIVEN:

$$v_i = 11.7630 \frac{\text{m}}{\text{s}}$$

$$a = -9.8 \frac{\text{m}}{\text{s}^2}$$

$$d = 0$$

$$t = 2.4006 \text{ s}$$

$$v_f = ?$$



$$v_f^2 = v_i^2 + 2ad$$

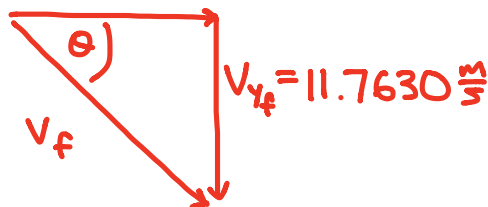
$$v_f^2 = v_i^2$$

$$v_f = \pm v_i$$

$$= \pm 11.7630 \frac{\text{m}}{\text{s}}$$

MOVING DOWNWARDS
WHEN IT HITS THE
GROUND.

$$v_x = 14.0186 \frac{\text{m}}{\text{s}}$$



$$v_f^2 = (14.0186)^2 + (11.7630)^2$$

$$v_f = \sqrt{(14.0186)^2 + (11.7630)^2}$$

$$= 18.3 \frac{\text{m}}{\text{s}}$$

$$\tan \theta = \frac{11.7630}{14.0186}$$

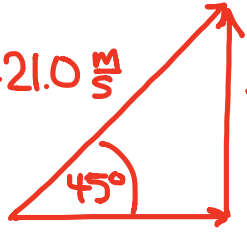
$$\theta = \tan^{-1} \left(\frac{11.7630}{14.0186} \right)$$

$$= 40.0^\circ$$

18.3 $\frac{\text{m}}{\text{s}}$ 40.0° BELOW THE HORIZONTAL

CAN ALSO BE SOLVED BY SYMMETRY.

4. a)



A right-angled triangle representing the decomposition of an initial velocity vector v_i into horizontal and vertical components. The hypotenuse is labeled $v_i = 21.0 \frac{m}{s}$. The angle between the hypotenuse and the horizontal base is 45° . The vertical side is labeled $v_{y_i} = 21.0 \sin 45^\circ = 14.8492 \frac{m}{s}$. The horizontal side is labeled $v_x = 21.0 \sin 45^\circ = 14.8492 \frac{m}{s}$.

$$v_i = 21.0 \frac{m}{s}$$
$$v_{y_i} = 21.0 \sin 45^\circ = 14.8492 \frac{m}{s}$$
$$v_x = 21.0 \sin 45^\circ = 14.8492 \frac{m}{s}$$

VERTICAL

GIVEN:


$$v_i = 14.8492 \frac{m}{s}$$

$$a = -9.8 \frac{m}{s^2}$$

$$d = 0$$

$$t = ?$$




$$d = v_i t + \frac{1}{2} a t^2$$

$$0 = v_i t + \frac{1}{2} a t^2$$

$$= t(v_i + \frac{1}{2} a t)$$

$$t = 0$$

$$v_i + \frac{1}{2} a t = 0$$

$$t = -\frac{2v_i}{a}$$

$$= -\frac{2(14.8492)}{-9.8}$$

$$= 3.0304 s$$

HORIZONTAL

GIVEN:

$$v = 14.8492 \frac{m}{s}$$

$$t = 3.0304 s$$

$$d = ?$$

$$d = vt$$

$$= (14.8492)(3.0304)$$

$$= 45 m$$

b) VERTICAL

GIVEN:

$$v_i = 14.8492 \frac{\text{m}}{\text{s}}$$

$$a = -9.8 \frac{\text{m}}{\text{s}^2}$$

$$d = 9.0 \text{ m}$$

$$t = ?$$



$$d = v_i t + \frac{1}{2} a t^2$$

$$0 = \frac{1}{2} a t^2 + v_i t - d$$

$$= \frac{1}{2} (-9.8) t^2 + 14.8492 t - 9.0$$

$$= -4.9 t^2 + 14.8492 t - 9.0$$

$$t = \frac{-14.8492 \pm \sqrt{14.8492^2 - 4(-4.9)(-9.0)}}{2(-4.9)}$$

$$= 0.8876 \text{ s} \quad \text{AND} \quad 2.1928 \text{ s}$$

ON THE WAY UP

ON THE WAY DOWN

c) HORIZONTAL

GIVEN:

$$v = 14.8492 \frac{\text{m}}{\text{s}}$$

$$t = 2.1928 \text{ s}$$

$$d = ?$$

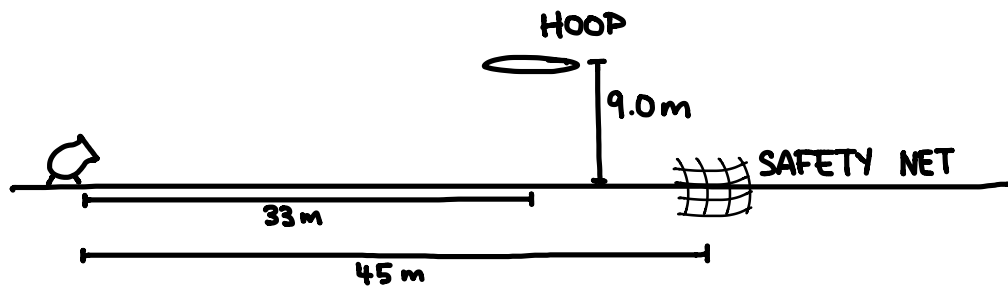
$$d = vt$$

$$= (14.8492)(2.1928)$$

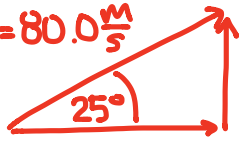
$$= 32.56 \text{ m}$$

$$\rightarrow 33 \text{ m}$$

d)



5. a)



A right-angled triangle representing the decomposition of an initial velocity vector v_i . The hypotenuse is labeled $v_i = 80.0 \frac{m}{s}$. The angle between the hypotenuse and the horizontal base is 25° . The vertical side represents the vertical component $v_{y,i}$.

$$v_{y,i} = 80.0 \sin 25^\circ = 33.8095 \frac{m}{s}$$
$$v_x = 80.0 \cos 25^\circ = 72.5046 \frac{m}{s}$$

VERTICAL

GIVEN:

$$v_i = 33.8095 \frac{m}{s}$$

$$a = -9.8 \frac{m}{s^2}$$

$$d = -60.0 m$$

$$t = ?$$

$$d = v_i t + \frac{1}{2} a t^2$$

$$0 = \frac{1}{2} a t^2 + v_i t - d$$

$$= \frac{1}{2} (-9.8) t^2 + 33.8095 t + 60.0$$

$$= -4.9 t^2 + 33.8095 t + 60.0$$

$$t = \frac{-33.8095 \pm \sqrt{33.8095^2 - 4(-4.9)(60.0)}}{2(-4.9)}$$

$$= -1.46 \cancel{+0} s \text{ AND } 8.3639 s$$

HORIZONTAL

GIVEN:

$$v = 72.5046 \frac{m}{s}$$

$$t = 8.3639 s$$

$$d = ?$$

$$d = vt$$

$$= (72.5046)(8.3639)$$

$$= 606.42 m$$

$$\rightarrow 610 m$$

b) VERTICAL

GIVEN:

$$v_i = 33.8095 \frac{\text{m}}{\text{s}}$$

$$a = -9.8 \frac{\text{m}}{\text{s}^2}$$

$$d = -60.0 \text{ m}$$

$$t = 8.3639 \text{ s}$$

$$v_f = ?$$



$$v_f^2 = v_i^2 + 2ad$$

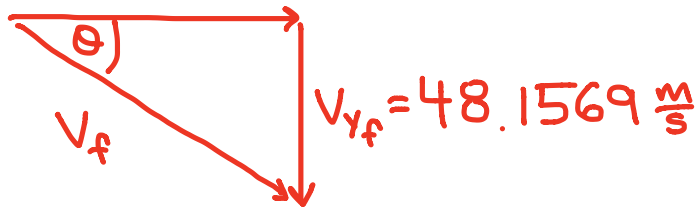
$$v_f = \pm \sqrt{v_i^2 + 2ad}$$

$$= \pm \sqrt{33.8095^2 + 2(-9.8)(-60.0)}$$

$$= \pm 48.1569 \frac{\text{m}}{\text{s}}$$

MOVING DOWNWARDS
WHEN IT HITS THE
GROUND.

$$v_x = 72.5046 \frac{\text{m}}{\text{s}}$$



$$v_f^2 = (72.5046)^2 + (48.1569)^2$$

$$v_f = \sqrt{(72.5046)^2 + (48.1569)^2}$$

$$= 87.0402 \frac{\text{m}}{\text{s}}$$

$$\tan \theta = \frac{48.1569}{72.5046}$$

$$\theta = \tan^{-1} \left(\frac{48.1569}{72.5046} \right)$$

$$= 33.59^\circ$$

87 $\frac{\text{m}}{\text{s}}$ 34° BELOW THE HORIZONTAL

c) VERTICAL

GIVEN:

$$v_i = 33.8095 \frac{\text{m}}{\text{s}}$$

$$a = -9.8 \frac{\text{m}}{\text{s}^2}$$

$$v_f = 0$$

$$d = ?$$



$$v_f^2 = v_i^2 + 2ad$$

$$0 = v_i^2 + 2ad$$

$$d = -\frac{v_i^2}{2a}$$

$$= -\frac{33.8095^2}{2(-9.8)}$$

$$= 58 \text{ m}$$

$$\text{HEIGHT ABOVE GROUND} = 60.0 + 58$$

$$= 118 \text{ m}$$

d) HORIZONTAL

GIVEN:

$$v = 72.5046 \frac{\text{m}}{\text{s}}$$

$$d = 76 \text{ m}$$

$$t = ?$$

$$d = vt$$

$$t = \frac{d}{v}$$

$$= \frac{76}{72.5046}$$

$$= 1.0482 \text{ s}$$

VERTICAL

GIVEN:

$$v_i = 33.8095 \frac{\text{m}}{\text{s}}$$

$$a = -9.8 \frac{\text{m}}{\text{s}^2}$$

$$t = 1.0482 \text{ s}$$

$$d = ?$$



$$d = v_i t + \frac{1}{2} a t^2$$

$$= (33.8095)(1.0482)$$

$$+ \frac{1}{2}(-9.8)(1.0482)^2$$

$$= 30. \text{ m}$$

$$\begin{aligned}\text{HEIGHT ABOVE GROUND} &= 60.0 + 30. \text{ m} \\ &= 90. \text{ m}\end{aligned}$$

$$\begin{aligned}\text{DISTANCE ABOVE TREE} &= 90. - 75 \\ &= 15 \text{ m}\end{aligned}$$