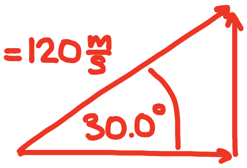
PROJECTILE MOTION (LAUNCHED AT ANGLES) - SOLUTIONS

1. a)



$v_{y_i} = 120 \sin 30.0°$

$= 60 \frac{m}{s}$

$v_x = 120 \cos 30.0°$

$= 103.9230 \frac{m}{s}$

$\rightarrow 1.0 \times 10^2 \frac{m}{s}$

b) VERTICAL

GIVEN:

$v_i = 60. \frac{m}{s}$

$a = -9.8 \frac{m}{s^2}$

$d = 0$

$t = ?$

$\uparrow +$

$\cancel{d}^{\,0} = v_i t + \frac{1}{2} a t^2$

$0 = v_i t + \frac{1}{2} a t^2$

$= t(v_i + \frac{1}{2} a t)$

$t = 0$

$v_i + \frac{1}{2} a t = 0$

$t = -\frac{2 v_i}{a}$

$= -\frac{2(60.)}{-9.8}$

$= 12.2449 \text{ s}$

12 s

c) HORIZONTAL

GIVEN:

$v = 103.9230 \frac{m}{s}$

$t = 12.2449 \text{ s}$

$d = ?$

$d = vt$

$= (103.9230)(12.2449)$

$= 1272.53 \text{ m}$

$\rightarrow 1300 \text{ m}$

2. a) HORIZONTAL

GIVEN:

$t = 4.3\,s$

$d = 55\,m$

$V_x = ?$

$V = \frac{d}{t}$

$= \frac{55}{4.3}$

$= 12.7907\,\frac{m}{s}$

VERTICAL

GIVEN:

$t = 4.3\,s$

$a = -9.8\,\frac{m}{s^2}$

$d = 0$

$V_i = ?$

$\uparrow +$

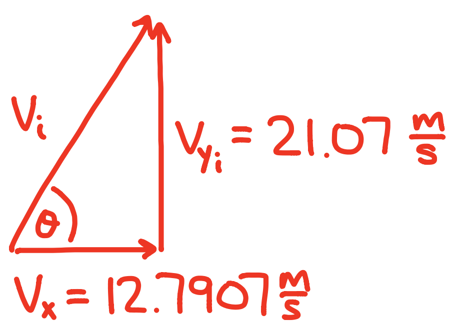
$\cancel{d}^{\,0} = V_i t + \frac{1}{2} a t^2$

$0 = V_i t + \frac{1}{2} a t^2$

$V_i = -\frac{1}{2} a t$

$= -\frac{1}{2}(-9.8)(4.3)$

$= 21.07\,\frac{m}{s}$



$V_i^2 = (12.7907)^2 + (21.07)^2$

$V_i = \sqrt{(12.7907)^2 + (21.07)^2}$

$= 24.65\,\frac{m}{s} \longrightarrow 25\,\frac{m}{s}$

$\tan\theta = \frac{21.07}{12.7907}$

$\theta = \tan^{-1}\left(\frac{21.07}{12.7907}\right)$

$= 59^\circ$

$25\,\frac{m}{s}$ 59° ABOVE THE HORIZONTAL

b) VERTICAL

GIVEN:

$t = 4.3\,s$

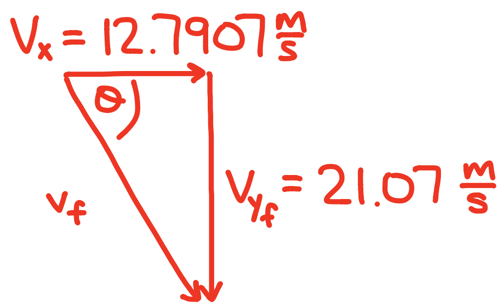
$a = -9.8\,\frac{m}{s^2}$

$d = 0$

$V_i = 21.07\,\frac{m}{s}$

$V_f = ?$

$$V_f = V_i + at$$
$$= 21.07 + (-9.8)(4.3)$$
$$= -21.07$$



$$V_i^2 = (12.7907)^2 + (21.07)^2$$
$$V_i = \sqrt{(12.7907)^2 + (21.07)^2}$$
$$= 24.65\,\frac{m}{s} \longrightarrow 25\,\frac{m}{s}$$

$$\tan\theta = \frac{21.07}{12.7907}$$
$$\theta = \tan^{-1}\left(\frac{21.07}{12.7907}\right)$$
$$= 59^\circ$$

$25\,\frac{m}{s}$ 59° BELOW THE HORIZONTAL

CAN ALSO BE SOLVED BY SYMMETRY.

c) VERTICAL

$a = -9.8\ \frac{m}{s^2}$

$v_f = 0$

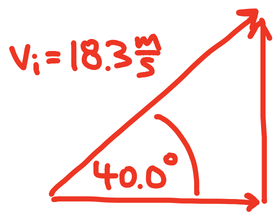
$v_i = 21.07\ \frac{m}{s}$

$t = 2.15\,s$

$d = ?$

$$\cancelto{0}{v_f^2} = v_i^2 + 2ad$$
$$0 = v_i^2 + 2ad$$
$$d = -\frac{v_i^2}{2a}$$
$$= -\frac{21.07^2}{2(-9.8)}$$
$$= \boxed{23\ m}$$

3. a)



$v_{y_i} = 18.3 \sin 40.0°$
$= 11.7630\ \frac{m}{s}$

$v_x = 18.3 \cos 40.0°$
$= 14.0186\ \frac{m}{s}$

VERTICAL

GIVEN:

$v_i = 11.7630\ \frac{m}{s}$

$a = -9.8\ \frac{m}{s}$

$d = 0$

$t = ?$

$$\cancelto{0}{d} = v_i t + \tfrac{1}{2}at^2$$
$$0 = v_i t + \tfrac{1}{2}at^2$$
$$= t\left(v_i + \tfrac{1}{2}at\right)$$

$t = 0$

$$v_i + \tfrac{1}{2}at = 0$$
$$t = -\frac{2v_i}{a}$$
$$= -\frac{2(11.7630)}{-9.8}$$
$$= 2.4006\,s$$
$$\rightarrow \boxed{2.4\,s}$$

b) HORIZONTAL

GIVEN:

$v = 14.0186\ \frac{m}{s}$

$t = 2.4006\ s$

$d = ?$

$d = vt$

$= (14.0186)(2.4006)$

$= 33.6532\ m$

$\rightarrow \boxed{34\ m}$

c) VERTICAL

GIVEN:

$V_i = 11.7630\ \frac{m}{s}$

$a = -9.8\ \frac{m}{s}$

$d = 0$

$t = 2.4006 s$

$V_f = ?$

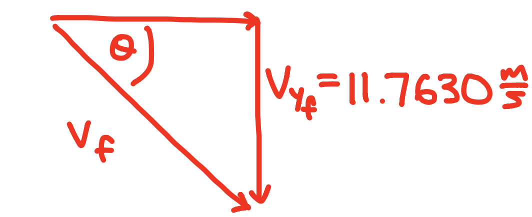
$V_f^2 = V_i^2 + \cancel{2ad}$ d=0

$V_f^2 = V_i^2$

$V_f = \pm V_i$

$= \pm 11.7630\ \frac{m}{s}$

MOVING DOWNWARDS WHEN IT HITS THE GROUND.



$V_f^2 = (14.0186)^2 + (11.7630)^2$

$V_f = \sqrt{(14.0186)^2 + (11.7630)^2}$

$= 18.3\ \frac{m}{s}$

$\tan\theta = \frac{11.7630}{14.0186}$

$\theta = \tan^{-1}\left(\frac{11.7630}{14.0186}\right)$

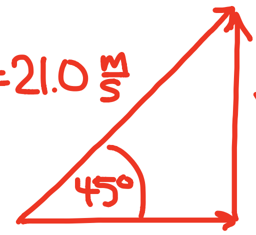
$= 40.0°$

18.3 $\frac{m}{s}$ 40.0° BELOW THE HORIZONTAL

CAN ALSO BE SOLVED BY SYMMETRY.

4. a)

$v_i = 21.0\,\frac{m}{s}$



$v_{y_i} = 21.0 \sin 45°$
$= 14.8492\,\frac{m}{s}$

$v_x = 21.0 \sin 45°$
$= 14.8492\,\frac{m}{s}$

VERTICAL

GIVEN:

$v_i = 14.8492\,\frac{m}{s}$
$a = -9.8\,\frac{m}{s^2}$
$d = 0$
$t = ?$

$\uparrow +$

$\cancel{d}^{\,0} = v_i t + \frac{1}{2}at^2$

$0 = v_i t + \frac{1}{2}at^2$

$= t(v_i + \frac{1}{2}at)$

$t = 0$

$v_i + \frac{1}{2}at = 0$

$t = -\frac{2v_i}{a}$

$= -\frac{2(14.8492)}{-9.8}$

$= 3.0304\,s$

HORIZONTAL

GIVEN:

$v = 14.8492\,\frac{m}{s}$
$t = 3.0304\,s$
$d = ?$

$d = vt$

$= (14.8492)(3.0304)$

$= 45\,m$

b) VERTICAL

GIVEN:

$v_i = 14.8492\ \frac{m}{s}$

$a = -9.8\ \frac{m}{s^2}$

$d = 9.0\ m$

$t = ?$

+

$$d = v_i t + \tfrac{1}{2}at^2$$

$$0 = \tfrac{1}{2}at^2 + v_i t - d$$

$$= \tfrac{1}{2}(-9.8)t^2 + 14.8492t - 9.0$$

$$= -4.9t^2 + 14.8492t - 9.0$$

$$t = \frac{-14.8492 \pm \sqrt{14.8492^2 - 4(-4.9)(-9.0)}}{2(-4.9)}$$

$$= 0.8\cancel{37}6\ s \quad \text{AND} \quad 2.1928\ s$$

ON THE WAY UP   ON THE WAY DOWN

c) HORIZONTAL

GIVEN:

$v = 14.8492\ \frac{m}{s}$

$t = 2.1928\ s$

$d = ?$

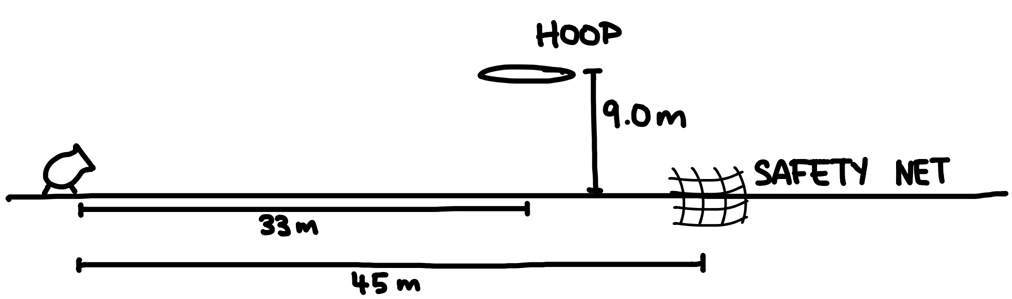
$$d = vt$$

$$= (14.8492)(2.1928)$$

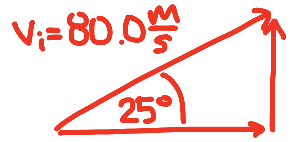
$$= 32.56\ m$$

$\rightarrow$ 33 m

d)



5. a)



$V_{y_i} = 80.0 \sin 25°$
$= 33.8095\ \frac{m}{s}$

$V_x = 80.0 \cos 25°$
$= 72.5046\ \frac{m}{s}$

VERTICAL

GIVEN:

$v_i = 33.8095\ \frac{m}{s}$
$a = -9.8\ \frac{m}{s^2}$
$d = -60.0\ m$
$t = ?$

+

$$d = v_i t + \frac{1}{2} a t^2$$
$$0 = \frac{1}{2} a t^2 + v_i t - d$$
$$= \frac{1}{2}(-9.8)t^2 + 33.8095t + 60.0$$
$$= -4.9t^2 + 33.8095t + 60.0$$

$$t = \frac{-33.8095 \pm \sqrt{33.8095^2 - 4(-4.9)(60.0)}}{2(-4.9)}$$

$$= \cancel{-1.4640}\ s \quad \text{AND} \quad 8.3639\ s$$

HORIZONTAL

GIVEN:

$v = 72.5046\ \frac{m}{s}$
$t = 8.3639\ s$
$d = ?$

$$d = vt$$
$$= (72.5046)(8.3639)$$
$$= 606.42\ m$$
$$\rightarrow \boxed{610\ m}$$

b) VERTICAL

GIVEN:

$v_i = 33.8095\ \frac{m}{s}$

$a = -9.8\ \frac{m}{s^2}$

$d = -60.0\ m$

$t = 8.3639\ s$

$v_f = ?$

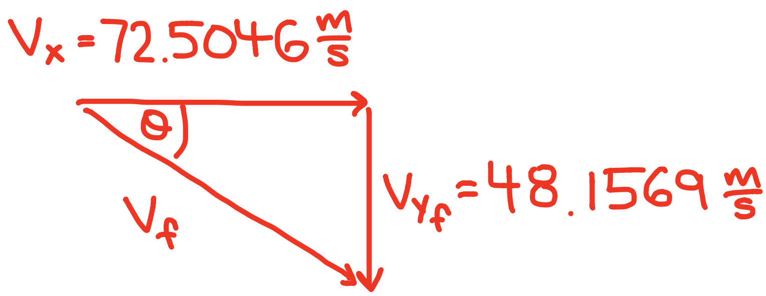
$$v_f^2 = v_i^2 + 2ad$$

$$v_f = \pm\sqrt{v_i^2 + 2ad}$$

$$= \pm\sqrt{33.8095^2 + 2(-9.8)(-60.0)}$$

$$= \pm 48.1569\ \frac{m}{s}$$

MOVING DOWNWARDS WHEN IT HITS THE GROUND.



$$v_f^2 = (72.5046)^2 + (48.1569)^2$$

$$v_f = \sqrt{(72.5046)^2 + (48.1569)^2}$$

$$= 87.0402\ \frac{m}{s}$$

$$\tan\theta = \frac{48.1569}{72.5046}$$

$$\theta = \tan^{-1}\left(\frac{48.1569}{72.5046}\right)$$

$$= 33.59^\circ$$

$87\ \frac{m}{s}$ 34° BELOW THE HORIZONTAL

c) VERTICAL

GIVEN:

$v_i = 33.8095 \frac{m}{s}$

$a = -9.8 \frac{m}{s^2}$

$v_f = 0$

$d = ?$

$$\cancel{v_f}^{2} = v_i^2 + 2ad$$
$$0 = v_i^2 + 2ad$$
$$d = -\frac{v_i^2}{2a}$$
$$= -\frac{33.8095^2}{2(-9.8)}$$
$$= 58 \text{ m}$$

HEIGHT ABOVE GROUND = 60.0 + 58

= 118 m

d) HORIZONTAL

GIVEN:

$V = 72.5046 \frac{m}{s}$

$d = 76$ m

$t = ?$

$$d = vt$$
$$t = \frac{d}{v}$$
$$= \frac{76}{72.5046}$$
$$= 1.0482 \text{ s}$$

VERTICAL

GIVEN:

$v_i = 33.8095 \frac{m}{s}$

$a = -9.8 \frac{m}{s^2}$

$t = 1.0482$ s

$d = ?$

$$d = v_i t + \tfrac{1}{2}at^2$$
$$= (33.8095)(1.0482) + \tfrac{1}{2}(-9.8)(1.0482)^2$$
$$= 30. \text{ m}$$

HEIGHT ABOVE GROUND $= 60.0 + 30.\ m$
$= 90.\ m$

DISTANCE ABOVE TREE $= 90. - 75$
$= 15\ m$